

-1 -1 j = J-1 عدد ۲۶۵، عد Tm Re X+jy **Complex Numbers for AC Circuits** ارتاع وكيم  $\sqrt{q} = 3$ J-q = J-1 Ja = 3,1

الرياضيات المستخدمة في هندسة الإلكترونيات لجمع المقاومات والتيارات أو الفولتية المستمرة تستخدم ما يسمى "الأرقام الحقيقية" إما كأعداد صحيحة أو كسور.

الكن الأعداد الحقيقية ليست هي النوع الوحيد من الأعداد الذي نحتاج إلى استخدامه خاصة عند التعامل مع المصادر والمتجهات الجيبية المعتمدة على التردد.

 بالإضافة إلى استخدام الأعداد العادية أو الحقيقية، تم تقديم الأعداد المركبة للسماح بحل المعادلات المعقدة باستخدام الأعداد التي تمثل الجذور التربيعية للأعداد السالبة، √-1.

●في هندسة الإلكترونيات يسمى هذا النوع من الأرقام "رقم وهمي" ولتمييز الرقم التخيلي عن الرقم الحقيقي، يتم استخدام الحرف "j" المعروف عادة في هندسة الإلكترونيات باسم العامل.j

•يتم وضع حرف j أمام الرقم الحقيقي للدلالة على عملية الرقم التخيلي الخاص به. ومن أمثلة الأرقام التخيلية j 100 ; j 100 إلخ.

ثم يتكون العدد المركب من جزأين مختلفين ولكنهما متر ابطان إلى حد كبير، "الرقم الحقيقي" بالإضافة إلى "الرقم التخيلي

تمثل الأرقام المركبة نقاطًا في مركب أو مستوى ثنائي الأبعاد يتم الرجوع إليها إلى محورين مختلفين. ويسمى المحور الأفقي "المحور الحقيقي" بينما يسمى المحور الرأسي "المحور التخيلي". يتم اختصار الأجزاء الحقيقية والتخيلية للعدد المركب بـ (Re(z و (m(z)على التوالي.

ويمكن جمع الأعداد المركبة التي تتكون من أرقام حقيقية (المكون النشط) وأرقام خيالية (المكون
 التفاعلي) وطرحها واستخدامها بنفس الطريقة التي يتم بها استخدام الجبر الأولي لتحليل دوائر التيار
 المستمر

القواعد والقوانين المستخدمة في الرياضيات لجمع أو طرح الأعداد التخيلية هي نفسها المستخدمة في
 الأعداد الحقيقية، j4 = j6 + 2 إو ما إلى ذلك.

الفرق الوحيد هو في الضرب لأن ضرب رقمين وهميين معًا يصبح رقمًا حقيقيًا سالبًا. يمكن أيضًا اعتبار الأعداد الحقيقية عددًا مركبًا ولكن بجزء وهمي صفري يسمى." jo

#### **Complex Numbers and Phasors**

The mathematics used in Electronics Engineering to add together resistances, currents or DC voltages use what are called (real numbers') either as integers or as fractions.

But real numbers are not the only kind of numbers we need to use especially when dealing with frequency dependent sinusoidal sources and vectors.

As well as using normal or real numbers, Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers,  $\sqrt{-1}$ .

In Electronics Engineering this type of number is called an "imaginary number" and to distinguish an imaginary number from a real number the letter "j" known commonly in Electronics Engineering as the j-operator, is used.

The letter **j** is placed in front of a real number to signify its imaginary number operation. Examples of imaginary numbers are: j3, j12, j100 etc. Then a complex number consists of two distinct but very much related parts, a "Real Number" plus an "Imaginary Number". 5 + 6j

#### **Complex Numbers and Phasors**

Complex Numbers represent points in a two dimensional complex or splane that are referenced to two distinct axes. The horizontal axis is called the "real axis" while the vertical axis is called the "imaginary axis". The real and imaginary parts of a complex number are abbreviated as Re(z)and Im(z), respectively.

Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyseDC Circuits

The rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, j2 + j4 = j6 etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a negative real number. Real numbers can also be thought of as a complex number but with a zero imaginary part labeled j0. (2+3j) + (5+2j) = 7+5j

The basic complex (imaginary) number is "i."

To avoid confusion we replace "i" with "j"

$$j = \sqrt{-1}$$

$$j^{2} = jj = \sqrt{-1}\sqrt{-1} = -1$$

$$j^{3} = jj^{2} = j(-1) = -j$$

$$j^{4} = j^{2}j^{2} = (-1)(-1) = +1$$

$$j^{5} = jj^{4} = j(+1) = j$$

Fi = -1

The j-operator has a value exactly equal to  $\sqrt{-1}$ , so successive multiplication of "j", (j x j) will result in j having the following values of, -1, -j and +1. As the j-operator is commonly used to indicate the anticlockwise rotation of a vector, each successive multiplication or power of "j, j<sup>2</sup>, j<sup>3</sup> etc, will force the vector to rotate through an angle of <u>90° anticlockwise</u> as shown below. Likewise, if the multiplication of the vector results in a -j operator then the phase shift will be -90°, i.e. a clockwise rotation.

1) ) = 1

کلم، نفرد بر زیلی دوران کاعری بال که لمبزامه

90° rotation:  $j^{1} = \sqrt{-1} = +j$ 180° rotation:  $j^{2} = (\sqrt{-1})^{2} = -1$ 270° rotation:  $j^{3} = (\sqrt{-1})^{3} = -j$ 360° rotation:  $j^{4} = (\sqrt{-1})^{4} = +1$ 

#### **Complex Numbers using the Rectangular Form**

We saw that a complex number is represented by a real part and an imaginary part that takes the generalized form of: Where: Z = X + j Y

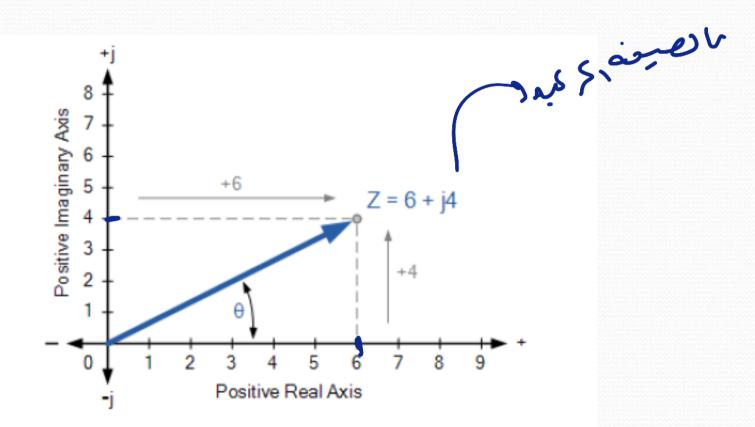
Z - is the Complex Number representing the Vector

