Dimensions[ ] الابعاد

The powers, to which the fundamental units of mass, length and time written as M, L and T are raised, which include their nature and not their Magnitude

$\left[\frac{M}{L^3}\right]$ الكثافة = كتلة / حجم

Density = mass/volume

= [M]/[L3]

= [$M^1L^{-3}T^0$]

الابعاد للكميه فيزيائيه هي تحتوي على جمع الكميات الفيزيائية الاساسية المكونة لها

Quantity الكمية	Dimension بعد	Unit وحدة	SI Symbol	Formula معادلة
Base units:				
length	[L]	meter	m	
mass	[M]	kilogram	kg	
temperature	θ [Θ]	kelvin	K	
time	[T]	second	s	
Derived units:				
area	$[L^2]$	square meter		m^2
volume	$[L^3]$	cubic meter		m^3
velocity	$[L\ T^{-1}]$	meter per second		$m\ s^{-1}$
acceleration	$[L\ T^{-2}]$	meter per second squared		$m\ s^{-2}$
density	$[M\ L^{-3}]$	kilogram per cubic meter		$kg\ m^{-3}$
force	$[M\ L\ T^{-2}]$	newton	N	$kg\ m\ s^{-2}$
pressure	$[M\ L^{-1}\ T^{-2}]$	pascal	Pa	$N\ m^{-2}$
stress	$[M\ L^{-1}\ T^{-2}]$	pascal	Pa	$N\ m^{-2}$
energy	$[M\ L^2\ T^{-2}]$	joule	J	$N\cdot m$

كميات ليس لها ابعاد لكن لها مقادير

Dimensionless quantities are those which do not have dimensions but have a fixed value.

- **Dimensionless quantities without units:** Pure numbers, π, e, sin θ, cos θ, tan θ, ratio etc.
- **Dimensionless quantities with units:** [$M^oL^oT^o$]: Angular displacement (radian) ,Joule's constant ($Jcal^{-1}$), etc.

Dimentions

Mass : [M]
length : [L]
Time : [T]

أساسي

Quantity	[Dimension]	unit
Area مساحة	L^2	m^2
Volume مجسم	L^3	m^3
Frequency = تردد = $\frac{1}{T}$	T^{-1}	s^{-1} = hertz = Hz $\frac{1}{s}$
Velocity = $\frac{\text{مسافة}}{\text{زمن}} = \frac{L}{T}$	$L \cdot T^{-1}$	$m \cdot s^{-1}$
تسارع Acceleration = $\frac{\text{السرعة}}{\text{الزمن}} = \frac{L}{T^2}$	$L \cdot T^{-2}$ L/T^2	$m \cdot s^{-2}$
كثافة Density = $\frac{\text{كتلة}}{\text{حجم}} = \frac{M}{L^3}$	$M \cdot L^{-3}$	$kg \cdot m^{-3}$
زخم Momentum = كتلة × سرعة $M \times \frac{L}{T}$	$M \cdot L \cdot T^{-1}$	$kg \cdot m \cdot s^{-1}$
قوة Force = كتلة × تسارع $M \times \frac{L}{T^2} = MLT^{-2}$	$M \cdot L \cdot T^{-2}$	$kg.m.s^{-2}$ = newton = N
طاقة Work, Energy = قوة × مسافة $MLT^{-2} \times L$	$M \cdot L^2 \cdot T^{-2}$	$kg \cdot m^2 \cdot s^{-2}$ = joule = J
Power = $\frac{\text{طاقة}}{\text{زمن}} = ML^2T^{3}$	$M \cdot L^2 \cdot T^{-3}$	$kg \cdot m^2 \cdot s^{-3}$ = watt = W
Pressure stress = $\frac{\text{قوة}}{\text{مساحة}} = \frac{MLT^{-2}}{L^2}$	$M \cdot L^{-1} \cdot T^{-2}$	$kg \cdot m^{-1} \cdot s^{-2}$ = pascal = Pa

