CHAPTER 1

The Wave Function

1.1 The Schrodinger equation

$x(t)$

$v = \frac{dx}{dt}$

$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

$F = ma$

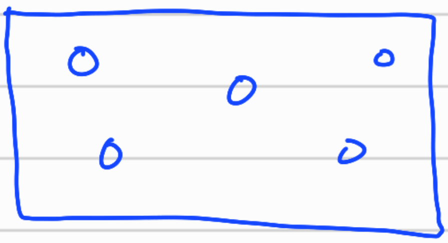
$T = \frac{1}{2} m v^2$

$F = -\frac{\partial V}{\partial x}$

$U = Fx$

باستخدام قوانين الميكانيكا الكلاسيكية من السهل
التعرف على موقع وحركة ومسار الجسم

✓ اذا اردنا معرفة موقع الكترون نتعامل مع الالكترون
كأنه موجة



$\Rightarrow \Psi(x,t)$

wave function

Schrodinger equation

$$\underbrace{i\hbar \frac{\partial \Psi}{\partial t}}_{\text{Total energy}} = \underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2}}_{\text{KE}} + \underbrace{V\Psi}_{\text{PE}}$$

Total energy

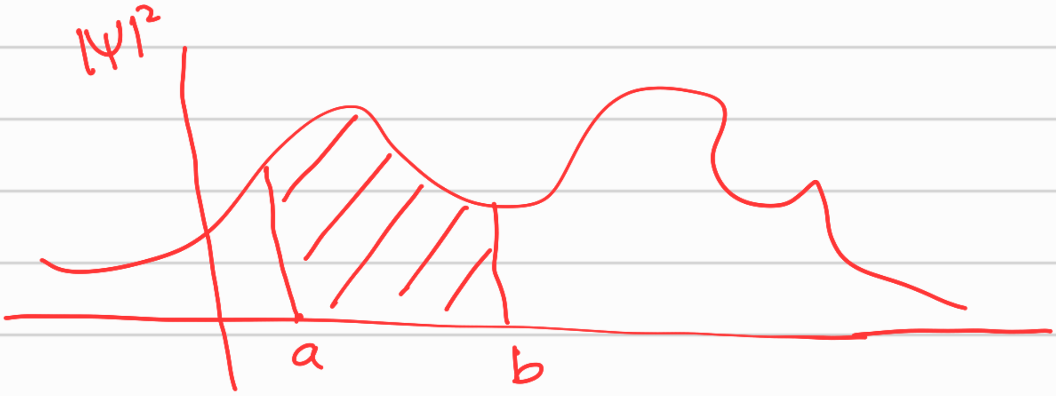
KE
طاقة حركية

طاقه الوضع
PE

$$i = \sqrt{-1} \qquad \hbar = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{2\pi}$$

$$\hbar = 1.0545 \times 10^{-34}\ Js$$

1.2 the statistical Interpretation



$$\int_a^b |\psi|^2 \, dx$$

التعبير الاحصائي عن
احتماليه وجود الجسم
في مكان ما في اي منطقة من
$a \to b$

$\int_a^b |\psi(x,t)|^2 \, dx$ = the Probability of finding the particle between a and b

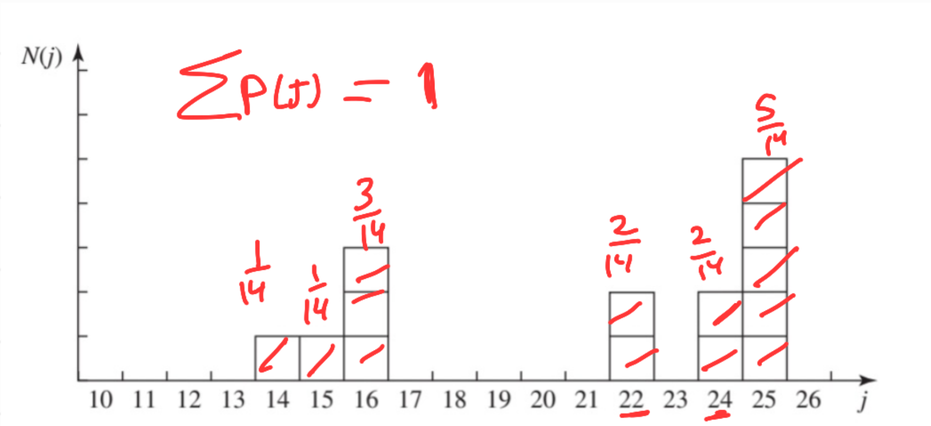
$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 1$$

the particle is at point C

1.3 Probability

* Discrete Variables

$N(14) = 1$
$N(15) = 1$
$N(16) = 3$
$N(22) = 2$
$N(24) = 2$
$N(25) = 5$



Question 1 If you selected one individual at random from this group, what is the **probability** that this person's age would be 15?