X - is the Real part or the Active component

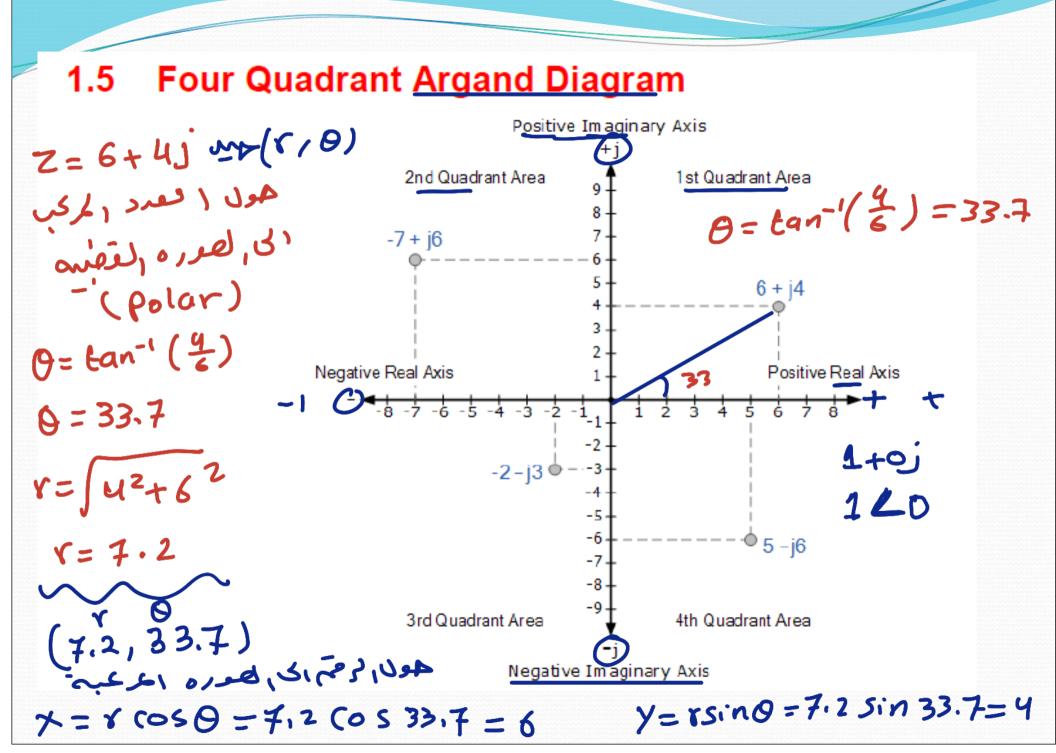
Y - is the Imaginary part or the Reactive component j - is defined by  $\sqrt{-1}$ In the rectangular form, a complex number can be represented as a point on a two-dimensional plane called the **complex** or **s-plane**.

So for example, Z = 6 + j4 represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis as shown.

### **Complex Numbers using the Complex or s-plane**



both the real and imaginary parts of a complex number in the rectangular form can be either a positive number or a negative number, then both the real and imaginary axis must also extend in both the positive and negative directions. This then produces a complex plane with four quadrants called an Argand Diagram as shown below.



On the Argand diagram, the horizontal axis represents all positive real numbers to the right of the vertical imaginary axis and all negative real numbers to the left of the vertical imaginary axis. All positive imaginary numbers are represented above the horizontal axis while all the negative imaginary numbers are below the horizontal real axis. This then produces a two dimensional complex plane with four distinct quadrants labeled, QI, QII, QIII, and QIV. The Argand diagram above can also be used to represent a rotating phasor as a point in the complex plane whose radius is given by the magnitude of the phasor will draw a full circle around it for every  $2\pi/\omega$  seconds. سیستندم مخطط (مکا مز لمتعبیر عهر کرتیس لرو ک Then we can extend this idea further to show the definition of a complex number in both the polar and rectangular form for rotations of 900.

$$0^{\circ} = \pm 360^{\circ} = +1 = 1 \angle 0^{\circ} = 1 + j0$$

$$+90^{\circ} = +\sqrt{-1} = +j = 1 \angle +90^{\circ} = 0 + j1$$

$$-90^{\circ} = -\sqrt{-1} = -j = 1 \angle -90^{\circ} = 0 - j1$$

$$\pm 180^{\circ} = (\sqrt{-1})^2 = -1 = 1 \angle \pm 180^{\circ} = -1 + j0$$

Complex Numbers can also have "zero" real or imaginary parts such as: Z = 6 + j0 or Z = 0 + j4. In this case the points are plotted directly onto the real or imaginary axis. Also, the angle of a complex number can be calculated using simple trigonometry to calculate the angles of right-angled triangles, or measured anti-clockwise around the Argand diagram starting from the positive real axis.

- Angles between <u>0 and 900</u> will be in the first quadrant (I).
- Angles ( $\theta$ ) between 90 and 180 o in the second quadrant (II).
- The third quadrant (III) includes angles between 180 and 270 o and

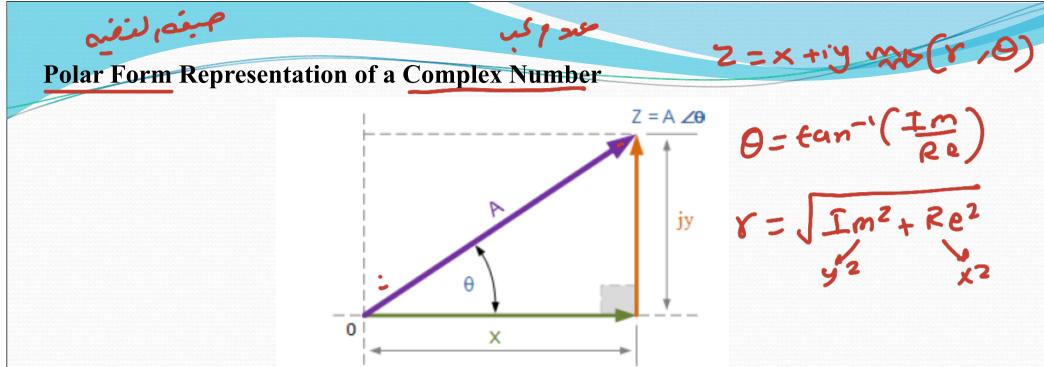
• The fourth and final quadrant ( $\underline{IV}$ ) which completes the full circle, includes the angles between 270 and 3600 and so on.

In all the four quadrants the relevant angles can be found from: **tan-1(imaginary component** ÷ **real component**)

#### **Complex Numbers using Polar Form**

Unlike rectangular form which plots points in the complex plane, the Polar Form of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as:  $Z = A \ge \pm \theta$ , where: Z is the complex number in polar form, A is the magnitude or modulo of the vector and  $\theta$  is its angle or argument of A which can be either positive or negative. The magnitude and angle of the point still remains the same as for the rectangular form above, this time in polar form the location of the point is represented in a "triangular form" as shown below.

