Logic:

Propositional Logic

المبارات المنطفيه

Agenda

- Propositions

- Connectives
  - Negation
  - Conjunction
  - Disjunction

- Truth Tables

Propositions المحمل الخبرية

- A ***proposition*** is a statement that is either true or false.

جملة تحتمل الصواب او الخطأ

ومثله على جمل خبرية

- Examples of propositions:
  a) The Moon is made of green cheese.   [FALSE] ✗   F
  b) Makkah is the Holy City of Islam.   [TRUE] ✓   T
  c) Madina is the capital of Saudi Arabia.   [FALSE] ✗
  d) $1 + 0 = 1$ [TRUE] ✓
  e) $0 + 0 = 2$ [FALSE] ✗

امثله على جمل ليست خبرية

- Examples that are **not** propositions.
  a) Sit down! أمر ✗
  b) What time is it? سؤال ✗
  c) $x + 1 = 2$ ✗
  d) $x + y = z$ ✗

لا نعتبر الجمله خبريه
اذا كان فيها سؤال
او امر او لا يمكن
تحديد صوابها من خطأ

الجمل الخبرية المركبة

Compound Propositions

الجملة الخبرية المركبة مكونه من اكثر من جملة موصوله بأدوات الوصل

- A ***compound proposition*** is constructed from other propositions using logical connectives.

ادوات الربط المنطقية

- Example:

  If p it is raining outside then q we are having dinner at home

  اذا اصطوت صوف نتعشى في المنزل

  If (P) then (qr)

  o Proposition p : it is raining outside
  o Proposition q : we are having dinner at home
  o A new proposition:

    If it is raining outside, then we are having dinner at home

- We call p and q ***propositional variables***

  متغيرات خبرية

Propositional Logic

- **Logical connectives**:
  - نفي Negation $\neg$
  - و Conjunction $\wedge$
  - أو Disjunction $\vee$
  - الزم Implication $\rightarrow$ (Conditional)
  - حرف شرطية Biconditional $\leftrightarrow$ (Equivalence)

NOT

AND

OR

IMPLIES يؤدي

IFF
IF and only
IF

P: It is raining

$\neg P$: It is not raining

It is Friday and it is raining

$$P \wedge q$$

It is Friday or it's raining

$$P \vee q$$

If it is raining then we will

Stay at home

$$P \rightarrow q$$

$p \wedge q$

$p \vee q$

Truth Tables جدول الصدق

- In order to analyze any compound proposition, we use truth tables.

لتحليل اي عبله مركب سنتخدم جدول الصدق T و F

- A ***truth table*** is a table which lists all possible values of the propositional variables in a compound proposition.
  - Each variable can be either True or False.
  - The number of rows in the table with n variables = 2^n

في جدول الصدق يتم وضع كل الاحتمالات الممكنه للجمل المركبة

ماعدد الصفوف في جدول صدق مكون من جمله مركبه من جملتين

$2^2 = 4$ صفوف

Compound Propositions: Negation النفي

- The **negation** of a proposition p is denoted by $\neg p$ and it is read as "not p"

$2^1 = 2$

- It has the following truth table:

p	¬p
T	F
F	T

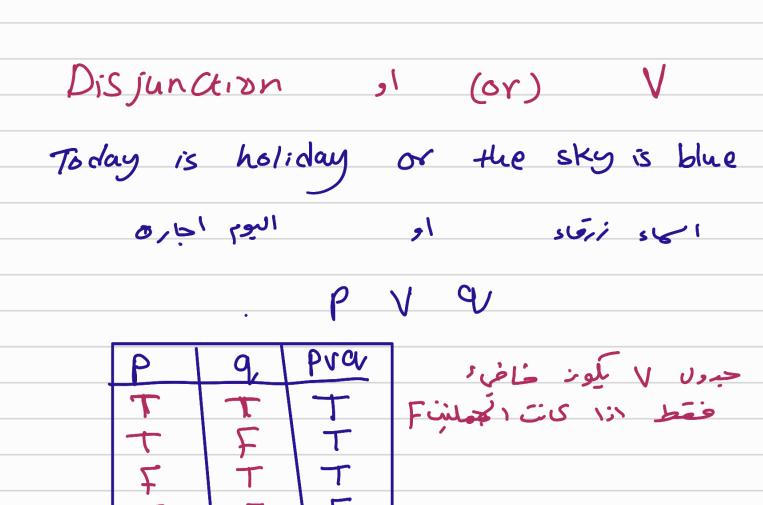- **Example**:
  p : The earth is round.
  $\neg p$: It is *not the case that* the earth is round
  or The earth is *not* round."

Negation   (التفي)   ¬

P : Today is holiday

¬P : Today is not holiday

P	¬P
T	F
F	T

جدول الصدق

Conjunction   (و)   (∧)

Today is holiday and the sky is blue
اليوم اجازه   و   السماء زرقاء

$2^2 = 4$

P ∧ q

P	q	P∧q
T	T	T
T	F	F
F	T	F
F	F	F

جدول ∧ تكون الجمله
T فقط اذا كانت
الجملتين T

Disjunction   او   (or)   V

Today is holiday or the sky is blue

اليوم اجازه   او   اسماء زرقاء

$$P \lor q$$

P	q	P∨q
T	T	T
T	F	T
F	T	T
F	F	F

يكون V خاطئ
فقط اذا كانت الجملتين F

Exclusive or   ⊕

(او) اي واحد منها وليس كلاهما

في مسابقة ربح شخص جائزه اما نكاره او غساله
وليس كلاهما .