$$P(15) = \frac{N(15)}{N}$$

$$= \frac{1}{14}$$

$$P(j) = \frac{N(j)}{N}$$

$P(j)$ احتمالية عمر معين

N العدد الكلي

$N(j)$ عدد أفراد عمر معين

Question 2 What is the **most probable** age?

25

Question 3 What is the **median** age?

الفئة اللي تأتي بالوسط

23

Question 4 What is the **average** (or **mean**) age?

$$\langle j \rangle = \frac{j\,N(j)}{N} = \sum j\,P(j)$$

$$\langle j \rangle = \frac{14(1) + 15(1) + 16(3) + 22(2) + 24(2) + 25(5)}{14}$$

$$\langle j \rangle = 21$$

$$14 \times \frac{1}{14} + 15 \times \frac{1}{14} + 16 \times \frac{3}{14} + 22 \times \frac{2}{14} + 24 \times \frac{2}{14} + 25 \times \frac{5}{14}$$

j $P(j)$

$= 21$

$$\langle j \rangle^2 = 21^2 = 441$$

Question 5 What is the average of the *squares* of the ages?

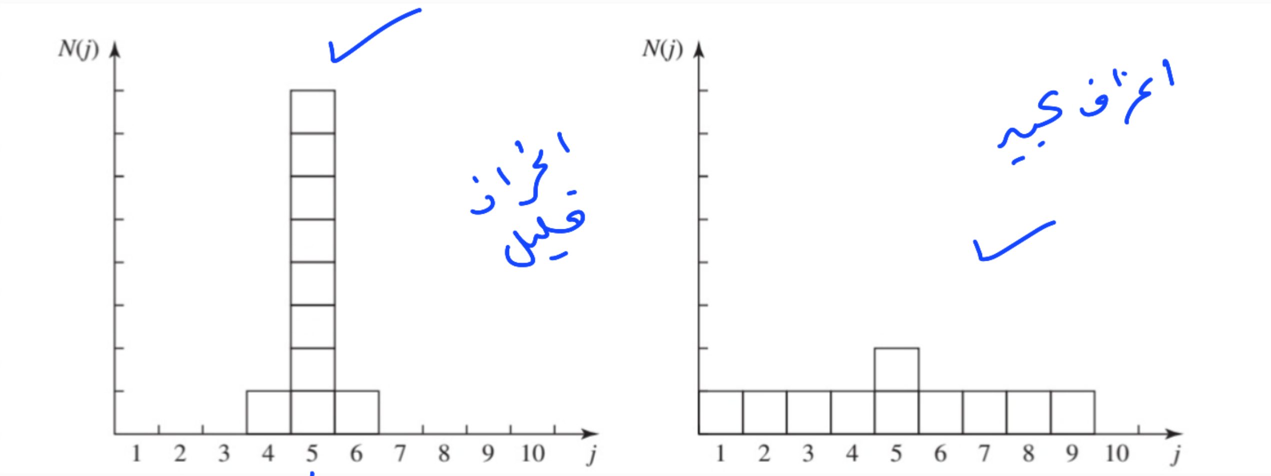
$$\langle j^2 \rangle = \sum j^2\,P(j)$$

$$\langle f(j) \rangle = \sum f(j)\,P(j)$$

$$= 14^2 \times \frac{1}{14} + 15^2 \times \frac{1}{14} + 16^2 \times \frac{3}{14} + 22^2 \times \frac{2}{14} + 24^2 \times \frac{2}{14} + 25^2 \times \frac{5}{14}$$

$$\langle j^2 \rangle = 459.6$$

$$\langle j \rangle^2 \neq \langle j^2 \rangle$$



mean
median = 5

mean
median = 5

standard deviation الانحراف المعياري

$$\sigma^2$$

تباين $\sigma \rightarrow$ variance

$$\boxed{\begin{aligned} \sigma^2 &= \langle j^2 \rangle - \langle j \rangle^2 \\ \sigma &= \sqrt{\langle j^2 \rangle - \langle j \rangle^2} \end{aligned}}$$