Derived

مشتقة

مساحة Area = $L \times L = [L^2]$

unit $m \times m = m^2$

حجم Volume = $L \times L \times L = [L^3]$

$m \times m \times m = m^3$

$m/s^2 = ms^{-2}$

تردد frequency = $\frac{1}{T} = [\frac{1}{T}] = T^{-1}$

Dimention analysis

① للتأكد من صحة المعادلات

LHS = RHS معادلة صحيحة

LHS ≠ RHS معادلة غير صحيحة

* Check if the following relation is corect

$\frac{L}{T}$

LT^{-1}

$v = v_0 + at$

$\frac{\text{سرعة}}{\text{زمن}} = \frac{L}{T^2} = LT^{-2}$

$[\frac{L}{T}] = [\frac{L}{T}] + [\frac{L}{\cancel{T^2}} \cancel{T}]$

Correct

LHS = RHS

$LT^{-1} = LT^{-1}$

② نستخدم تحليل الابعاد من اجل التعرف على الابعاد الخاصه بأي كميه فيزيائيه جديده

ma

MLT^2

$\frac{ML}{T^2}$

$d \times d$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{\left[\frac{M\cancel{L}}{T^2}\right]}{\left[L^{\cancel{2}}\right]} = \frac{M}{T^2 L} = M L^{-1} T^{-2}$$

③ يستخدم تحليل الابعاد في صياغه المعادلات الجديده

$$KE \propto m \; v$$

$$\left[\frac{M L^2}{T^2}\right] \propto [M]^1 \left[\frac{L}{T}\right]^2$$

$$K \propto m \, v^2$$

استخدمت معلومات الابعاد لصياغة معادلة

Dimension Analysis

تحليل الابعاد

Dimensional analysis is the study of the relation between physical quantities based on their units and dimensions.

دراسة العلاقة بين الكميات بالاعتماد على الابعاد و الوحدات

Dimensional analysis can be used:

للتأكد من صحة المعادلة

١ 1- To check if a relationship is incorrect. A simple rule applies here we add or subtract quantities only If they have the same dimensions (we do not add centimeters and hours). This implies that the quantities on each side of an equation must also be the same.

يستخدم لتحديد ابعاد اي كمية

٢ -Dimensional analysis can also be used to specify the dimensions of a physical quantity.

٣ -Dimensional analysis can help phrase a physical relationship which we can see in the following examples. This can be applied by following steps:

تستخدم لصياغة العلاقات باتباع الخطوات التالية

a) **Covert the proportional formula to an equation setting each variable on the right-hand-side (R.H.S) to a power of unknown value.**

في الطرف اليمين نضع لكل كمية اس غير معروف

b) **Analysing the dimensions of both sides of the equation in terms of base dimensions (into brackets) with respect to all mathematical operations.**

مقارنة الاسس على كلا الطرفين للعمليات الرياضية

c) **Substitute all base units for variables**

تعويض الكميات الاساسية

d) **Equate the powers on both sides of the equation to each other.**

e) **Rewrite the equation in the new form.**

طرفي معادلة تساوي الاسس على الطرفين

اعادة كتابة المعادلة

تأكد اذا كانت هذه المعادله صحيحه ام لا

Example 11: Check if the following equation is correct using dimensional analysis:

$V = v_0 + ½ a t^2$

Where v is the speed of an object after a time t, v_0 is the objects initial speed, and the object undergoes an acceleration a.

Solution:

Let's do a dimensional check to see if this equation is correct, note that the numerical factors, like ½ do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of speed are $\left[\frac{L}{T}\right]$ and the dimensions of acceleration are $\left[\frac{L}{T^2}\right]$

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2]$$

$$\stackrel{?}{=} \left[\frac{L}{T}\right] + [L]$$

$$\text{Dimensions of } L.H.S. \neq R.H.S$$

The dimensions are incorrect on the right-hand side, we have the sum of the quantities whose dimensions are not the same. We conclude that an error was made in the derivation of the equation.