Z=X+0j auges suble نفع کا محور X ی زہ 180 کے 1821 کہ 100' ل + 0=2 لَعَعَ كَار عور ال التخلسه مقط < -90290 1981 8.1  $\theta = \tan^{-1}\left(\frac{Im}{I^2}\right)$ 0 (360)



As the polar representation of a point is based around the triangular form, we can use simple geometry of the triangle and especially trigonometry and Pythagoras's Theorem on triangles to find both the magnitude and the angle of the complex number. As we remember from school, trigonometry deals with the relationship between the sides and the angles of triangles so we can describe the relationships between the sides as:  $A^2 = x^2 + y^2$ 

توبل ۲ معر (۲،۵) معد (۲،۵) رور (۲،۵)  $A = \sqrt{x^2 + y^2}$ Also,  $x = A.\cos\theta$ ,  $y = A.\sin\theta$ X=r Cos O Using trigonometry again, the angle  $\theta$  of A is given as follows. y = rsino

 $\Delta = \tan^{-1} y$ 

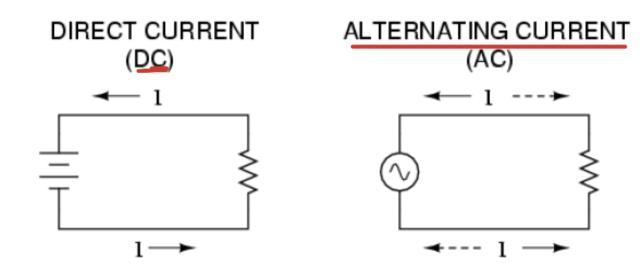
# Types of current

There are two main types of current flow in a circuit in our world. One is Direct Current (DC), which is a constant stream of electrons in one direction in other words the electric charge (current) only flows in one direction . The other is Alternating Current, which is a stream of charges that reverses direction in other words an electric current that reverses its direction many times a second at regular intervals, typically used in power supplies. Most of the digital electronics that you build will use DC. However, it is important to understand some AC concepts. Most homes are wired for AC.

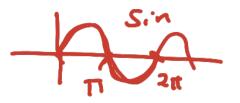
AC also has some useful properties, such as being able to convert voltage levels with a single component (a transformer), which is why AC was chosen as the primary means to transmit electricity over long distances.

## es of current

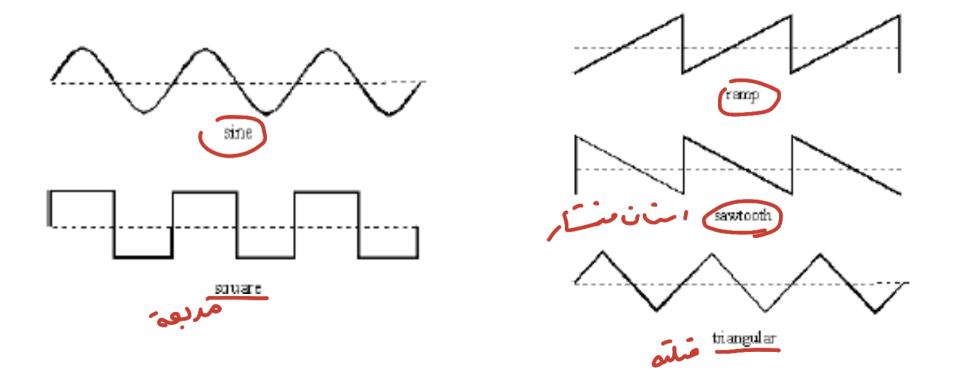
- Direct Current (DC), which is a constant stream of electrons in one direction in other words the electric charge (current) only flows in one direction. (Ac)
- Alternating Current, which is a stream of charges that reverses direction in other words an electric current that reverses its direction many times a second at <u>regular intervals</u>, typically used in power supplies.
- Most of the digital electronics that you build will use OC. However, it is important to understand some AC concepts. Most homes are wired for AC ACalso has some useful properties, such as being able to convert voltage levels with a single component (a transformer), which is why AC was chosen as the primary means to transmit electricity over long distances.



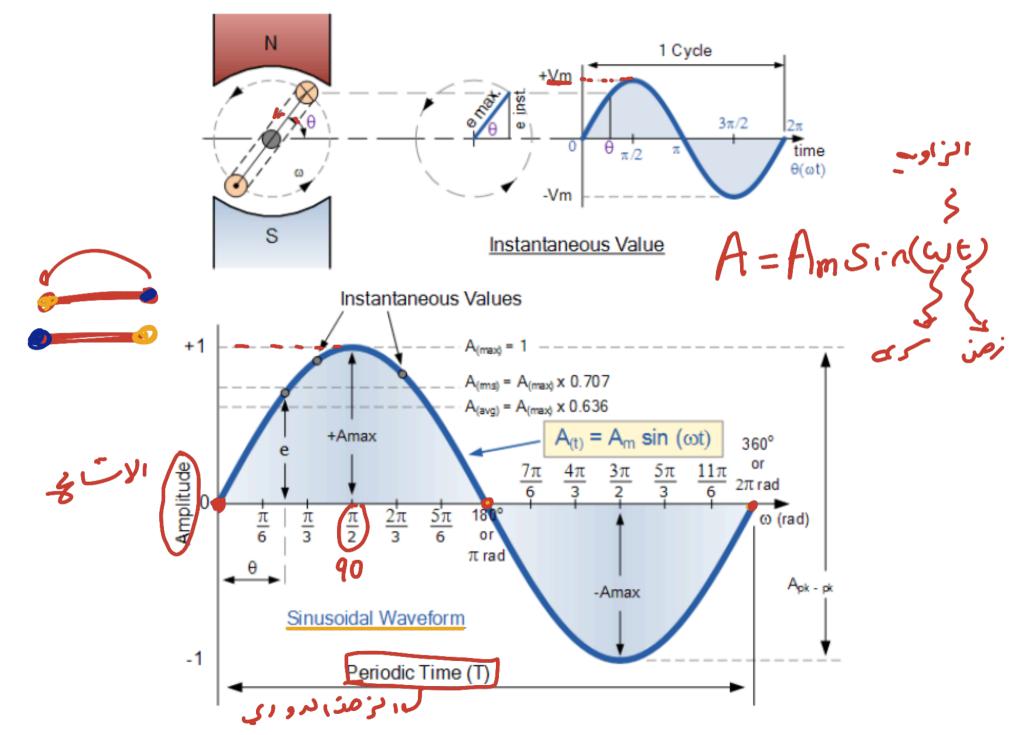
#### الموجات الدرامي Types of Periodic Waveform



A waveform is a representation of how alternating current (AC) varies with time. The most familiar AC waveform is the **sine wave**, which derives its name from the fact that the current or voltage varies with the sine of the elapsed time. Other common AC waveforms are: the square wave, the ramp, the <u>saw tooth</u> wave, and the triangular wave. Their general shapes are shown below



### **Sinusoidal Waveform**



## ( الترد د ) <u>Frequency</u> F

- التردد : عدد المدورزت غير التمانيم The frequency of a supply is the number of times a cycle appears in one second and that frequency is measured in Hertz.
  - As one cycle of induced emf is produced each full revolution of the coil through a magnetic field comprising of a north and south pole as shown above, if the coil rotates at a constant speed a constant number of cycles will be produced per second giving a constant frequency.
- By increasing the speed of rotation of the coil the frequency will also be increased.
- the frequency output from an AC generator is:

 $f \alpha N$ , and  $f \alpha P$  $\therefore f = N \times P$  in cycles/min

As frequency is measured in Hertz

AND NP ...

