Tarikh No.

CHAPTER 1

System of Linear Equations.

→ linear equation x one variable

$$x + 2 = 5$$
$$x = 5 - 2$$
$$x = 3 \quad \text{solution}$$

$$2x + 1 = 9$$

$$2x = 9 - 1$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4 \quad \text{solution}$$

$$\frac{1}{3}x - 1 = 11$$

$$\frac{1}{3}x = 11 + 1$$

$$3 \times \frac{1}{3}x = 12 \times 3$$

$$x = 36$$

Tarikh No.

Solve

$$3x + 2 = x + 4$$

$$3x - x = 4 - 2$$

$$\frac{2x}{2} = \frac{2}{2}$$

$x = 1$ ✓ solution

linear equation in $\overset{(n)}{2}$ variables
3

$$ax + by = c$$

$3x + 2y = 5$ ✓

$3x_1 + 2x_2 = 5$ ✓

Coefficients

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b$$

معادله خطیه ۴ متغیرات:

linear equation in 4 variables

non linear $xy = 5$

$\sin x + y = 5$

$e^y + x = 3$

هذه المعادلات لاتعتبر معادلات خطيه

Tarikh No.

Parametric Representation

$$X + y = 10$$

$$X = 10 - y$$

$$y = t$$

t any value

$$X = 10 - t$$

$t = 0$	$X = 10 - 0 = 10$
$t = 1$	$X = 10 - 1 = 9$

many solutions

$$X = 10 - t$$

Solution

x	y
5	5
9	1
1	9
9.9	0.1

set of solution

x	y = t
10	0
9	1

Ex 2 Page (3)

Solve the linear equation

$$X_1 + 2X_2 = 4$$

$$X_1 = 4 - 2X_2$$

$$X_1 = 4 - 2t$$

free parameter

$X_2 = t$

t is any real number

X_1	$X_2 = t$
4	0
0	2
2	1

$t = 0$ $\quad X_1 = 4 - 2(0) = 4$

$t = 2$ $\quad X_1 = 4 - 2(2) = 0$

$t = 1$ $\quad X_1 = 4 - 2(1) = 2$

Tarikh No.

Ex 3 Solve the linear equation

$$3x + 2y - z = 3$$

$$\frac{3x}{3} = \frac{3 - 2y + z}{3}$$

$$x = 1 - \frac{2}{3}y + \frac{1}{3}z$$

y and z are free variables

Parameters: $y \to s$, $z \to t$

$$x = 1 - \frac{2}{3}s + \frac{1}{3}t$$

s and t are any real number

x	y = s	z = t
1	0	0
	3	3
1	1	2

$$x = 1 - \frac{2}{3}(0) + \frac{1}{3}(0)$$

$$x = 1$$

$$x = 1 - \frac{2}{3}(3) + \frac{1}{3}(3)$$

$$x = 1 - 2 + 1 = 0$$

$$x = 1 - \frac{2}{3}(1) + \frac{1}{3}(2)$$

$$= 1 - \frac{2}{3} + \frac{2}{3}$$

$$= 1$$

Tarikh No.

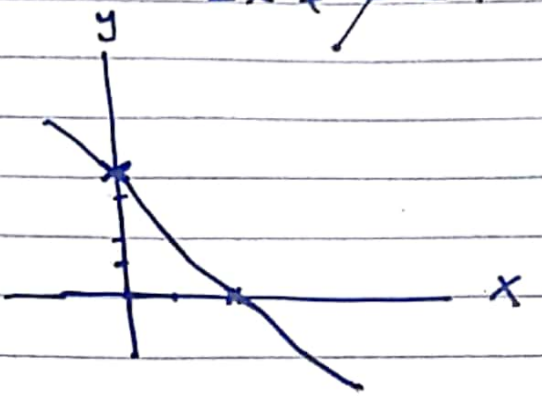
طريقة رسم البيان لمعادلة

linear equation

$2x + y = 4$

$x = 0$ $\quad 2(0) + y = 4$ $\quad \boxed{y = 4}$

$y = 0$ $\quad 2x + 0 = 4$ $\quad \frac{2x}{2} = \frac{4}{2}$ $\quad \boxed{x = 2}$



مراجعة

Tarikh No.

