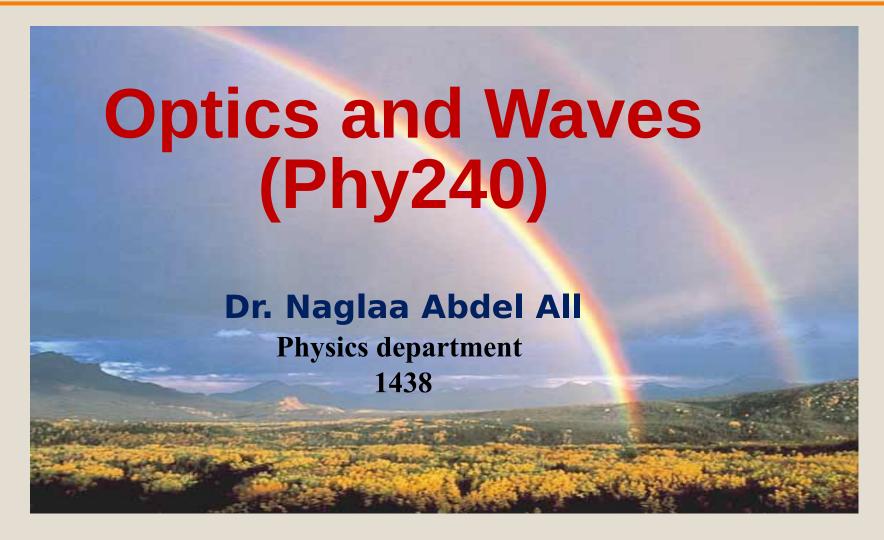
KINGDOM OF SAUDI ARABIA AL-IMAM MUHAMMAD BIN SAUD ISLAMIC UNIVERSITY FACULTY OF SCIENCES DEPARTMENT OF PHYSICS



المملكة العربية السعودية جامعة الإمام محمد بن سعود الإسلامية كلية العلوم قسم الفيزياء



The Course Contents



قانون البصريات الهندسية و تكوين الصور Chapter 1: The laws of geometric optics and image formation

تداخل موجات الصوت **Chapter 2: Interference of light waves**

أناط التشويش و الاستقطاب

Chapter 3: Diffraction Patterns and Polarization

Chapter 4: Wave Motion

Chapter 5: Sound Waves

Chapter 6: Superposition and standing waves

Book: Physics for Scientists and Engineers (with modern physics) –by Raymond A. Serway, and John W. Jewett – Brooks Cole – 6th Edition



Chapter 1

The laws of geometric optics and image formation

3

:Chapter Outline

- Reflection blussel
- Refraction الانكسار
- Dispersion and Prism
- Total internal reflection
- > Images formed by flat mirror
- Images formed by spherical mirrors
- > Images formed by refraction,
- > Thin lenses

In optics, geometric optics deals with the study of light behavior based on the principles of geometry. Here's a brief explanation:

.في البصريات، تتعامل البصريات الهندسية مع دراسة سلوك الضوء استنادًا إلى مبادئ الهندسة

Law of Reflection:

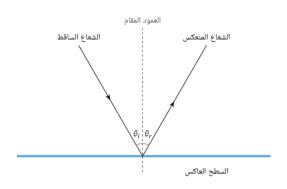
:قانون الانعكاس

States that the angle of incidence is equal to the angle of reflection.

ينص على أن زاوية السقوط تكون مساوية لزاوية الانعكاس

When light reflects off a surface, the incident ray, reflected ray, and normal to the surface all lie in the same plane.

عندما ينعكس الضوء عن سطح، تكون الشعاع الساقط والشعاع المنعكس والعمود إلى السطح في نفس السطح.



Law of Refraction Snill's law:

:قانون الانكسار

Describes how light bends as it passes from one medium to another.

يصف كيفية انحناء الضوء عندما يمر من وسط إلى آخر

Snell's Law quantifies this behavior, stating that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a particular pair of media.

قانون سنيل يحدد هذا السلوك، حيث يكون نسبة جيب تمام الزاوية الساقطة إلى جيب تمام الزاوية المنعكسة ثابته لزوج معين من الوسائط.

Image Formation:

:تكوين الصور

Describes how images are formed by lenses and mirrors.

In a concave mirror or converging lens, real and inverted images can be formed. In a convex mirror or diverging lens, virtual and upright images are formed.

في المرآة القاعسة أو العدسة المقعرة، يمكن تكوين صور حقيقية ومقلوبة. في المرآة المحدبة أو العدسة المتشعبة، تكون الصور افتراضية.

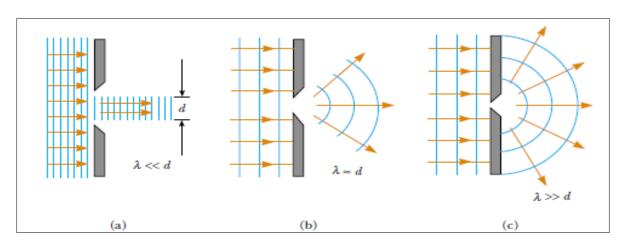
These laws and principles are fundamental in understanding how light interacts with various optical elements, leading to the formation of images.

هذه القوانين والمبادئ أساسية لفهم كيفية تفاعل الضوء مع مختلف العناصر البصرية، مما يؤدي إلى تكوين الصور

The field of geometric optics:

نتحدث هنا عن مجال البصريات

involves the study of the propagation of light, with the assumption that light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are non-uniform in either space or time.



يتضمن دراسة انتشار الضوء، مع افتراض أن الضوء يسير في اتجاه ثابت على خط مستقيم أثناء مروره عبر وسط متجانس ويغير اتجاهه عندما يلتقي بسطح وسط مختلف أو إذا كانت الخصائص البصرية للوسط غير متجانسة إما في الفضاء أو في الزمن.