التردد تكون تابت

اعلنه تابه درآ

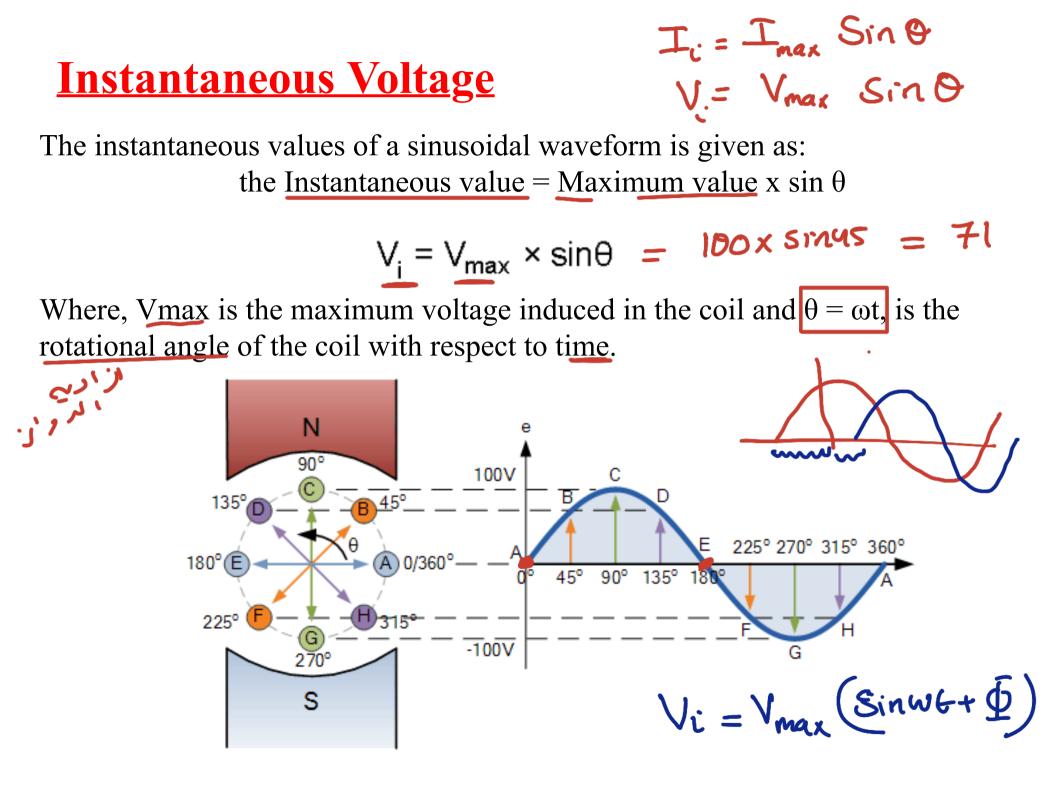
Frequency, 
$$(f) = \frac{m}{60}$$
 Hz  
Where: N is the speed of rotation in r.p.m. P is the number of "pairs of poles" and 60 converts it into seconds. Scientists describe the cycle of switching directions as the Frequency. Frequency is measured in Hertz (Hz).

#### الزهن لمروري

### الرحن اللازم لاتناء حورة

The Period, (T) is the length of time in seconds that the waveform takes to repeat itself from start to finish in other words the time taken for an AC Waveform to complete one full pattern from its positive half to its negative half and back to its zero baseline again. This can also be called the Periodic Time of the waveform for sine waves, or the Pulse Width for square waves. The Frequency, (f) is the number of times the waveform repeats itself within a one second time period (the number of complete cycles that are produced within one second (cycles/second)). Frequency is the reciprocal of the time period, (f = 1/T) with the unit of frequency being the Hertz, (Hz).

$$\begin{aligned} \mathcal{F} &= \frac{1}{T} &= \frac{1}{T} \\ \mathcal{T} &= \frac{1}{\sqrt{2}} \\ \mathcal{T} &= \frac{1}{T} \\ \mathcal{F} &= \frac{1}{T} \end{aligned}$$



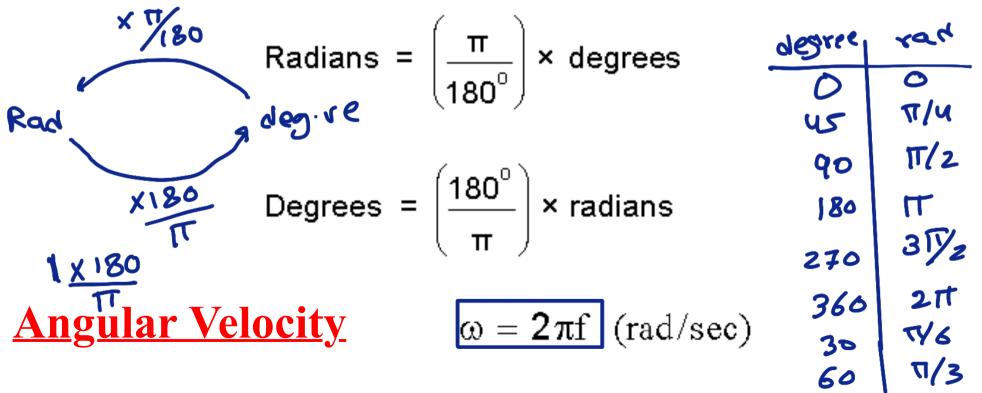
### **Definition of a Radian**

The Radian, (rad) is defined mathematically as a quadrant of a circle where the distance subtended on the circumference equals the radius (r) of the circle.

