determine the size of the matrix.

1. $\begin{bmatrix} 1 & 2 & -4 \\ 3 & -4 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ 3×3.

6. $[-1]$ 1×1.

In Exercises 9–14, determine whether the matrix is in row-echelon form. If it is, determine whether it is also in reduced row-echelon form.

9. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ row-echelon form

11. $\begin{bmatrix} 2 & 0 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ *not* in row-echelon

13. $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$ *not* in row-echelon form

In Exercises 15–22, find the solution set of the system of linear equations represented by the augmented matrix.

21. $\begin{bmatrix} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

$$\begin{aligned} x_1 + 2x_2 \qquad + x_4 &= 4 \\ x_2 + 2x_3 + x_4 &= 3 \\ x_3 + 2x_4 &= 1 \\ x_4 &= 4 \end{aligned}$$

Solve this system by back-substitution.

$$x_3 = 1 - 2x_4 = 1 - 2(4) = -7$$

$$x_2 = 3 - 2x_3 - x_4 = 3 - 2(-7) - 4 = 13$$

$$x_1 = 4 - 2x_2 - x_4 = 4 - 2(13) - 4 = -26$$

So, the solution is: $x_1 = -26$, $x_2 = 13$, $x_3 = -7$, and $x_4 = 4$.

In Exercises 23–36, solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

24. $2x + 6y = 16$

$-2x - 6y = -16$

The augmented matrix for this system is

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix}.$$

Use Gauss-Jordan elimination as follows.

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow$$ R1/2→R1

$$\begin{bmatrix} 1 & 3 & 8 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow$$ 2R1+R2→R2

$$\begin{bmatrix} 1 & 3 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

Converting back to system of linear equations

$x + 3y = 8.$

<u>y= t</u> t is any real number $x + 3t = 8$ then <u>x = 8− 3t</u>

25. $-x + 2y = 1.5$

$2x - 4y = 3$

The augmented matrix for this system is

$$\begin{bmatrix} -1 & 2 & 1.5 \\ 2 & -4 & 3 \end{bmatrix}.$$

$$\begin{bmatrix} -1 & 2 & 1.5 \\ 2 & -4 & 3 \end{bmatrix} \Rightarrow$$ -R1→R1

$$\begin{bmatrix} 1 & -2 & -\frac{3}{2} \\ 2 & -4 & 3 \end{bmatrix} \Rightarrow$$ -2R1+R2→R2

$$\begin{bmatrix} 1 & -2 & -\frac{3}{2} \\ 0 & 0 & 6 \end{bmatrix}$$

$0 = 6$ no solution.

35.
$$\begin{aligned} 3x + 3y + 12z &= 6 \\ x + y + 4z &= 2 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4 \end{aligned}$$

The augmented matrix for this system is

$$\begin{bmatrix} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{bmatrix}.$$

Gaussian elimination produces the following.

$$\begin{bmatrix} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 4 & 2 \\ 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{bmatrix}$$

-1(R1)+R2→ R2

-2(R1)+R3→ R3

R1+R4→ R4

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 12 & 6 \\ 0 & 3 & 12 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 3 & 12 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R2→R4

-(R2)+R3→R3

1/3 R2→R2

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-(R2)+R1→R1

Z= t   t is any real number   x = 0 ,   y=2-4t