1.3.2 Continuous variables

Probability $= \rho(x)\,\underline{dx}$

احتماليه اختيار عدد في فتره معينه dx

Probability density $a \longrightarrow b$

$$P_{ab} = \int_a^b \rho(x)\,dx$$

$$\int_{-\infty}^{\infty} \rho(x)\,dx = 1$$

$$\langle f(x) \rangle = \int_{-\infty}^{+\infty} f(x)\,\rho(x)\,dx$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

حل تمارين كم (١) شابتر (١)

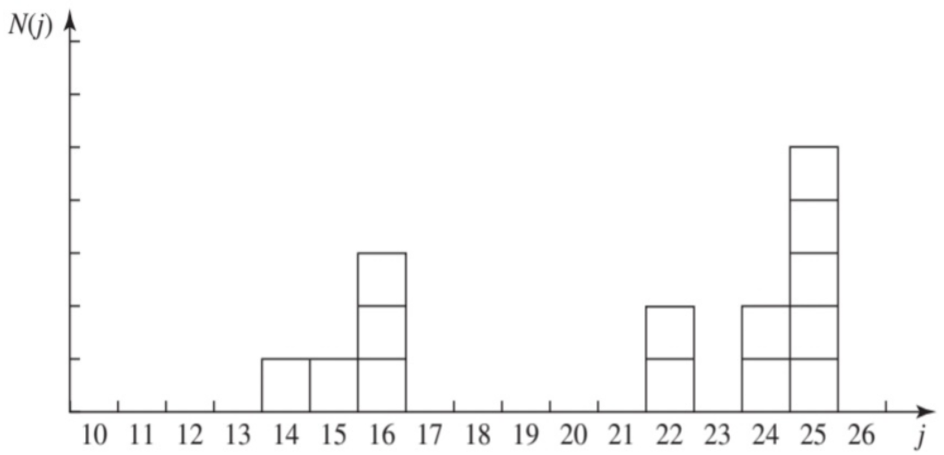
1.1 + 1.2 + 1.3

Problem 1.1 For the distribution of ages in the example in Section 1.3.1:

(a) Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.

(b) Determine Δj for each j, and use Equation 1.11 to compute the standard deviation.

(c) Use your results in (a) and (b) to check Equation 1.12.



j	$N(j)$
14	1
15	1
16	3
22	2
24	2
25	5
	14

a) $\langle j^2 \rangle \qquad \langle j \rangle^2$

$$\langle j \rangle = \sum j P(j) = \sum \frac{j N(j)}{N}$$

$$\langle j \rangle = \frac{14(1)+15(1)+16(3)+22(2)+24(2)+25(5)}{14}$$

$$= 21$$

$$\langle j \rangle^2 = 21^2 = 441$$

$$\langle j^2 \rangle = \sum j^2 P(j) = \sum j^2 \frac{N(j)}{N}$$

$$\langle j^2 \rangle = \frac{14^2(1)+15^2(1)+16^2(3)+22^2(2)+24^2(2)+25^2(5)}{14}$$

$$\langle j^2 \rangle = 459.57$$

b)

$$\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$$

(1.11) $$\sigma^2 = \langle \Delta j^2 \rangle = \frac{\sum (\Delta j)^2 N(j)}{N}$$

j	$\Delta j = j - \langle j\rangle$	$N(j)$
14	$14-21 = -7$	1
15	$15-21 = -6$	1
16	$16-21 = -5$	3
22	$22-21 = 1$	2
24	$24-21 = 3$	2
25	$25-21 = 4$	5

$$\sigma^2 = \frac{\sum \Delta j^2 \, N(j)}{N}$$

$$= \frac{(-7)^2 1 + (-6^2)1 + (-5)^2(3) + 1(2) + 3^2(2) + 4^2(5)}{14}$$

$\sigma^2 = 18.571$ $\sigma = 4.309$

c)

$$\sigma^2 = \langle j^2\rangle - \langle j\rangle^2$$

$$= 459.571 - 441$$

$$\sigma^2 = 18.571$$

$$\sigma = 4.309$$

Problem 1.2

σ^2

(a) Find the standard deviation of the distribution in Example 1.2.