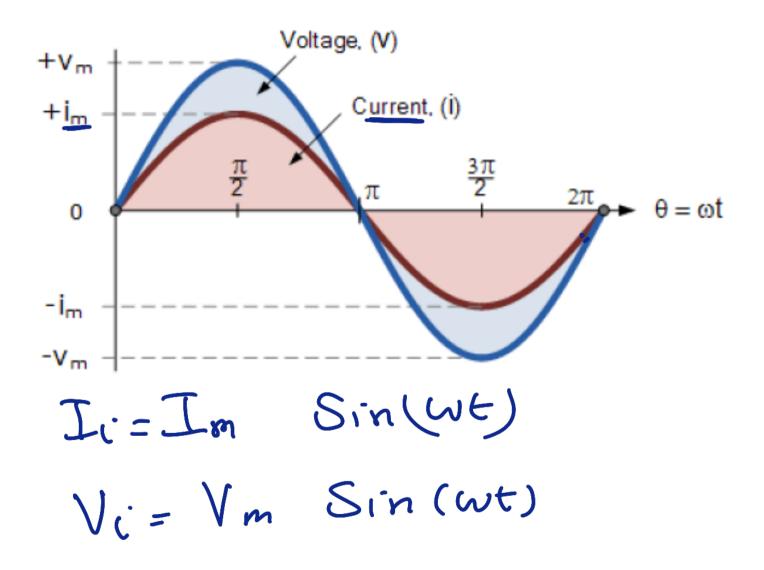
 $f = \frac{\omega}{2\pi} \quad \omega = 2\pi f$ 

Since the circumference of a circle is equal to  $2\pi \times radius$ , there must be  $2\pi$  radians around a 360 ocircle, so 1 radian =  $3600/2\pi = 57.30$ .

### **Relationship between Degrees and Radians**

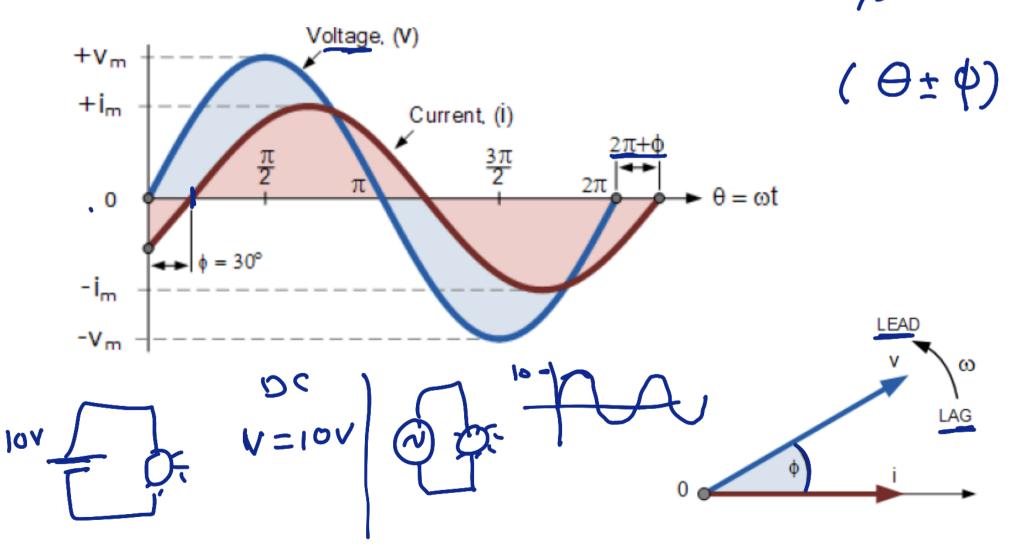


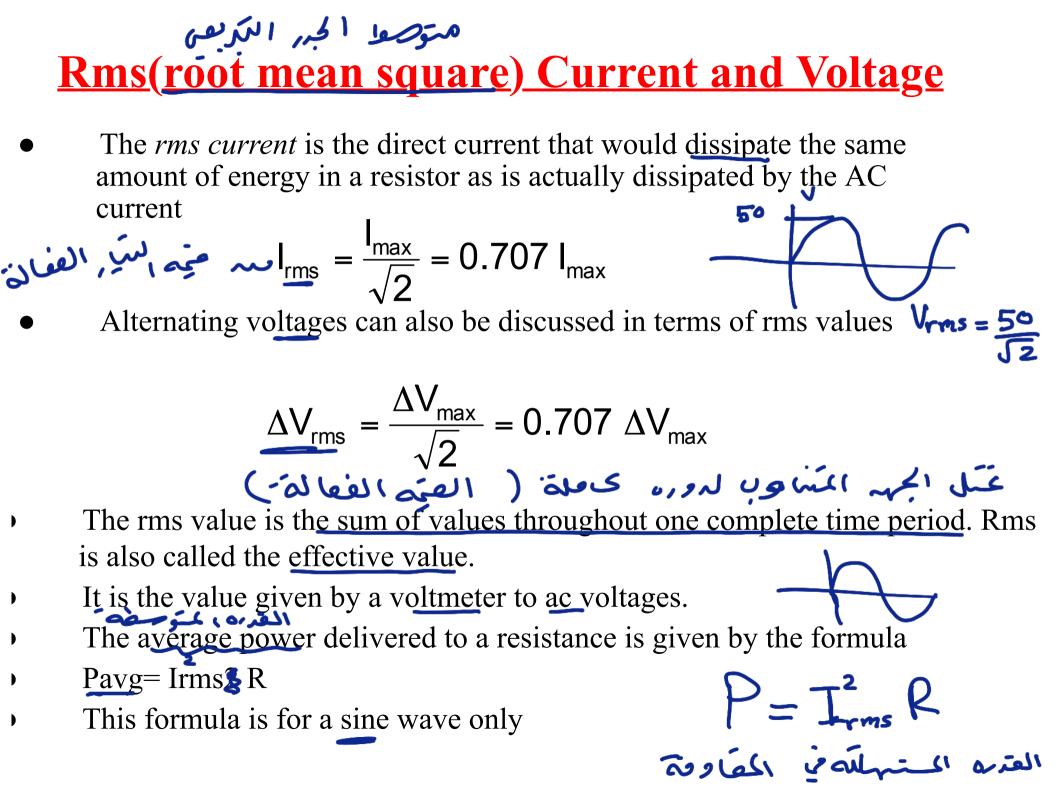




### **Phase Difference of a Sinusoidal Waveform**

The voltage waveform above starts at zero along the horizontal reference axis, but at that <u>same instant of time</u> the current waveform is <u>still negative</u> in value and does not cross this reference axis <u>until 30</u>° later. Then there exists a **Phase difference** between the two waveforms.





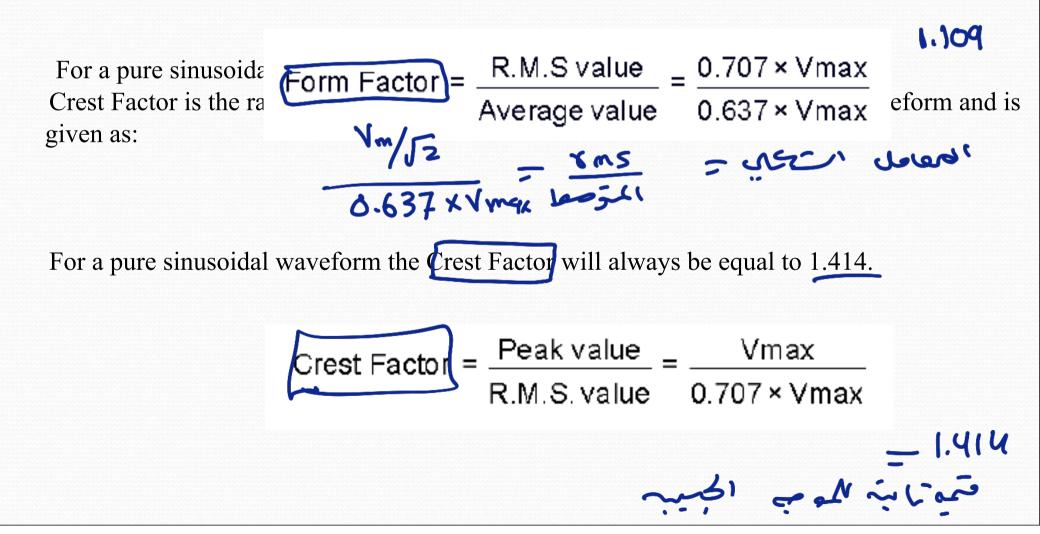
### Form Factor and Crest Factor هممدارت تعمل صلومات عمر تشكل الإعلام لمرجع

اعمامل ال

Both **Form Factor** and **Crest Factor** can be used to give information about the actual shape of the AC waveform.