A plane wave of wavelength λ is incident on a barrier in which there is an opening of lambda (λ)

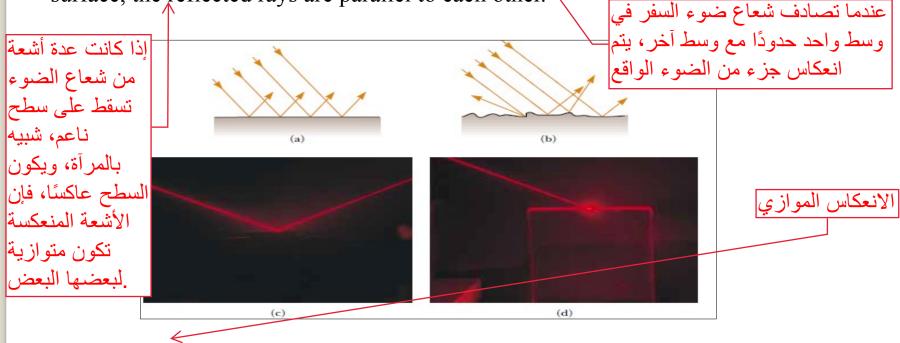
diameter d.

- (a) When $(\lambda << d)$, the rays continue in a straight-line path.
 - اما اذا كانت طول الموجة مساوي تقريبا من القطر, تنتشر الموجات بمجرد مرورها من الفتحة
- (b) When $(\lambda \approx d)$, the rays spread out after passing through the opening.
- في حال كانت طول الموجة أكبر من القطر تتصرف الفتحة كمصدر نقطي يطلق موجات كروية,
- (c) When $(\lambda >> d)$, the opening behaves as a point source emitting spherical waves.

Reflection:

When a light ray traveling in one medium encounters a boundary with another medium, part of the incident light is reflected.

• If the several rays of a beam of light incident on a smooth, mirror-like, reflecting surface, the reflected rays are parallel to each other.



(a) specular reflection, where the reflected rays are all parallel to each other,

(b) <u>diffuse reflection</u>, where the reflected rays travel in random directions.

الانعكاس المنتشر

(c) and (d) Photographs of specular and diffuse reflection using laser light.

- Reflection of light from such a smooth surface is called **specular reflection**.
- If the reflecting surface is rough, the surface reflects the rays not as a parallel set but in various directions. Reflection from any rough surface is known as diffuse reflection.

• The Law of reflection

Consider a light ray traveling in air and incident at an angle on a flat, smooth Surface. Specular Reflection

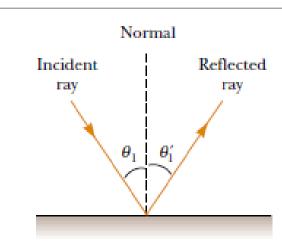
 \checkmark The incident and reflected rays make angles θ_1 and θ_1 , where the angles are measured between the normal and the rays.

(The normal is a line drawn perpendicular to the surface at the point where the incident ray strikes the surface.)

Experiments and theory show that:

The angle of reflection equals the angle of incidence

$$\theta_1'=\theta_1$$



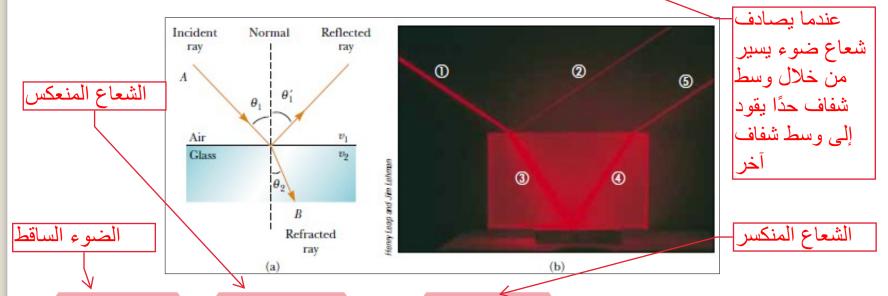


Refraction:

الانكسار

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium:

part of the energy is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be refracted.



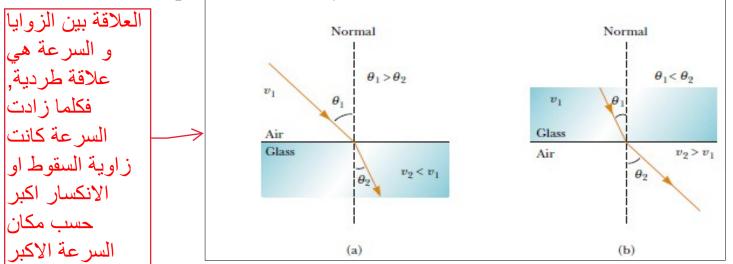
The incident ray, the reflected ray, and the refracted ray all lie in the same plane. The angle of refraction, θ_2 , depends on the properties of the two media and on the angle of incidence through the relationship $\sin \theta_2$

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant}$$

(a) When the light beam moves from air (speed is high) into glass (speed is lower) $(v_1>v_2)$, the light slows down on entering the glass and its path is bent toward the normal.

(b) When the beam moves from glass into air $(v_1 < v_2)$, the light speeds up on entering

the air and its path is bent away from the normal.



• When light travels in air, its speed is 3.00 ×10⁸ m/s but speed is reduced to approximately 2 ×10⁸ m/s when the light enters a block of glass. When the light re-emerges into air, its speed instantaneously increases to its original value of 3×10⁸ m/s.

معامل الانكسار و يكون مختلف من مادة الى اخرى

Index of Refraction:

تكون سرعة الضوء اكبر ما يمكن في الفراغ او بالهواء لذلك نعرف معامل الانكسار انه النسبة بين سرعة الضوء بالهواء الى سرعة الضوء بالوسط الثاني

,the speed of light in any material is *less* than its speed in vacuum. In fact light travels at its maximum speed in vacuum. It is convenient to define the index of refraction *n* of a medium to be the ratio

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

The index of refraction is a dimensionless number greater than unity because v is always less than c. Furthermore, n is equal to unity for vacuum معامل الانكسار يكون اكبر من 1

 $v_{\rm I}$ $1\lambda_2$ و معامل الانكسار للهواء يساوي 1

✓ As light travels from one medium to another, its frequency does not change but its wavelength does.

$$v_1 = f\lambda_1$$
 and $v_2 = f\lambda_2$
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$
 وسط لاخر, التردد لا يتغير glaa Abdel All, PHY 240

glaa Abdel All, PHY 240

If medium 1 is vacuum, or for all practical purposes air, then $n_1=1$. Hence, it then the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda}{\lambda_n}$$

where λ is the wavelength of light in vacuum and λ_n is the wavelength of light in the medium whose index of refraction is n.

we see that because n > 1, $\lambda_n < \lambda$

Snell's law of refraction

$$\frac{\sin\,\theta_2}{\sin\,\theta_1} = \frac{v_2}{v_1}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		Liquids at 20°C	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF ₂)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO ₂)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H ₂ O)	1.309	Gases at 0°C, 1 atm	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.00045

Example 1:

A beam of light of wavelength 550 nm traveling in air is incident on a slab of transparent material. The incident beam makes an angle of 40.0° with the normal, and the refracted beam makes an angle of 26.0° with the normal. Find the index of refraction of the material.