Ex 11

$$v = v_0 + \frac{1}{2}at^2$$

$$\left[\frac{L}{T}\right] = \left[\frac{L}{T}\right] + \left[\frac{L}{\cancel{T^2}}\right]\left[\cancel{T^2}\right]$$

$$\left[\frac{L}{T}\right] = \left[\frac{L}{T}\right] + [L]$$

LHS $\neq$ RHS   This relation is not correct

v = velocity
v_0 = velocity
a = acceleration
t = time

E 12 (i)

$$\frac{1}{2}mv^2 = mgh$$

$$[M]\left[\frac{L}{T}\right]^2 = [M]\left[\frac{L}{T^2}\right][L]$$

$$\left[\frac{ML^2}{T^2}\right] = \left[\frac{ML^2}{T^2}\right]$$

LHS = RHS   the equation is correct

m: mass $[M]$
v: velocity $\left[\frac{L}{T}\right]$
g: acceleration $\left[\frac{L}{T^2}\right]$
h: highth L

Ex 12 (ii)

$$t = 2\pi\sqrt{\frac{l}{g}}$$

$$[T] = \sqrt{\left[\frac{\cancel{L}}{\frac{\cancel{L}}{T^2}}\right]}$$

$$[T] = \sqrt{[T^{\cancel{2}}]}$$

$[T] = [T]$
LHS = RHS
this equation is correct

t: time $[T]$
l: length $[L]$
g: acceleratio $\left[\frac{L}{T^2}\right]$

Example 12

Check if the following equations are correct using dimensional analysis:

Let us consider the equation given below,

$$\frac{1}{2}mv^2 = mgh$$

The dimensions of the LHS and the RHS are calculated

$$\text{LHS: } [M]\left[LT^{-1}\right]^2 = [M][L^2T^{-2}] = [ML^2T^{-2}]$$

$$\text{RHS: } [M][LT^{-2}][L] = [ML^2T^{-2}]$$

As we can see, the dimensions of the LHS and the RHS are the same. Hence, the equation is consistent.

(ii) $t = 2\pi\sqrt{l/g}$

Here, Dimensions of L.H.S, $t = [T^1] = [M^0L^0T^1]$

Dimensions of the terms on R.H.S

Dimensions of (length) $= [L^1]$

Dimensions of g (acc due to gravity) $= [L^1T^{-2}]$

2π being constant have no dimensions.

Hence, the dimensions of terms $2\pi\sqrt{l/g}$ on R.H.S

$= (L^1/\ L^1T^{-2}]\ )^{1/2} = [T^1] = [M^0L^0T^1]$

Thus, the dimensions of the terms on both sides of the relation are the same i.e., $[M^0L^0T^1]$.Therefore, the relation is correct.

Example 13: Using dimensional analysis try to obtain the units of work?

Solution:

Work equals force by distance: $\mathrm{W} = Fx$

Force is given by Newton's Second Law: $F = ma$

$$\mathrm{W} = Fx \Rightarrow \mathrm{W} = max$$

Using dimensional analysis:

$$M\frac{L}{T^2}L = ML^2T^{-2}$$

The unit of work in SI units: $kg\ m^2/s^2$

Work = Force x distance

$W = F\ x$

$W = m a x$

$W = M \frac{d}{T^2} d$

$W = \left[\frac{M d^2}{T^2}\right] = \left[M d^2 T^{-2}\right]$

unit $= kg\ m^2/s^2 = kg\ m^2 s^{-2}$

Example 14: **To keep an object moving in a circle at constant speed requires a force called the "centripetal force ". Use the dimensional analysis to predict the formula of centripetal force F, if you know that F depends on its mass m, its speed v, and the radius r of its circular path.**