Form Factor is the ratio between the average value and the RMS value and is given as:

معامل لغية



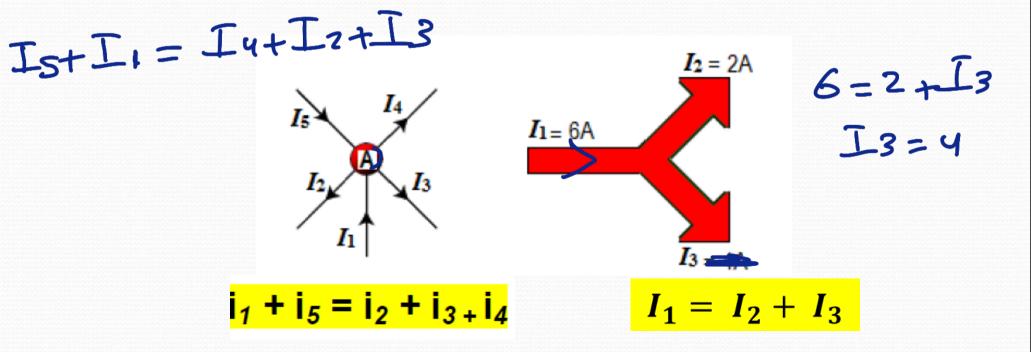
### Kirchhoff's circuit laws

قوانس كمرز

Kirchhoff's circuit laws are two equalities that deal with the conservation of charge and energy in electrical circuits, and were first described in 1845 by Gustav Kirchhoff. These "laws" are widely used in the analysis and design of electronic circuits.

القادوب المول (حفط الكنة) • Kirchhoff's current law (KCL) : The current entering any junction in a circuit network is equal to the current leaving that ما يونه تحديث الدول : عنداي مفطه تفري محموع ليكرات الداخله سيادي محمولا التيارات الخارجة junction.

منعنالهاية معنا التحنه



**Kirchhoff's Voltage Law (KVL)** 

v1 + v2 + v3 + v4 = 0

The Kirchhoff's second law stated that; In any closed path (or circuit) in a network, the algebraic sum of the IR product is equal to the EMF in that path. In other words, in any closed loop, the algebraic sum of the EMF applied is equal to the algebraic sum of the voltage drops in the elements.  $\sum_{i=1}^{n} IR = \sum_{i=1}^{n} E$ 

قادوم حيرتون التاني مجري الحبور ( RT) ت، مرج لوزه الدامن للبط، بات ( EMF) في الرا لمفلق

Kirchhoff's

The sum of all the voltages around a loop of circuit elements is equal to zero.

Ęμf

 $V_{m} = V_{m} Sin(wt)$   $I = I_{m} Sin(et)$  $V = Vm Sin(akt \phi)$   $F = Im Sin(w f + \phi)$  $V = V_{m} s_{in} (\omega t \mp \phi) = V_{rms} \angle \phi$  $V_{rmS} = \frac{V_m}{\sqrt{2}}$   $T_{rmS} = \frac{T_m}{\sqrt{2}}$ convert the following to plasor form  $50 \text{ Sinwt} = \frac{50}{\sqrt{2}} \angle 0$  $70 \text{ Sin}(wt+27) = \frac{70}{\sqrt{2}} \angle 27$  $45 \cos \omega t = 45 \sin(\omega t + 90) = \frac{45}{\sqrt{2}} \angle 90$ 

 $I_{1} = 10 \, \sin(\omega \epsilon + 60) \approx 10 \, L \, 60$   $I_{2} = 20 \, \sin(\omega \epsilon + 45) \, \approx 20245$  $I_{1+}I_{2} = 29.77 \angle 49.99$ = 29.77 Sin(Wt+49.99]

# مترنات محبر Passive Components in AC Circuits

The three main passive components used in any circuit are the: Resistor, the Capacitor and the Inductor. All three of these passive components have one thing in common, they limit the flow of electrical current through a circuit but in very different ways.

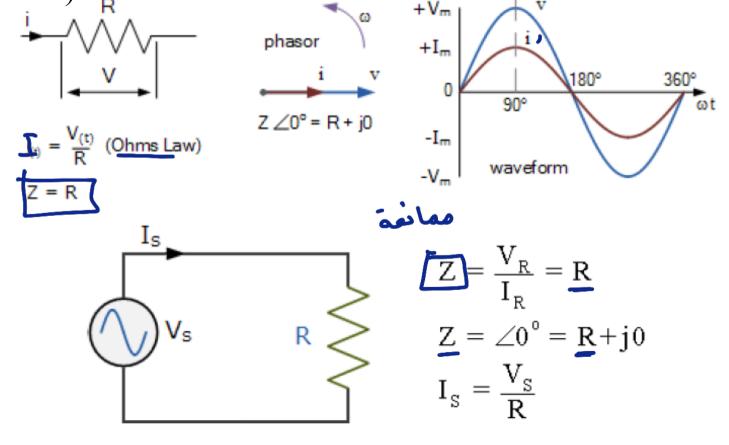
Electrical current can flow through a circuit in either of two ways. If it flows in one steady direction only it is classed as direct current, (DC). If the electrical current alternates in both directions back and forth it is classed as alternating current, (AC). Although they present an impedance within a circuit, passive components in AC circuits behave very differently to those in DC circuits.

Passive components consume electrical energy and therefore cannot increase or amplify the power of any electrical signals applied to them, simply because they are passive and as such will always have a gain of less than one. Passive components used in electrical and electronic circuits can be connected in an infinite number of ways as shown below, with the operation of these circuits depending on the interaction between their different electrical properties.