:Solution

Using Snell's law of refraction with these data, and taking $n_1 = 1$ for air, we have

$$n_{1} \sin \theta_{1} = n_{2} \sin \theta_{2}$$

$$n_{2} = \frac{n_{1} \sin \theta_{1}}{\sin \theta_{2}} = (1.00) \frac{\sin 40.0^{\circ}}{\sin 26.0^{\circ}}$$

$$= \frac{0.643}{0.438} = 1.47$$

Transparent material

Example 35.4 Angle of Refraction for Glass

A light ray of wavelength 589 nm traveling through air is incident on a smooth, flat slab of crown glass at an angle of 30.0° to the normal, as sketched in Figure 35.15. Find the angle of refraction.

Solution We rearrange Snell's law of refraction to obtain

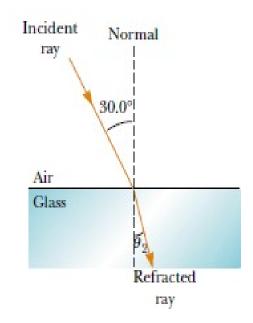
$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

From Table 35.1, we find that $n_1 = 1.00$ for air and $n_2 = 1.52$ for crown glass. Therefore,

$$\sin \theta_2 = \left(\frac{1.00}{1.52}\right) \sin 30.0^\circ = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = 19.2^\circ$$

Because this is less than the incident angle of 30°, the refracted ray is bent toward the normal, as expected. Its



Example 35.5 Laser Light in a Compact Disc

A laser in a compact disc player generates light that has a wavelength of 780 nm in air.

(A) Find the speed of this light once it enters the plastic of a compact disc (n = 1.55).

Solution We expect to find a value less than 3.00×10^8 m/s because n > 1. We can obtain the speed of light in the plastic by using Equation 35.4:

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \,\text{m/s}}{1.55}$$
 $v = 1.94 \times 10^8 \,\text{m/s}$

(B) What is the wavelength of this light in the plastic?

Solution We use Equation 35.7 to calculate the wavelength in plastic, noting that we are given the wavelength in air to be $\lambda = 780$ nm:

$$\lambda_n = \frac{\lambda}{n} = \frac{780 \text{ nm}}{1.55} = 503 \text{ nm}$$

Dispersion and Prism:

تشتت الضوء والمنشور

التشتت يعني تغير معامل الانكسار مع اختلاف الطول الموجي للضوء عند عبوره خلال مادة معينة

Dispersion means the index n varies with the wavelength of the light

passing through the material.4

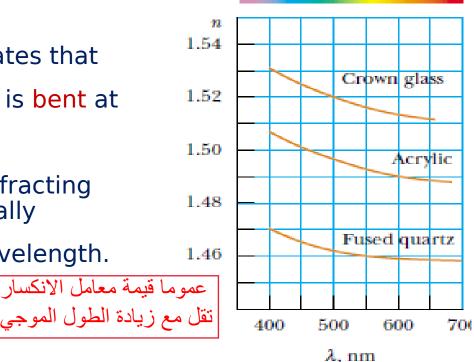
- Snell's law of refraction indicates that
 - light of different wavelengths is bent at

different

angles when incident on a refracting

 The index of refraction generally material.

decreases with increasing wavelength.



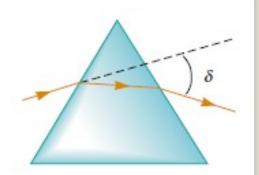
violet light bends more than red light does when passing

violet light bends more than red light does when passing into a refracting material.

 To understand the effects that dispersion can have on light, consider

?what happens when light strikes a prism

 A ray of single-wavelength light incident on the prism from the left emerges refracted from its original direction of travel by an angle 3, called the angle of deviation.



a beam of **white light** (a combination of all visible wavelengths) is incident on a prism. The rays that emerge spread out in **a series of** .colors known as the **visible spectrum**

✓ These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet. Clearly, the angle of deviation δ depends on wavelength.

Newton showed that each color has a particular angle of deviation and that the colors can be recombined to form the

The dispersion of light into a spectrum is demonstrated most vividly in nature by the formation of a rainbow.

- White light enters a glass prism at the upper left.
- A reflected beam of light comes out of the prism just below the incoming beam.
- The beam moving toward the lower right shows distinct colors.
- Different colors are refracted at different angles because the index of refraction of the glass depends on wavelength.
- Violet light deviates the most; red



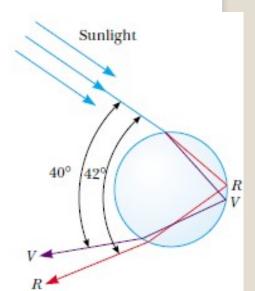
how a rainbow is formed?

A ray of sunlight (which is white light) passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows

- Sunlight at first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least.
- At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air.
- The rays leave the drop such that the angle between the incident white light and the most intense returning violet ray is 40° and the angle between the

white light and the most intense returning red ray is **This small angular difference between the returning rays causes** 42°.

us to see a colored bow.



Example 35.7 Measuring n Using a Prism

Although we do not prove it here, the minimum angle of deviation δ_{min} for a prism occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces, as shown in Figure 35.25. Obtain an expression for the index of refraction of the prism material.

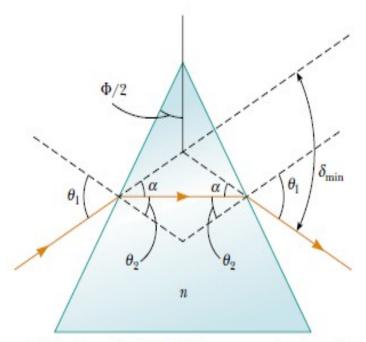


Figure 35.25 (Example 35.7) A light ray passing through a prism at the minimum angle of deviation δ_{min} .