(b) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

$$P(x) = \frac{1}{2\sqrt{hx}} \qquad (0 \le x \le h)$$

a)

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x \cdot P(x)\, dx$$

$$\langle x \rangle = \int_0^h x \frac{1}{2\sqrt{hx}} = \int_0^h \frac{x^1}{\sqrt{x}} \frac{1}{2\sqrt{h}}$$

$$\langle x \rangle = \int_0^h \frac{x^{1/2}}{2\sqrt{h}}\, dx = \frac{1}{2\sqrt{h}} \int_0^h x^{1/2}\, dx$$

$$\langle x \rangle = \frac{1}{2\sqrt{h}} \left[\frac{x^{3/2}}{3/2} \right]_0^h$$

$$\langle x \rangle = \frac{1}{\cancel{2}\sqrt{h}} \frac{\cancel{2}}{3} x^{3/2} \Big]_0^h$$

$$\langle x \rangle = \frac{1}{3\sqrt{h}} \left[h^{3/2} - 0^{3/2} \right]$$

$$\langle x \rangle = \frac{1}{3} \frac{h^{3/2}}{h^{1/2}} = \frac{h}{3}$$

$$\langle x \rangle = \frac{h}{3}$$

$$\langle x \rangle^2 = \frac{h^2}{9}$$

$$\langle x^2 \rangle = \int_0^h x^2 \cdot \rho(x)$$

$$= \int_0^h x^2 \frac{1}{2\sqrt{hx}}$$

$$= \frac{1}{2\sqrt{h}} \int_0^h \frac{x^2}{x^{1/2}} = \frac{1}{2\sqrt{h}} \int_0^h x^{3/2}\,dx$$

$$= \frac{1}{2\sqrt{h}} \left. \frac{x^{5/2}}{5/2} \right]_0^h = \frac{1 \cdot \cancel{2}}{\cancel{2}\sqrt{h}\,5}\left[h^{5/2} - 0\right]$$

$$= \frac{1}{5\,h^{1/2}} h^{5/2} = \frac{h^2}{5}$$

$$\langle x^2 \rangle = \frac{h^2}{5}$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{h^2}{5} - \frac{h^2}{9}$$

✓ $$\sigma^2 = h^2\left(\frac{1}{5} - \frac{1}{9}\right) = \frac{4h^2}{45}$$

✓ $$\sigma = \sqrt{\frac{4}{45}}\,h$$

Problem 1.3 Consider the **gaussian** distribution

$$\rho(x) = Ae^{-\lambda(x-a)^2},$$

where A, a, and λ are positive real constants. (The necessary integrals are inside the back cover.)

(a) Use Equation 1.16 to determine A.

(b) Find $\langle x\rangle$, $\langle x^2\rangle$, and σ.

(c) Sketch the graph of $\rho(x)$.

$$\int_{-\infty}^{+\infty} \rho(x)\, dx = 1$$

$$\rho(x) = A e^{-\lambda(x-a)^2}$$

$$1 = \int_{-\infty}^{\infty} \rho(x)\, dx = A\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\, dx$$

بالتعويض مكانه

$$u = x - a$$

$$du = dx$$

$$1 = A\int_{-\infty}^{\infty} e^{-\lambda u^2}\, du$$

$$\int_{0}^{\infty} e^{-au^2}\, du = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$1 = 2A\int_{0}^{\infty} e^{-\lambda u^2}\, du$$

$$1 = \cancel{2}A\left[\frac{1}{\cancel{2}}\sqrt{\frac{\pi}{\lambda}}\right]$$

$$1 = A\frac{\sqrt{\pi}}{\sqrt{\lambda}} \qquad A = \frac{\sqrt{\lambda}}{\sqrt{\pi}} = \sqrt{\frac{\lambda}{\pi}}$$

b) $$\langle x \rangle = \int_{-\infty}^{+\infty} x \, P(x) \, dx$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x \sqrt{\frac{\lambda}{\pi}} \, e^{-\lambda (x-a)^2} \, dx$$

$u = x - a \qquad x = u + a$ بتعويض

$du = dx$

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} x \, e^{-\lambda u^2} \, du$$