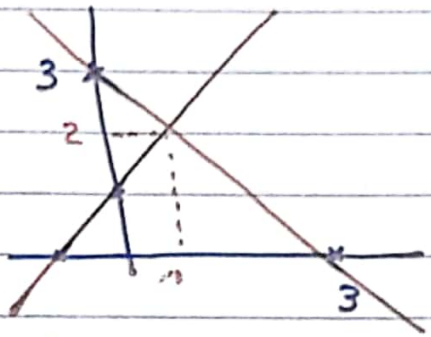
How to solve system of linear equation

* using graph

Ex4 (page 5)

(a) $x + y = 3$

$x - y = 1$



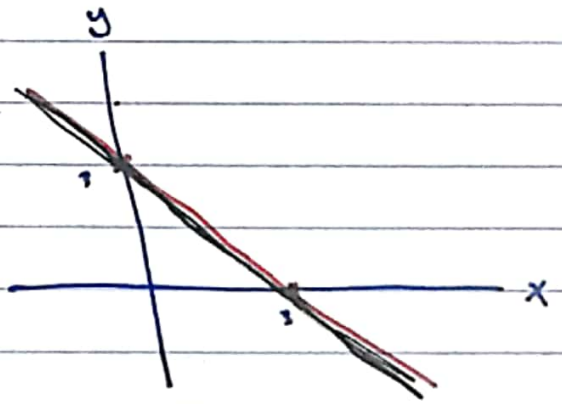
Consistent

هل المعادله هو نقطه تقاطع المستقيمين

Solution (1, 2)

(b) $x + y = 3$

$2x + 2y = 6$



consistent

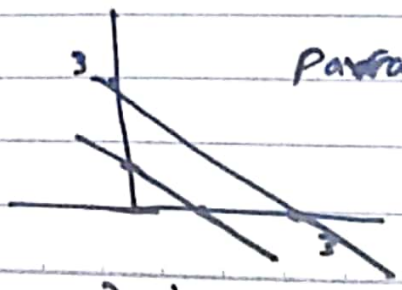
هذا تطابق المستقيمين فان هناك عدد لانهائي من الحلول

Infinit number of solutions

too many solutions

(c) $x + y = 3$

$x + y = 1$



Parallel

No solution

inconsistent

اذا كان المستقيمات متوازيين (لا يوجد حل)

Tarikh No.

Solving linear equations by back-Substitution

Solve $2y - 3x = 4$ ---①

$2x = -2$ ---②

$\frac{2x}{2} = \frac{-2}{2}$

$\boxed{x = -1}$ نعوض قيمة x في المعادلة ①

$2y - 3(-1) = 4$

$2y + 3 = 4$

$2y = 4 - 3$ $\quad 2y = 1$

$\boxed{y = \frac{1}{2}}$

$x = -1$ $\quad y = \frac{1}{2}$

$(-1, \frac{1}{2})$

Tarikh No.

Ex 6 Page 6

$$x - 2y + 3z = 9 \quad \text{---} ①$$

$$y + 3z = 5 \quad \text{---} ②$$

$$z = 2 \quad \text{---} ③$$

3 var

2 var

1 var

$\boxed{z = 2}$

نعوض في معادلة 2

$$y + 3(2) = 5$$

$$y + 6 = 5 \qquad y = 5 - 6 = -1$$

$\boxed{y = -1}$

نعوض في معادلة

$$x - 2y + 3z = 9$$

$$x - 2(-1) + 3(2) = 9$$

$$x + 2 + 6 = 9$$

$$x + 8 = 9 \qquad \boxed{x = 1}$$

$x = 1 \qquad y = -1 \qquad z = 2$

Tarikh No.