Solution Using the geometry shown in Figure 35.25, we find that $\theta_2 = \Phi/2$, where Φ is the apex angle and

$$\theta_1 = \theta_2 + \alpha = \frac{\Phi}{2} + \frac{\delta_{\min}}{2} = \frac{\Phi + \delta_{\min}}{2}$$

From Snell's law of refraction, with $n_1 = 1$ because medium 1 is air, we have

$$\sin \theta_1 = n \sin \theta_2$$

$$\sin \left(\frac{\Phi + \delta_{\min}}{2}\right) = n \sin(\Phi/2)$$

$$n = \frac{\sin \left(\frac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)}$$
(35.9)

Hence, knowing the apex angle Φ of the prism and measuring δ_{\min} , we can calculate the index of refraction of the prism material. Furthermore, we can use a hollow prism to determine the values of n for various liquids filling the prism.

Total internal reflection:

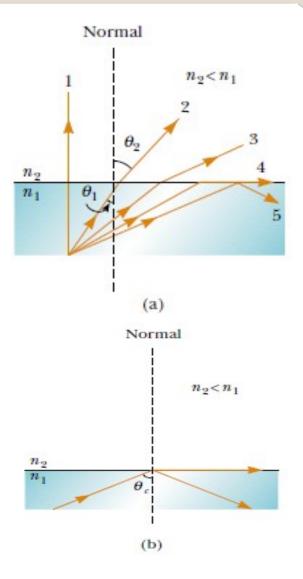
الانعكاس الكلى الداخلي

(a) Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2 , where $n_2 < n_1$.

As the angle of incidence θ_1 increases, the angle of refraction θ_2 increases until θ_2 is 90° (ray 4).

For even larger angles of incidence, total internal reflection occurs (ray 5).

(b) The angle of incidence producing an angle of refraction equal to 90° is the critical angle θ_c .



For angles of incidence **greater than** θ_{ϵ} , the beam is entirely reflected at the boundary, (ray 5)

<u>Critical angle for total internal</u>

reflection We can use Snell's law of refraction to find the critical angle. When θ_1

$$= \theta_c$$
, $\theta_2 = 90^\circ$

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

$$\sin\,\theta_c = \frac{n_2}{n_1} \qquad (\text{for } n_1 > n_2)$$

يظهر الانعكاس الكلي الداخلي اذا انتقل الضوء من وسط معامل انكساره اكبر من معامل انكسار

- This equation can be used only when n_1 is greater than n_2
- Total internal reflection occurs only when light is directed from a medium of a given index of refraction toward a medium of lower index of refraction.

 $\sin\theta_c > 1$; this is a meaningless result because the sine of an angle .can never be greater than unity

Example 35.8 A View from the Fish's Eye

Find the critical angle for an air-water boundary. (The index of refraction of water is 1.33.)

Solution We can use Figure 35.26 to solve this problem, with the air above the water having index of refraction n_2 and the water having index of refraction n_1 . Applying

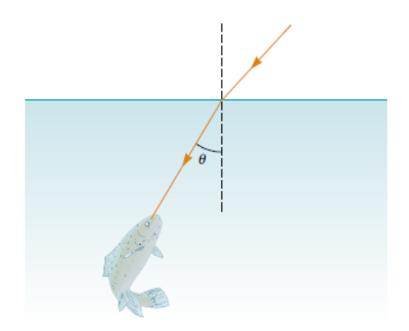


Figure 35.28 (Example 35.8) What If? A fish looks upward toward the water surface.

Equation 35.10, we find that

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1}{1.33} = 0.752$$

$$\theta_c = 48.8^{\circ}$$

What If? What if a fish in a still pond looks upward toward the water's surface at different angles relative to the surface, as in Figure 35.28? What does it see?

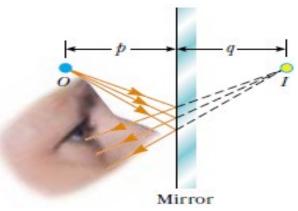
Answer Because the path of a light ray is reversible, light traveling from medium 2 into medium 1 in Figure 35.26a follows the paths shown, but in the *opposite* direction. A fish looking upward toward the water surface, as in Figure 35.28, can see out of the water if it looks toward the surface at an angle less than the critical angle. Thus, for example, when the fish's line of vision makes an angle of 40° with the normal to the surface, light from above the water reaches the fish's eye. At 48.8°, the critical angle for water, the light has to skim along the water's surface before being refracted to the fish's eye; at this angle, the fish can in principle see the whole shore of the pond. At angles greater than the critical angle, the light reaching the fish comes by means of internal reflection at the surface. Thus, at 60°, the fish sees a reflection of the bottom of the pond.

5. Images formed by flat mirror:

We considering the simplest possible mirror, the flat mirror.

لصور التي تتكون عند الستخدام مرآة مستوية

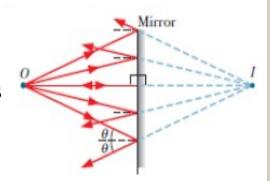
- a point source of light placed at O.
- a distance **p** in front of a flat mirror which is called the **object distance**.
- Light rays leave the source and are reflected from the mirror.



- An image formed by reflection from a flat mirror.
- The image point I is located behind the mirror a perpendicular distance q from the mirror (the image distance).
- The image distance **q** has the same magnitude as the object distance **p**.

Images are classified as real or virtual:

- A real image is formed when light rays pass through and diverge from the image point.
- virtual image is formed when the light rays do not pass through the image point but only appear to diverge from that point.



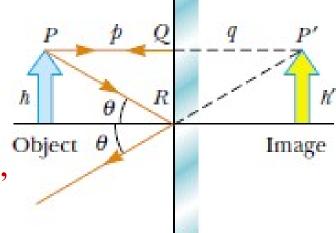
The image of an object seen in a flat mirror is always virtual.

Real images can be displayed on a screen (as at a movie).

A geometric construction that is used to examine the properties of the

images of extended objects formed by flat mirrors.

The image distance q behind the mirror is equal to the object distance p in front of a flat mirror.
 |p| = |q|



The object height h equals the image height h'

$$h = h'$$

• Lateral Magnification of an image as follows:

$$M \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h}$$

For a flat mirror, M = 1 for any image because h' = h.