قوة مركزية

تنبؤ

القوة المركزية تعتمد على السرعة، الكتلة، نصف القطر

Solution: Using the steps mentioned in previous slide,

١-Covert the proportional formula to an equation setting each variable on the right-hand-side (R.H.S) to a power of unknown value:

$$F = m^a v^b r^c$$

٢- Analyse the dimensions of both sides of the equation in terms of base dimensions (into brackets) with respect to all mathematical operations:

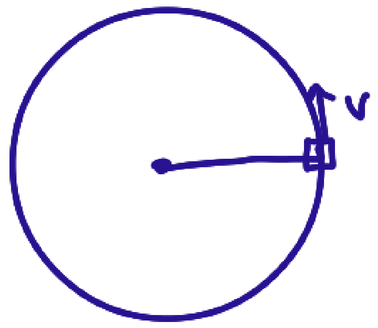
$$F = MLT^{-2} \qquad , m = M \quad , v = LT^{-1} \quad , r = L$$

٣ -Substitute all base units for variables:

$$L.H.S. \Rightarrow \qquad MLT^{-2} = M^a \ (LT^{-1})^b \ L^c \qquad \Leftarrow R.H.S.$$

$$= M^a L^b T^{-b} L^c$$

$$MLT^{-2} = M^a L^{b+c} T^{-b}$$



4-Equate the powers on both sides of the equation to each other:

4-Equate the powers on both sides of the equation to each other:

$$a = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots. \quad (1)$$

$$b + c = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

$$-b = -2 \Rightarrow b = 2 \ldots\ldots\ldots. \quad (3)$$

Substituting 3 in 2

$$2 + c = 1$$

$$c = -1$$

5-Rewrite the equation in the new form:

$$F = mv^2r^{-1} \quad \longrightarrow \quad F = \frac{mv^2}{r}$$

Ex 14

قوه مركزية

$F \begin{cases} \to m & \text{mass} & [M] \\ \to v & \text{velocity} & \left[\frac{L}{T}\right] \\ \to r & \text{radius} & [L] \end{cases}$

1

$$F = m^a v^b r^c$$

كتابه المعادله ووضع اسس مجهوله للمتغيرات

2

$$F = m^a v^b r^c$$

نقوم بتعويض الابعاد الخاصه بجميع المتغيرات

$$\left[\frac{ML}{T^2}\right] = \left[M^a \frac{L^b}{T^b} L^c\right]$$

3

نقوم بمساواه الاسس لكل كمية طرفى كل اس مجهول

$$\left[M L T^{-2}\right] = \left[M^a L^{b+c} T^{-b}\right]$$

$M = M^a$ $\boxed{a = 1}$ ✓

$T^{-2} = T^{-b}$ $\boxed{b = 2}$

$L = L^{2+c}$

$1 = 2 + c$ $1 - 2 = c$

$\boxed{c = -1}$

4

اعاده كتابه المعادله مع وضع الاس

$$\boxed{F = m v^2 r^{-1}}$$

$$F = \frac{m v^2}{r}$$

Homework

True or False questions

1. The SI system has only three base units.

2. The definition of the standard for length is based on the speed of light in vacuum**.**

3. The new definition of the second is based on the period of the Earth's rotation.

Multichoice Questions:

4. Which one among the following physics quantities is a derived quantity:

a- The mass of the proton

b- The radius of the Earth

c- The speed of sound

d- The period of oscillation of a simple pendulum

5. Given that the force can be written as $F = ma$, where m is the mass and a is the acceleration, the SI unit for the force is:

a- $kg\ m\ s$

b- $kg\ m/s$

c- $kg\ m/s^2$

d- $kg\ m\ s^2$

Problems:

6. The distance between two cities is d=525 km. Given that 1mile=1.609 km, convert this distance into miles.

7. A plant can grow 2.5 inches in 5 days, what is its growth rate in millimetres per hour. (1 in=2.54 cm)