The system is in row-echelon form

$$y + x = 5 \quad \leftarrow 2$$
$$y = -2 \quad \leftarrow 1$$

اذا كان نظام المعادلات بـ (row-echelon) فان
طريقة الحل المناسبة التعويض الخلفي back-substitution

System of linear equations

ROW-ECHELON Form (REF)

$$-x + y - z = 0$$
$$y + z = 3$$
$$\frac{1}{2} z = 0$$

Elimination حذف (Gaussian)

Solve

$$2x + y = 4$$
$$x - y = 2$$
$$3x = 6$$
$$x = \frac{6}{3}$$
$$x = 2 \qquad x - y = 2$$
$$2 - y = 2$$
$$-y = 0$$
$$y = 0$$

$$x = 2 \qquad y = 0$$

$$2(x - y = 1)$$
$$-2x + 2y = -2$$
$$2x - 2y = 2$$
$$0 = 0$$

To many solutions

اذا كانت الاجابة بعد الحذف عباره

$$3 = 0$$

No Solution

Tarikh No.

Ex 7 (Page 7)

$$x - 2y + 3z = 9 \quad \text{---} ①$$
$$-x + 3y = -4 \quad \text{---} ②$$
$$2x - 5y + 5z = 17 \quad \text{---} ③$$

① ②

$$\cancel{x} - 2y + 3z = 9$$
$$\cancel{-x} + 3y = -4$$

x حذف

$$y + 3z = 5 \quad \text{---} (A)$$

② ③

$$2(-x + 3y = -4)$$
$$2x - 5y + 5z = 17$$
$$-7x + 6y = -8$$

x حذف

$$y + 5z = 9 \quad \text{---} (B)$$

Ⓐ Ⓑ

$$-(y + 3z = 5)$$
$$y + 5z = 9$$
$$-y - 3z = -5$$
$$2z = 4$$
$$z = \frac{4}{2}$$
$$z = 2$$

y جاء

$$y + 3(2) = 5$$
$$y + 6 = 5$$
$$y = 5 - 6$$
$$y = -1$$

$$x - 2(-1) + 3(2) = 9$$
$$x + 2 + 6 = 9$$
$$x + 8 = 9$$
$$x = 9 - 8$$
$$x = 1$$

Ex8 (Page 9)

solve

$$x_1 - 3x_2 + x_3 = 1 \quad \text{....①}$$
$$2x_1 - x_2 - 2x_3 = 2 \quad \text{---②}$$
$$x_1 + 2x_2 - 3x_3 = -1 \quad \text{---③}$$

① ②

$-2(x_1 - 3x_2 + x_3 = 1)$ خذ

$2x_1 - x_2 - 2x_3 = 2$ x_1

$$-2x_1 + 6x_2 - 2x_3 = -2$$
$$2x_1 - x_2 - 2x_3 = 2$$

$$5x_2 - 4x_3 = 0 \quad \text{----.} \ (A)$$

① ③

$-(x_1 - 3x_2 + x_3 = 1)$ خذ

$x_1 + 2x_2 - 3x_3 = -1$ x_1

$$-x_1 + 3x_2 - x_3 = -1$$
$$x_1 + 2x_2 - 3x_3 = -1$$

$$5x_2 - 4x_3 = -2 \quad \text{-----} \ (B)$$

$$5x_2 - 4x_3 = 0$$
$$5x_2 - 4x_3 = -2$$
$$-5x_2 + 4x_3 = 0$$
$$0 = -2$$

No solution inconsistent system

Tarikh No.

Ex9 Page (10)

$$x_2 - x_3 = 0 \quad \text{---(1)}$$
$$x_1 - 3x_3 = -1 \quad \text{---(2)}$$
$$-x_1 + 3x_2 = 1 \quad \text{---(3)}$$

(1) (2)

$$-3(x_2 - x_3 = 0)$$
$$x_1 + 0 - 3x_3 = -1$$
$$+\ 0 + -3x_2 + 3x_3 = 0$$

$$x_1 - 3x_2 = -1 \quad \text{---(A)}$$

أخذ معادلة A مع معادلة 3 مباشرة

$$-x_1 + 3x_2 = 1$$
$$x_1 - 3x_2 = -1$$

$0 = 0$ infinite number of solutions

$x_3 = t$ ✓

$x_1 - 3(t) = -1$

$x_1 - 3t = -1$

$x_1 = 3t - 1$ ✓

$x_2 - t = 0$

$x_2 = t$ ✓

$x_1 = 3t - 1 \qquad x_2 = t \qquad x_3 = t$