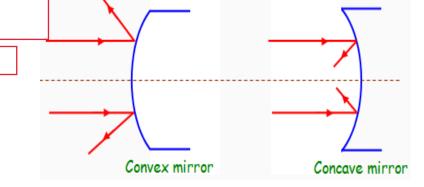
- We conclude that the image that is formed by a flat mirror has the following Properties:
- The image is as far behind the mirror as the object is in front.
- The image is unmagnified, virtual, and upright
- The image has front-back reversal.

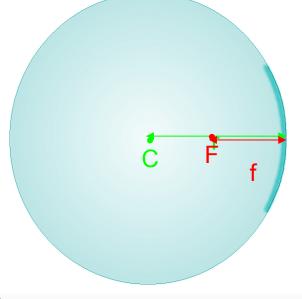
6. Images formed by spherical mirror

A spherical mirror has the shape of a section of a sphere. This type of mirror focuses incoming parallel rays.

Concave Mirrors

Convex Mirrors





C: the center point of the sphere

r: radius of curvature (the radius of the sphere)

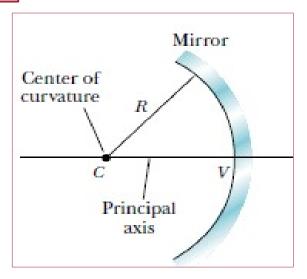
F: the focal point of the mirror (halfway between C and the sphere)

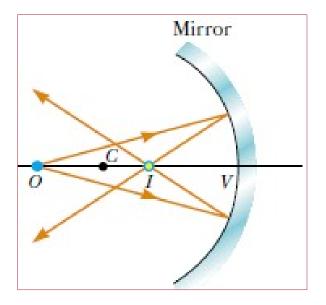
f: the focal distance, f = r/2

Concave Mirrors

light is reflected from the inner (concave surface).

The mirror has a radius of curvature R, and its center of curvature is point C. Point V is the center of the spherical section, and a line through C and V is called the principal axis of the mirror.



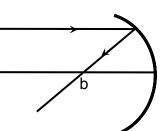


- A point object placed at point *O* in front of a concave spherical mirror,
 - where O is point on the principal axis farther than R from the mirror surface.
- Reflected rays forms a real image at *I*.

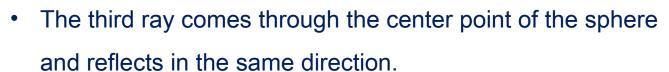
Ray Diagrams for Concave Mirrors

انواع الاشعه مهمة جدا خصوصا

The ray comes in parallel to the optical axis and reflects through the focal point.



The second ray comes through the focal point and reflects parallel to the optical axis.



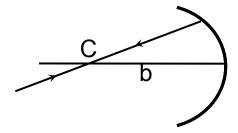
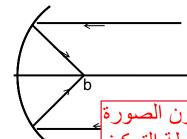
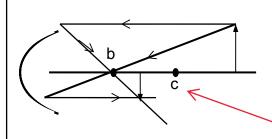


Image formed by Concave Mirrors



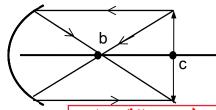
When the object is located at infinite, the image is real, inverted, and very very small at focal point.

عندما يكون الجسم موجودًا في اللانهائية، تكون الصورة حقيقية، مقلوبة، وصغيرة جدًا جدًا عند نقطة التركيز



When the object is located at distance larger than twice of the focal point, the image is real, inverted, and small.

عندما يكون الجسم على مسافة أكبر من ضعف نقطة التركيز، تكون الصورة حقيقية، مقلوبة، وصغيرة



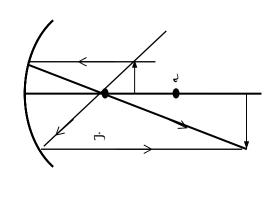
When the object is located at distance equal twice of the focal point, the image is real,

inverted, and equal the object.

عندما يكون الجسم على مسافة تساوي ضعف نقطة التركيز، تكون الصورة حقيقية، مقلوبة، وتساوي الجسم

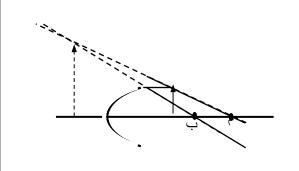
PHY 240

Image formed by Concave Mirrors



When the object is located between the focal point and curvature center of a concave mirror surface, the image is real, inverted, and enlarged.

عندما يكون الجسم موجودًا بين نقطة التركيز ومركز الانحناء لسطح مرآة مقعرة، تكون الصورة حقيقية، مقلوبة، ومكبرة



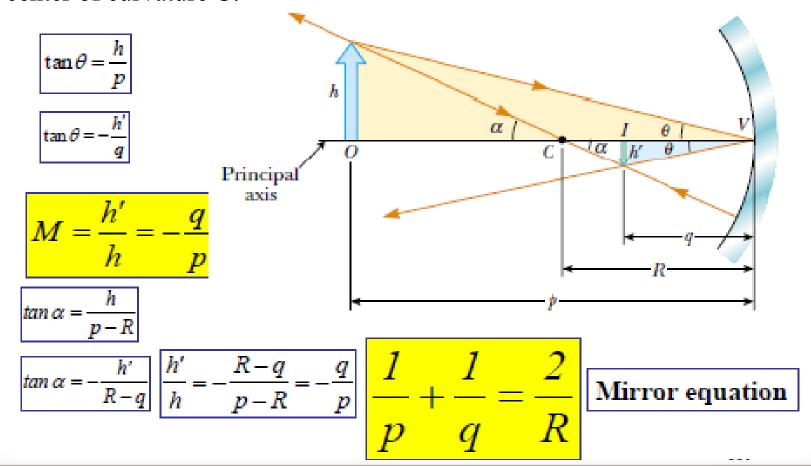
When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged.