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{\infty} (u+a) \, e^{-\lambda u^2} \, du$$

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \left[\underbrace{\int_{-\infty}^{\infty} u \, e^{-\lambda u^2} \, du}_{\text{odd function}} + a \int_{-\infty}^{\infty} e^{-\lambda u^2} \, du \right]$$

odd function

= zero

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \left[0 + 2a \int_{0}^{\infty} e^{-\lambda u^2} \, du \right]$$

$$\langle x \rangle = \cancel{\sqrt{\frac{\lambda}{\pi}}} \left[\cancel{2} a \, \frac{1}{\cancel{2}} \cancel{\sqrt{\frac{\pi}{\lambda}}} \right]$$

$$\langle x \rangle = a$$

$$\langle x^2\rangle = \int_{-\infty}^{+\infty} x^2\, P(x)\, dx$$

$$= \int x^2 \sqrt{\frac{\lambda}{\pi}}\, e^{-\lambda(x-a)^2}\, dx$$

$u = x - a$

$du = dx$

$x = u + a$

$(u+a)^2 = u^2 + 2au + a^2$

$$= \sqrt{\frac{\lambda}{\pi}} \int (u+a)^2\, e^{-\lambda u^2}\, du$$

فك التربيع

$$= \sqrt{\frac{\lambda}{\pi}} \left[\int_{-\infty}^{+\infty} u^2 e^{-\lambda u^2} \underbrace{+ 2a \int u\, e^{-\lambda u^2}}_{\text{zero}} + \underbrace{a^2 \int_{-\infty}^{\infty} e^{-\lambda u^2}\, du}_{} \right]$$

$$= \sqrt{\frac{\lambda}{\pi}} \left[2\int_0^{\infty} u^2 e^{-\lambda u^2}\, du + 2a^2 \int_0^{\infty} e^{-\lambda u^2}\, du \right]$$

$$= \cancel{\sqrt{\frac{\lambda}{\pi}}} \left[\cancel{2}\, \frac{1}{\underset{2}{\cancel{4}}\lambda} \cancel{\sqrt{\frac{\pi}{\lambda}}} + \cancel{2} a^2 \frac{1}{\cancel{2}} \cancel{\sqrt{\frac{\pi}{\lambda}}} \right]$$

$$\langle x^2\rangle = \frac{1}{2\lambda} + a^2$$

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\sigma = \sqrt{\frac{1}{2\lambda} + \cancel{a^2} - \cancel{a^2}}$$

$$\sigma = \sqrt{\frac{1}{2\lambda}}$$

1.4 Normalization

ψ wave function

1- يجب ان تحقق معادلة شرودنغر ψ

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial t^2} - \frac{i}{\hbar} V \psi$$

$\psi \quad \psi^*$

$i \quad -i$

$$\frac{\partial \psi^*}{\partial t} = \frac{-i\hbar}{2m}\frac{\partial^2 \psi}{\partial t^2} + \frac{i}{\hbar} V \psi$$

2- $\int |\psi^2| \, dx$ probability density

$$\int_{-\infty}^{\infty} |\psi^2| \, dx = 1$$

Normalized $\displaystyle\int_{-\infty}^{\infty} |\psi^2| \, dx = \int_{-\infty}^{\infty} \psi \psi^* \, dx = 1$

ψ non-normalized $\displaystyle\int_{-\infty}^{\infty} |\psi^*| \neq 1$

$$\int_{-\infty}^{\infty} \underline{A}\psi = 1$$

Normalization : نحسب الثابت A الذي يجعل $\displaystyle\int_{-\infty}^{\infty} \psi \psi^* =$

$$\int_{-\infty}^{\infty} |\psi|^2 = 1 \qquad \frac{d}{dt}\int_{-\infty}^{\infty} |\psi|^2 = 0$$

Problem 1.4 At time $t = 0$ a particle is represented by the wave function

$$\Psi(x, \overset{t}{0}) = \begin{cases} A(x/a), & 0 \leq x \leq a, \\ A(b-x)/(b-a), & a \leq x \leq b, \\ 0, & \text{otherwise}, \end{cases}$$

where A, a, and b are (positive) constants.