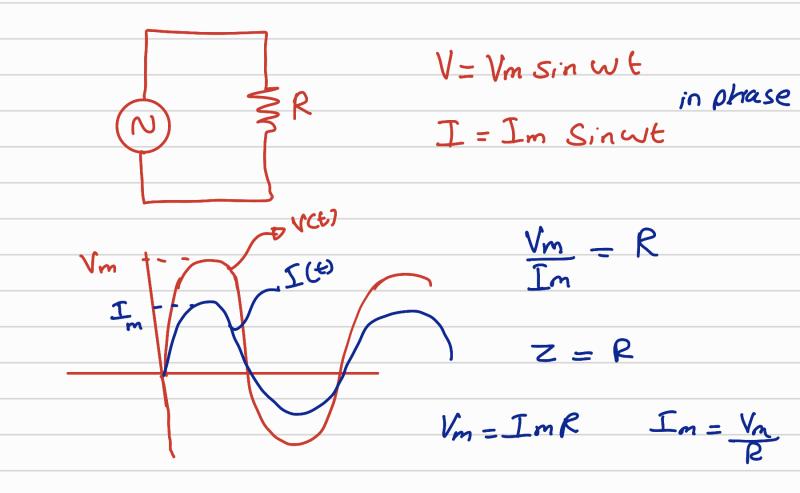
### Purely Resistive Circuit

Resistors regulate, impede or set the flow of current through a particular path or impose a voltage reduction in an electrical circuit as a result of this current flow. Resistors have a form of impedance which is simply termed *resistance*, R with the resistive value of a resistor being measured in Ohms,  $\Omega$ . Resistors can be of either a fixed value or a variable value (potentiometers).

V=IR



المكر والجهد لعلان اك اكل قيمه من من الوفر The current and the voltage reach their maximum values at the same time نغب الطور The current and the voltage are said to be *in phase* The direction of the current has no effect on the behavior of the ای در اید ای مونم علی الحل المعاومة resistor The rate at which electrical energy is dissipated in the circuit is given where i is the *instantaneous current*  $P = I_i^2 R = V_i I_i + V_i I_i$ by:  $\mathbf{y} = \mathbf{r} P = i^2 R$ rms values will be used when discussing AC currents and voltages • AC ammeters and voltmeters are designed to read rms values □ Many of the equations will be in the same form as in <u>DC</u> circuits R=2 Ohm's Law for a resistor, R, in an AC circuit  $\Box \Delta \mathbf{V}_{\rm rms} = \mathbf{I}_{\rm rms} \mathbf{R}$ □ Also applies to the maximum values of v and i



**EXAMPLE 14.1** The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10  $\Omega$ . Sketch the curves for *v* and *i*. a.  $v = 100 \sin 377t$ b.  $v = 25 \sin(377t + 60^\circ)$ 

a) 
$$V = 100 \text{ sin} (377t)$$
  
 $V = Vm \text{ sin} (wt)$   $I = Im \text{ sin} (xt)$   
 $Im = \frac{Vm}{R} = \frac{100}{10} = 10 \text{ A}$   $I = 10 \text{ sin} (377t)$   
a)  $V = 25 \text{ sin} (377t + 60)$   $I = \frac{V}{R}$   
 $I = 2.5 \text{ sin} (377t + 60) = \frac{25}{10}$ 

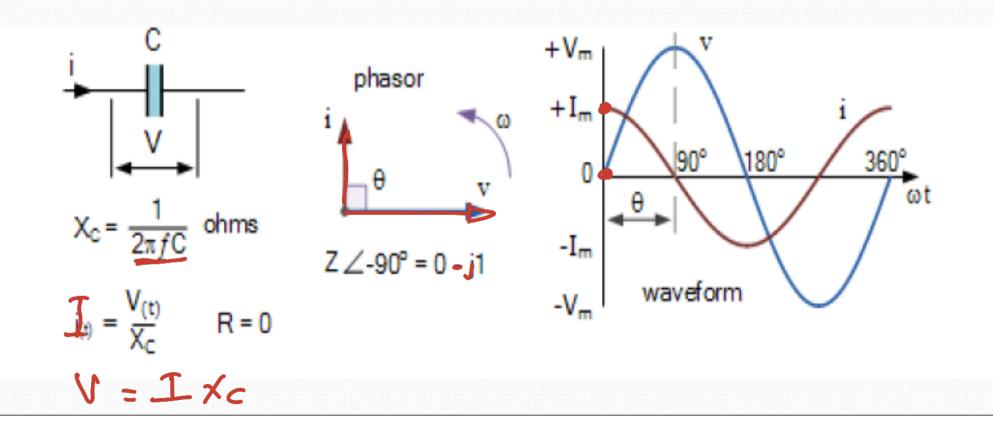
#### تنزيف الطاقة The capacitor is a component which has the ability or "capacity" to store energy in the form of an electrical charge like a small battery. The capacitance value of a capacitor is measured in Farads, F.

At DC a capacitor has infinite (open-circuit) impedance, (XC) while at very high frequencies a capacitor has zero impedance (short-circuit).

**v** Capacitive Circuit

Pure

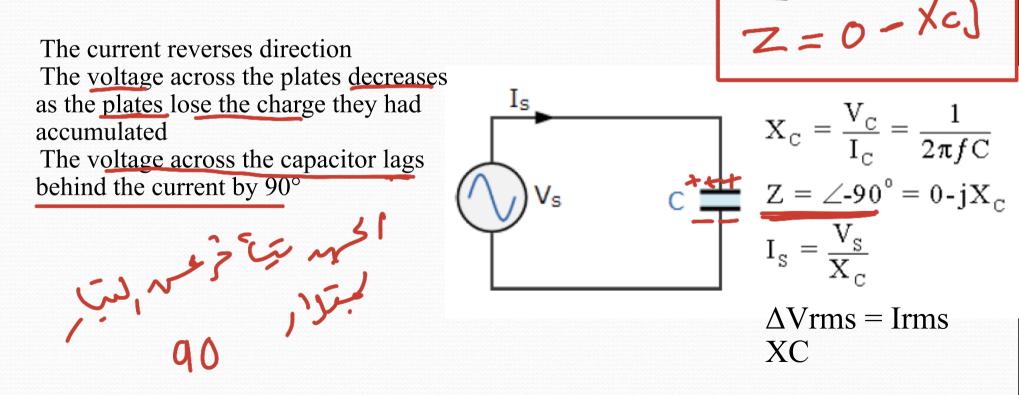
عذم لميود التردر عابي تكرما لغه الملحت فيد (داره فق)