عندما يكون الجسم موجودًا بين قطة التركيز وسطح مرآة مقعرة، تكون الصورة وهمية، مستقيمة،

Concave Mirrors

Mirror equation in terms of radius of curvature

The image formed by a spherical concave mirror when the object O lies outside the center of curvature C.



when the object is very far from the mirror:

- Light rays reflect from a concave mirror through the focal point F.
- The image point is halfway between the center of curvature and the center point on the mirror.
- the image distance q = R/2 = f, where f is the focal length of the mirror.

$$\frac{l}{p} + \frac{l}{q} = \frac{2}{R}$$

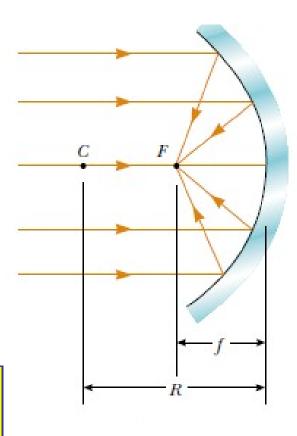
If the object is very far From the mirror: p is so much greater than R that p can be said to approach infinity.

$$f \to \infty \Leftrightarrow \frac{1}{p} \to 0$$

$$f = \frac{R}{2}$$
f: focal length

The mirror equation can be expressed in terms of the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



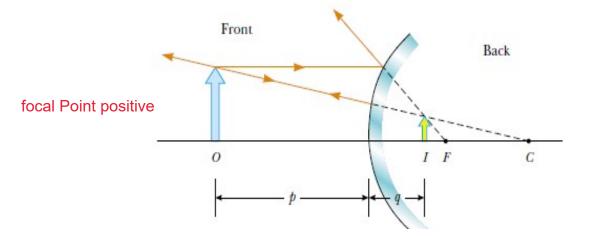
focal point negative

Focal point always positive

Image formed by Convex Mirrors

• Light is reflected from the outer, convex surface.

This is sometimes called a diverging mirror because the rays from any point on an object diverge after reflection as though they were coming from some point behind the mirror.



تنعكس الضوء من السطح الخارجي المحدب. ويُطلق عليها في بعض الأحيان اسم مرآة متشعبة لأن الأشعة الخارجة من أي نقطة على الجسم تتشعب بعد الانعكاس كما لو كانت قادمة من نقطة خلف المرآة

When the object is in front of a convex mirror, the image is: virtual, upright, and reduced in size.

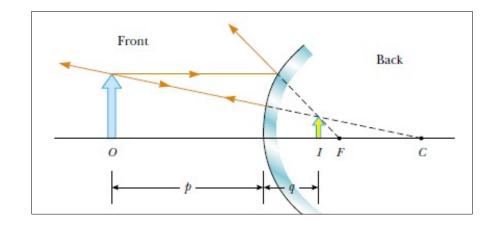
الصورة وهمية, معتدلة, و مصغرة we can use the same Equations for either concave or

convex mirrors

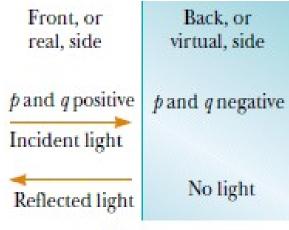
$$M = \frac{h'}{h} = -\frac{q}{p}$$

$$\frac{I}{p} + \frac{I}{q} = \frac{2}{R}$$

$$\frac{l}{p} + \frac{l}{q} = \frac{l}{f}$$



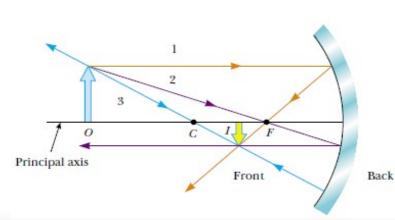
Sign conventions for object and image distances,



Convex or concave mirror

Sign Conventions for Mirrors Positive When Quantity Negative When Object location (p) Object is in back Object is in front of mirror (real object) of mirror (virtual object) Image is in front of Image location (q) Image is in back of mirror (real image) mirror (virtual image) Image height (h')Image is upright Image is inverted Focal length (f)Mirror is concave Mirror is convex and radius (R)

Image is upright

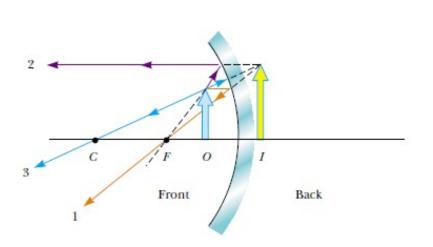


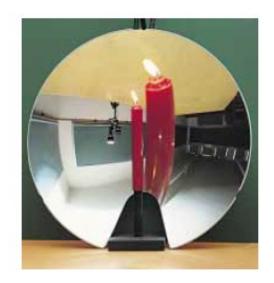
Magnification (M)

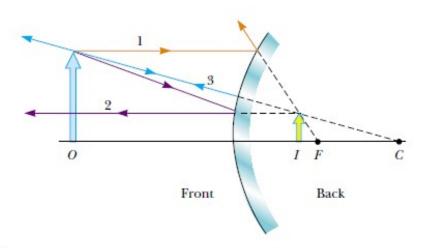


Image is inverted

Ray diagrams for spherical mirrors, along with corresponding photographs of the images of candles







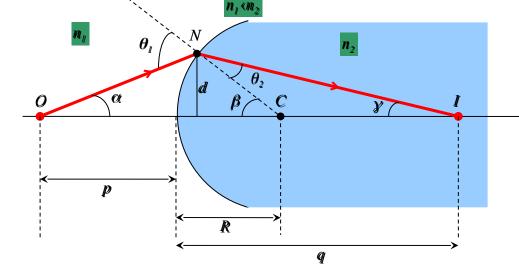


7. Images formed by refraction

Consider two transparent media having indices of refraction n_1 and n_2 , where the boundary between the two media is a spherical surface of

radius R.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Because θ_1 and θ_2 are assumed to be small, we can use the small angle approximation $\sin \theta \approx \theta$ (angle in radians)

7. Images formed by refraction

$$n_{1}\theta_{1} = n_{2}\theta_{2}$$

$$\theta_{1} = \alpha + \beta$$

$$\beta = \theta_{2} + \gamma$$

$$n_{1}\alpha + n_{1}\beta = n_{2}\beta - n_{2}\gamma$$

$$\hat{a}$$

$$\hat{a} + \hat{c} = \hat{b}$$

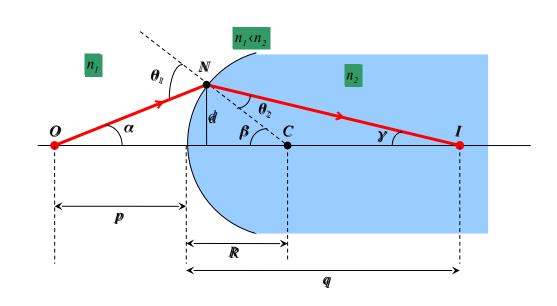
7. Images formed by refraction

$$\tan \alpha \approx \alpha \approx \frac{d}{p}$$

$$\tan \beta \approx \beta \approx \frac{d}{R}$$

$$tan \gamma \approx \gamma \approx \frac{d}{q}$$

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$



$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

For a fixed object distance p, the image distance q is independent of the angle that the ray makes with the axis.