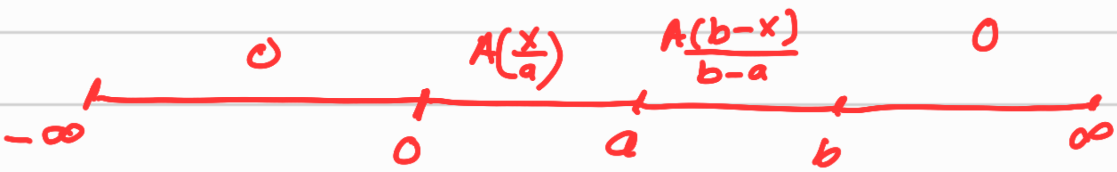
(a) Normalize Ψ (that is, find A, in terms of a and b).

(b) Sketch $\Psi(x, 0)$, as a function of x.

(c) Where is the particle most likely to be found, at $t = 0$?

(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.

(e) What is the expectation value of x? القيمة المتوقعة $\langle x \rangle = \int x \psi^2 dx$



a) $$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

$$= \int_{-\infty}^{0} |\psi|^2 dx + \int_{0}^{a} |\psi|^2 dx + \int_{a}^{b} |\psi|^2 dx + \int_{b}^{\infty} |\psi|^2 dx = 1$$

$$= \cancel{\int_{-\infty}^{0} 0} + \int_{0}^{a} A^2 \frac{x^2}{a^2} dx + \int_{a}^{b} \frac{A^2 (b-x)^2}{(b-a)^2} dx + \cancel{\int_{b}^{\infty} 0} = 1$$

$$= \frac{A^2}{a^2} \int_{0}^{a} x^2 dx + \frac{A^2}{(b-a)^2} \int (b-x)^2 dx = 1$$

$$1 = \frac{A^2}{a^2}\left[\frac{x^3}{3}\right]_0^a + \frac{A^2}{(b-a)^2}\left[\frac{(b-x)^3}{-3}\right]_a^b$$

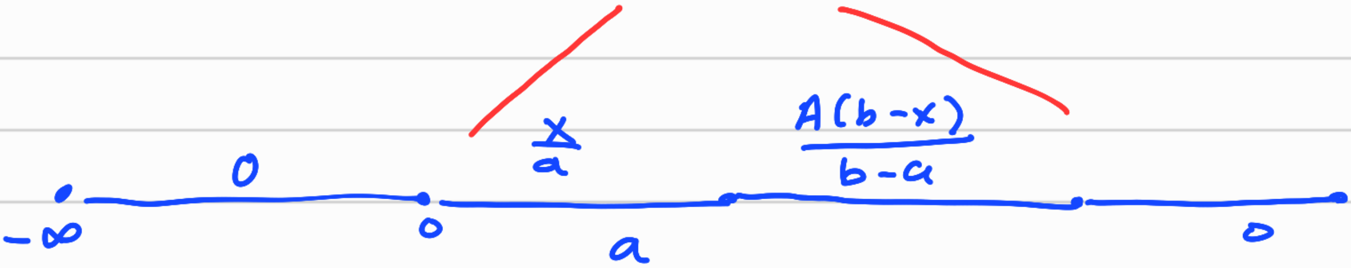
$$1 = \frac{A^2}{a^2}\left[\frac{a^3}{3}\right] + \frac{A^2}{-3(b-a)^2}\left[\cancel{(b-b)^3} - (b-a)^3\right]$$

$$= \frac{A^2}{\cancel{a^2}}\,\frac{\overset{a}{\cancel{a^3}}}{3} + \frac{A^2}{3\cancel{(b-a)^2}}\,\overset{(b-a)}{\cancel{(b-a)^3}} = 1$$

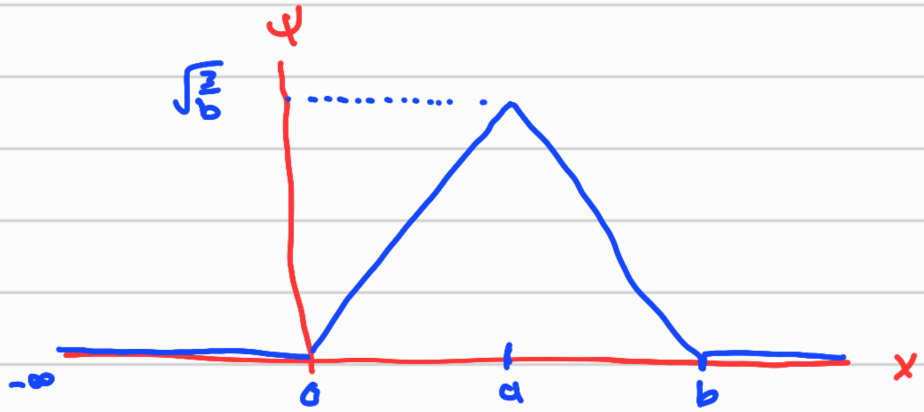
$$1 = \cancel{\frac{A^2 a}{3}} + \frac{A^2 b}{3} - \cancel{\frac{A^2 a}{3}}$$