P	q	P⊕q
T	T	F
T	F	T
F	T	T
F	F	F

Compound Propositions: Negation

انفي الجمل التالية

- Negate the following propositions:
  - It is raining today.
    - It is **not** raining today.

  - 2 is a prime number.
    - 2 is **not** a prime number

  not

  - There are other life forms on other planets in the universe.
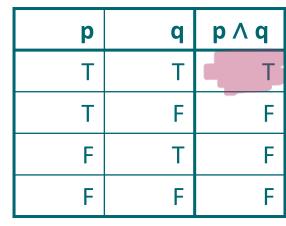   - **It is not the case that** there are other life forms on other planets in the universe.

Conjunction (و) ∧

- The ***conjunction*** of propositions p and q is denoted by $p \wedge q$ and it is read as p and q.

2^2

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

- It has this truth table:

P   q   P∧q   I am at home and its raining

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

and

Disjunction (او)   (or)   (∨)

- The **disjunction** of propositions p and q is denoted by $p \vee q$ and it is read as p or q.

- It has this truth table:

p	q	p ∨q
T	T	T
T	F	T
F	T	T
F	F	F

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining."

or

The Connective Or in English

- In English 'or' has two distinct meanings.

  - "Inclusive Or": ✓
    - Example: "Students who have taken CS202 or Math120 may take this class,"
    - We assume that students need to have taken one of the prerequisites but may have taken both. This is the meaning of disjunction. For p ∨q to be true, either one or both of p and q must be true.

      ويجوز وقوع احد شيء

      $p \vee q$

  - "Exclusive Or" ⊕
    - Example: "Soup or salad comes with this meal"
    - We do not expect to be able to get both soup and salad.

      مع صذه لوجبه تحصل على شوربه او سلطة

- "Inclusive or" means "either this, or that, or both." $p \vee q$
- "Exclusive or" means "either this or that, but **not** both." $p \oplus q$
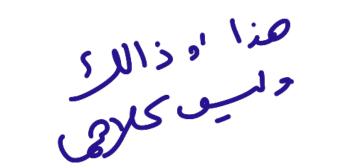
  احدحما وليس كلاهصها

Exclusive OR (XOR) ⊕

XOR

- The **exclusive OR** of propositions p and q is denoted by $p \oplus q$ and it is read as p xor q.

- It has this truth table:

صذا او ذالك
وليس كلاهما

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

- One of p and q must be true, but not both.
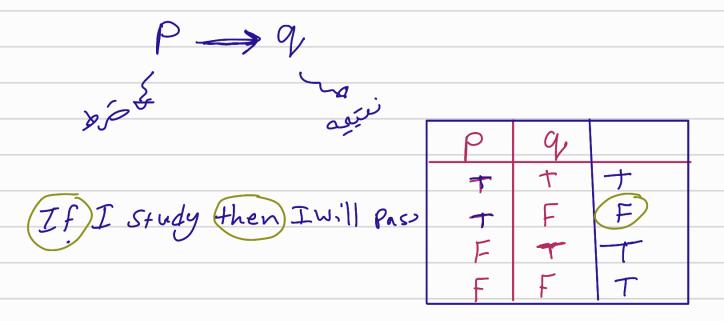
Implication

- If p and q are propositions, then $p \rightarrow q$ is a ***conditional statement*** or ***implication*** which is read as "if p, then q".

- It has this truth table:

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

- **Example**: If p denotes "I am at home." and q denotes "It is raining." then $p \rightarrow q$ denotes "If I am at home then it is raining."

- In $p \rightarrow q$, p is the *hypothesis* and q is the *conclusion*.

$$P \implies q$$

شرط   نتيجة

(If) I study (then) I will Pass

P	q	
T	T	T
T	F	F
F	T	T
F	F	T

Understanding Implication

- In $p \rightarrow q$ there does not need to be any connection between the hypothesis or the conclusion. The "meaning" of $p \rightarrow q$ depends only on the truth values of p and q.

- These implications are perfectly fine but would not be used in ordinary English.
  - If the clouds are made of cotton candy, then I have more money than Bill Gates.
  - if UQU is opened every Friday then 2 is a prime.

Understanding Implication (cont)

- One way to view the logical conditional is to think of an obligation or contract.
  - "If I am elected, then I will lower taxes."
  - "If you get 100% on the final, then you will get an A."

$$T \longrightarrow F$$

$$T \longrightarrow F \quad \times$$

✓ اذا اذا تم انتخابي صوف اخفض الضرائب
✓ اذا حصلت على 100% سوف اعطيك A

- If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

Different Ways of Expressing $p \to q$

if p, then q p **implies** q

if p, q p **only if** q

q **unless** $\neg p$ q **when** p

q **if** p

q **whenever** p p **is sufficient for** q

q **follows from** p q **is necessary for** p

شرط ضروري

a necessary condition for p **is** q

a sufficient condition for q **is** p

شرط كافي

if p then q
If p q

IF I Study I will PASS

I will Pass when I Study

I will Pass unless I didnt

انا صوف انجح الاذا لم ادرس

What is Next?

- **Read** Chapter 1 until 1.1.3 of Rosen's book.