Sign convention for refracting surfaces

- p is positive if object is in front of surface (real object)
- p is <u>negative</u> if object is in <u>back</u> of surface (virtual object)
- q is positive if image is in back of surface (real image)
- q is <u>negative</u> if image is in <u>front</u> of surface (virtual image)
- R is <u>positive</u> if center of curvature is in <u>back</u> of convex surface.
- R is negative if center of curvature is in <u>front</u> of concave surface.

Flat refracting surfaces

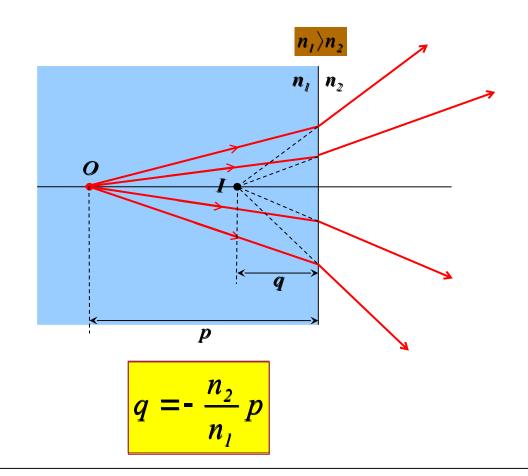
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

If a refracting surface is flat, then R is infinite

if
$$R \to \infty$$

$$\frac{n_1}{p} + \frac{n_2}{q} = 0$$

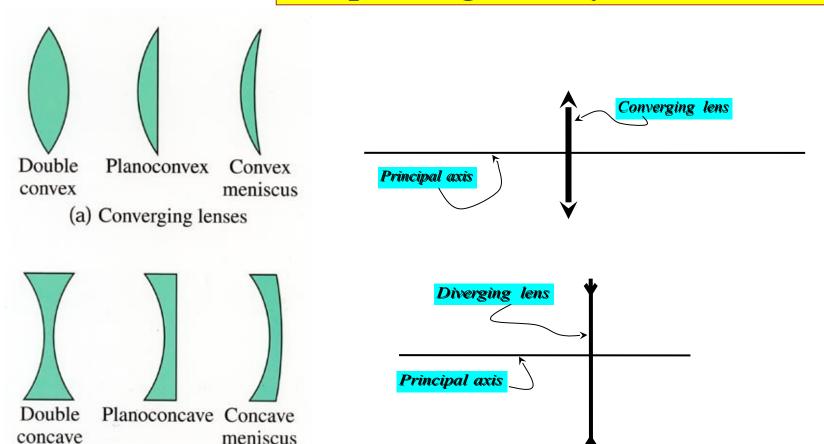
$$\frac{n_1}{p} = -\frac{n_2}{q}$$



the image formed by a flat refracting surface is on the same side of the surface as the object.

8. Thin lenses

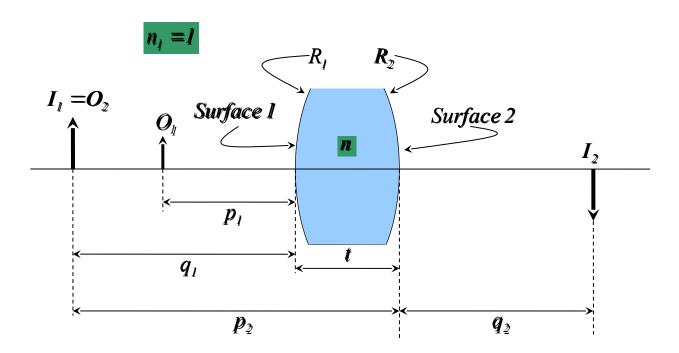
Simplified geometry for a thin lens



(b) Diverging lenses

8. Thin lenses

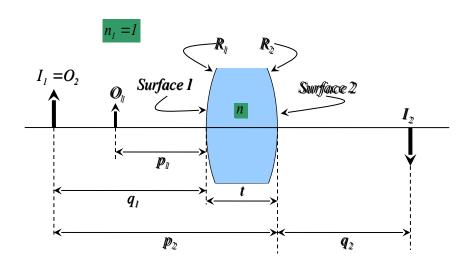
- Light passing through a lens experiences refraction at two surfaces.
- The image formed by one surface serves as the object for the second surface.



$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

The image I₁ formed by surface 1 satisfies the Equation:

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}$$



We apply the same equation to surface 2, taking $n_1 = n$ and $n_2 = 1$ (we make this switch in index because the light rays approaching surface 2 are in the material of the lens, and this material has index n.):

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

$$\frac{p_2 \rangle 0}{q_1 \langle 0} \} p_2 = -q_1 + t$$

t: the thickness of the lens

:For thin lenses, we can neglect t

$$p_2 = -q_1$$

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

$$p_2 = -q_1$$

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

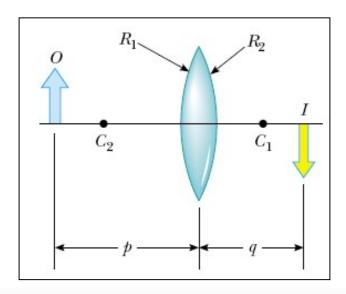
$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}$$

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

:For thin lens

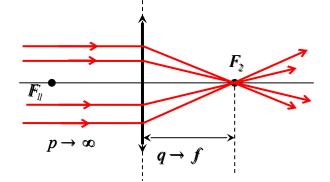
$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This equation is valid only for paraxial rays and only when the lens thickness is much less than R_1 and R_2 .

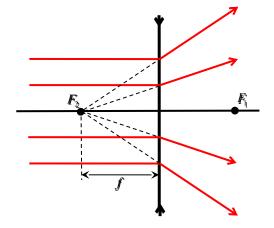


The <u>focal length</u> f of a thin lens is the image distance that corresponds to an infinite object distance.