$$1 = \frac{A^2 b}{3}$$

$$\frac{3}{b} = A^2 \qquad A = \sqrt{\frac{3}{b}}$$



b)



c) at $x = a$

d) $$P = \int_{-\infty}^{a} |\psi(x,0)|^2 \, dx$$

$$P = \cancel{\int_{-\infty}^{0} 0 \, dx} + \int_{0}^{a} A^2 \frac{x^2}{a^2} \, dx$$

$$P = \int_{0}^{a} \sqrt{\frac{3}{b}}^{\,2} \frac{x^2}{a^2}$$

$$P = \frac{3}{ba^2} \int_{0}^{a} x^2 = \frac{3}{ba^2} \left[\frac{x^3}{3} \right]_0^a$$

$$= \frac{3}{ba^2} \frac{a^3}{3} = \frac{a}{b}$$

$$P = \frac{a}{b}$$

$\Rightarrow$ in case $a = b$ $\quad P = \frac{a}{b} = \frac{a}{a} = 1$

in case $b = 2a$ $\quad P = \frac{a}{b} = \frac{a}{2a} = \frac{1}{2}$

$$\begin{cases} a = b & P = 1 \\ b = 2a & P = 1/2 \end{cases}$$

e)

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,|\psi|^2\,dx \qquad A = \sqrt{\frac{3}{b}}$$

$$= \int_0^a x\,\frac{A^2x^2}{a^2}\,dx + \int_a^b x\,\frac{A^2(b-x)^2}{(b-a)^2}\,dx$$

$$= \frac{3}{ba^2}\int_0^a x^3\,dx + \frac{3}{b(b-a)^2}\int_a^b \underbrace{x}_{u}\,\underbrace{(b-x)^2\,dx}_{dv}$$

تكامل بالاجزاء

$$\frac{3}{ba^2}\left.\frac{x^4}{4}\right]_0^a + \frac{3}{b(b-a)^2}\left[\frac{x(b-x)^3}{-3} - \int\frac{(b-x)^3}{-3}\right]$$

$u = x \qquad dv = (b-x)^2$

$du = dx \qquad V = \frac{(b-x)^3}{-3}$

$uv - \int v\,du$

$$\frac{3a^4}{4ba^2} + \frac{3}{b(b-a)^2}\left[\frac{a(b-a)^3}{-3} - \frac{(b-x)^4}{12}\right]_a^b$$

$$\frac{3a^2}{4b} + \frac{3}{b(b-a)^2}\left[\frac{a(b-a)^3}{-3} + \frac{(b-a)^4}{12}\right]$$

$$\frac{3a^2}{4b} - \frac{3a(b-a)^3}{3b(b-a)^2} + \frac{3(b-a)^4}{12b(b-a)^2}$$

$$\frac{3a^2}{4b} + \frac{a(b-a)}{b} + \frac{(b-a)^2}{4b}$$

$$\frac{3\cancel{a^2}}{\cancel{4}b} + \frac{4ab}{4b} - \frac{4\cancel{a^2}}{\cancel{4}b} + \frac{b^2}{4b}\left(-\frac{2ab}{4b} + \frac{\cancel{a^2}}{\cancel{4}b}\right)$$

$$= \frac{2ab}{4b} + \frac{b^2}{4b}$$

1.5 Momentum

$$\vec{p} = mv = m\frac{dx}{dt}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,|\psi(x,t)|^2\,dx$$

$$\langle v \rangle = \frac{d\langle x \rangle}{dt} = \frac{d}{dt}\int_{-\infty}^{\infty} x\,\psi^2\,dx$$