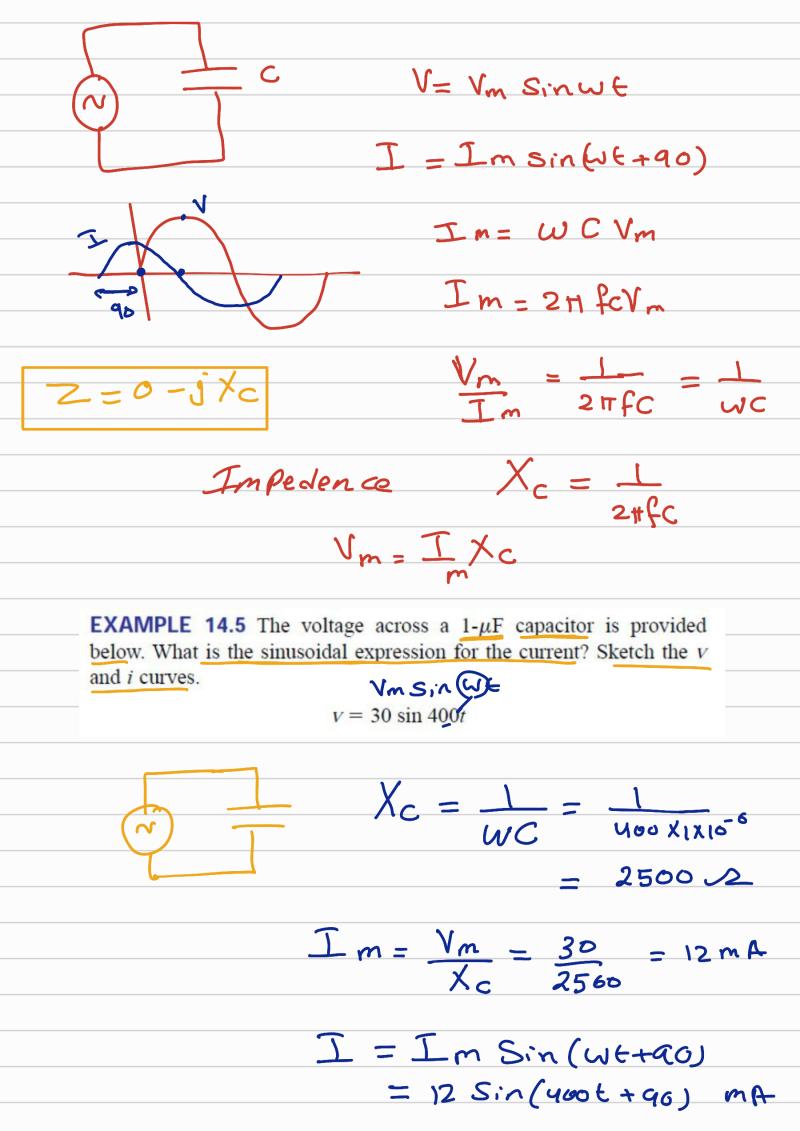


The current starts out at a large value and charges the plates of the capacitor خي البراي مح معط نف لانه لانه لا بوج مسلف لانه عنه وجرستان مي لحق There is initially no resistance to hinder the flow of the current while the plates are not charged

in Provident and the series last

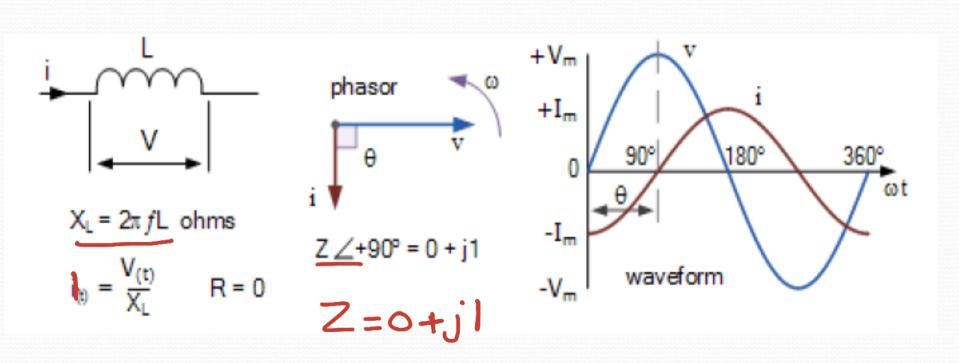
As the charge on the plates increases, the voltage across the plates increases and the current flowing in the circuit decreases

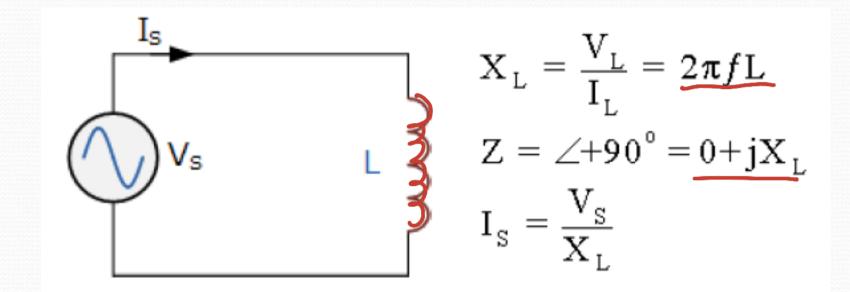




### **Purely Inductive Circuit**

- An inductor is a coil of wire that induces a magnetic field within itself or within a central core as a direct result of current passing through the coil. The inductance value of an inductor is measured in Henries, H.
- At DC an inductor has zero impedance (short-circuit), while at high frequencies an inductor has infinite (open-circuit) impedance. عدما يكون المتردد عاي تلوز معانف الحبرما يكي عدما تلون المتردد فنخف تتون (يديوج حمايف)





- Consider an AC circuit with a source and an inductor
- The current in the circuit is impeded by the back emf of the inductor
- The voltage across the inductor always leads the current by 90°
   ۹٥ من من مه

### **Inductive Reactance and Ohm's Law**

• The effective resistance of a coil in an AC circuit is called its *inductive reactance* and is given by

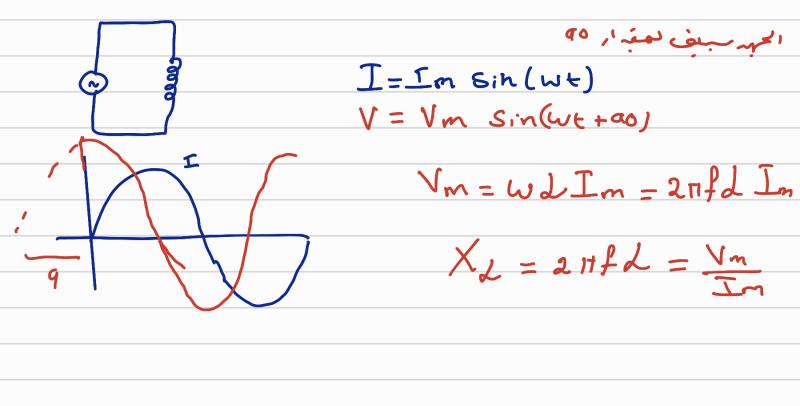
 $\times$  = 2 $\pi$ fL

When f is in Hz and L is in H, XL will be in ohms

- Ohm's Law for the inductor
- $\Delta Vrms = Irms XL$

Xx = SL

V=IXX



**EXAMPLE 14.3** The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and *i* curves. a.  $i = 10 \sin 377t$ 

 $X_{d} = Wd = 377 (0.1) = 37.7$ a)  $V m = Im X_{L} = 10 (37.7) = 377 V$  $V = 377 \sin(773t + 90)$ 