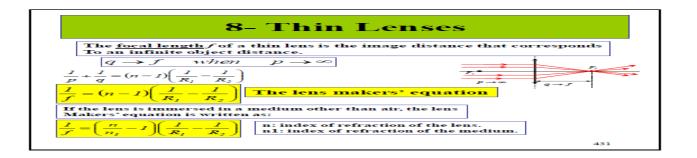
$$q \to f$$
 when $p \to \infty$



$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_{\parallel}} - \frac{1}{R_{2}}\right)$$



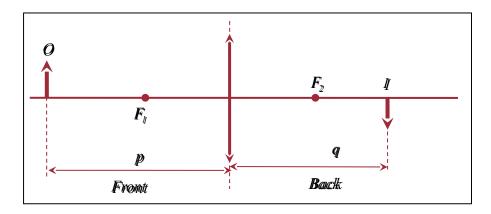
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
 The lens makers' equation



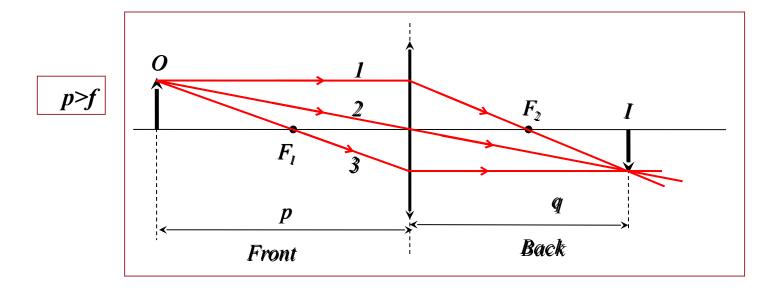
$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad and \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Thin lens equation

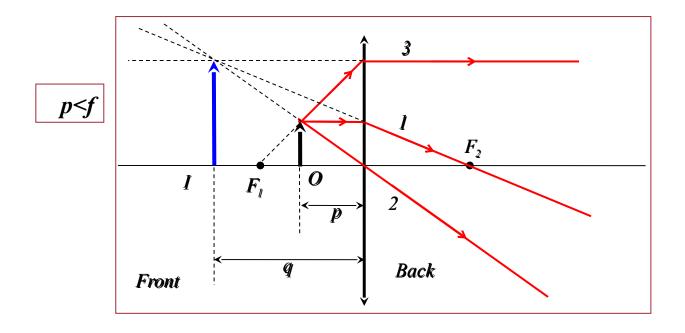


Ray diagrams for thin lenses



- <u>Ray 1</u> is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through the focal point on the front side of the lens and emerges from the lens parallel to the principal axis.

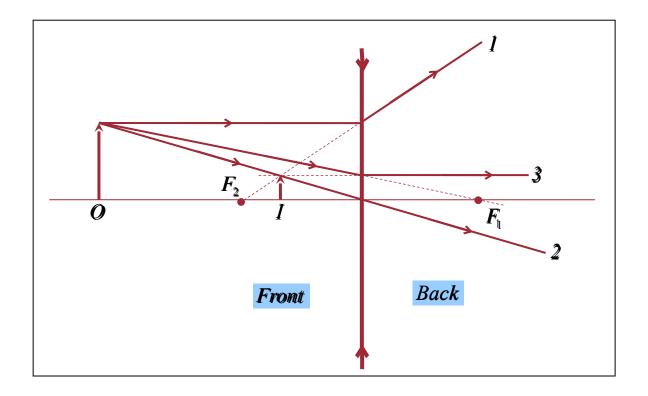
Ray diagrams for thin lenses



Ray 3 is drawn as if coming from the focal point and emerges from the lens parallel to the principal axis.

Ray diagrams for thin lenses

Diverging lens



8. Thin lenses

```
p is positive if object is in front of lens (real object).p is negative if object is in back of lens (virtual object).
```

q is <u>positive</u> if image is in <u>back</u> of lens (real image).
q is <u>negative</u> if image is in <u>front</u> of lens (virtual image).

 R_1 and R_2 are <u>positive</u> if center of curvature is in <u>back</u> of lens. R_1 and R_2 are <u>negative</u> if center of curvature is in <u>front</u> of lens.

f is positive if the lens is converging.
f is negative if the lens is diverging.

Magnification of image

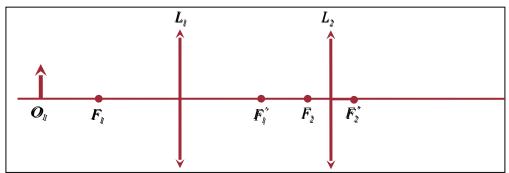
$$M = \frac{h'}{h} = -\frac{q}{p}$$

- M is positive, the image is upright and on the same side of the lens as the object.
- M is negative, the image is inverted and on the side of the lens opposite the object.

Combination of thin lenses

If two thin lenses are used to form an image, the system can be treated in the following manner:

- 1. the image formed by the first lens is located as if the second lens were not present.
- 2. the image formed by the first lens is serving as the object for the second lens.
- 3. A ray diagram is drawn for the second lens.
- 4. the second image formed is the final image of the system.



Combination of thin lenses

If the image formed by the first lens lies on the <u>back side</u> of the second .Lens, that image is treated as a <u>virtual object</u> for the second lens (p < 0)

.The same procedure can be extended to a system of three or more lenses

:The overall magnification of a system of thin lenses

$$M = \frac{\text{Final image's height}}{\text{Object's height}}$$

Example: Consider a system of three thin lenses:

$$M = \frac{\text{Final image's height}}{\text{Object's height}} = \frac{h_3'}{h_1}$$

$$M = \frac{h_1'}{h_1} \times \frac{h_2'}{h_1''} \times \frac{h_3''}{h_2''}$$

Combination of thin lenses

 $h'_1 = h_2$: the image formed by the first lens is serving as the object for the second lens.

 $h_2' = h_3$: the image formed by the second lens is serving as the object for the third lens

$$M = \frac{h_1'}{h_1} \times \frac{h_2'}{h_2} \times \frac{h_3'}{h_3} \qquad M = M_1 \times M_2 \times M_3 \text{ (for a system of 3 lenses)}$$

In general, the overall magnification of a system of n thin lenses equals :the product of the magnifications of the separate lenses

$$M = M_1 \times M_2 \times \dots \times M_n$$