$$\langle v \rangle = \frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{m}\int \psi^* \frac{\partial \psi}{\partial x}\,dx$$

$$p = m\frac{d\langle x \rangle}{dt} = \cancel{m}\left(-\frac{\hbar}{2\cancel{m}}\int \psi^* \frac{\partial \psi}{\partial x}\,dx\right)$$

$$P = -i\hbar \int \psi^* \frac{\partial \psi}{\partial x}\, dx$$

شكل آخر لمعادلة القيمة المتوقعة للموقع والزخم

$$\langle x \rangle = \int \psi^* [x] \psi \, dx$$

operator

$$\langle P \rangle = \int \psi^* \left[-i\hbar \frac{\partial}{\partial x}\right] \psi \, dx$$

operator

operator

$$\langle x \rangle \Longrightarrow [x]$$

$$\langle P \rangle \Longrightarrow \left[-i\hbar \frac{\partial}{\partial x}\right]$$

$$\langle Q \rangle \Longrightarrow [Q]$$

Kinetic energy T

$$T = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$p^2 = m^2 v^2$ $\quad v^2 = \frac{p^2}{m^2}$

$T = \frac{1}{2} m v^2$

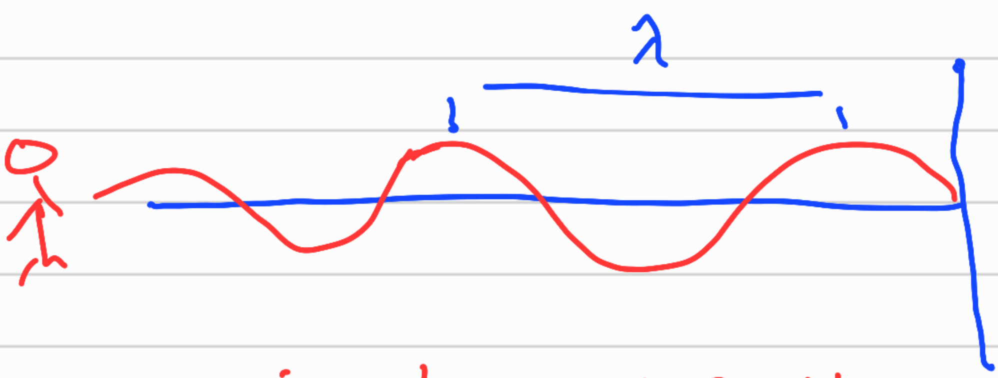
$T = \frac{1}{2} \cancel{m} \frac{p^2}{m^{\cancel{2}}}$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int \psi^* \frac{\partial^2 \psi}{\partial x^2} \, dx$$

$i^2 = -1$

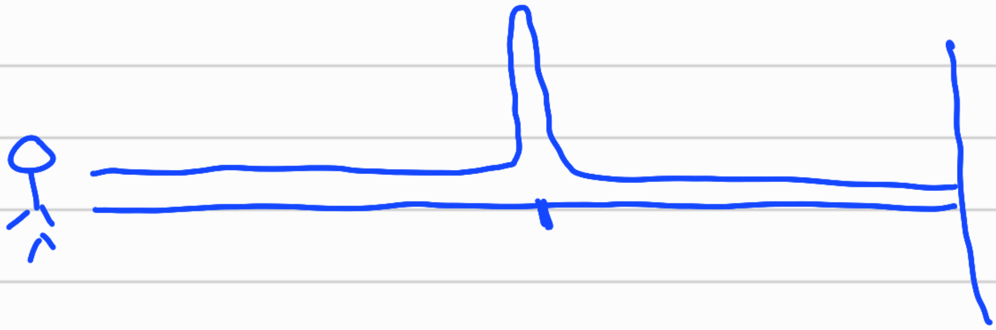
1.6 The uncertainity Principle

فرضية الشك عدم الدقة



→ we can't determine the exact Position of the wave

→ yes we can determine the wavelength



✓ We able to define more accurat position of the wave

✓ We can't accurately find the wavelength

اذا ركزنا القياس على موقع الجسم سنحصل
على قياس غير دقيق للزخم والعكس صحيح

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

x : Position $\qquad$ p : momentum

= de Broglie formula

علاقة مترابطة بين طول الموجه وزخمها

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

كلما كان طول الموجه أقل كان الزخم أكبر