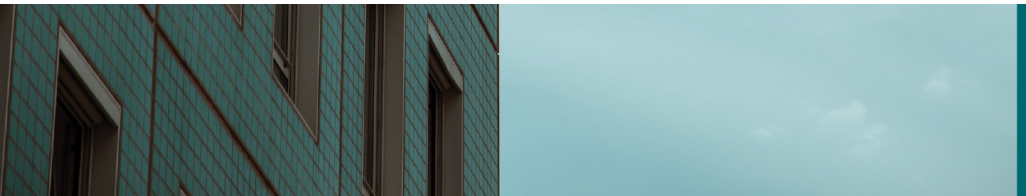
جامعة أم القرى
UMM AL-QURA UNIVERSITY

Logic:

Propositional Logic Exercises



Acknowledgment: All course slides are either referenced to Rosen Book online presentations (with certain amendments) or are personally developed by the instructors.

Exercise 2 pp 13 @[KB 8ed]

2. **Which** of these are **propositions**? **What** are the **truth values** of those that are propositions?

a) Do not pass go. x Not proposition

b) What time is it? No

c) There are no black flies in Maine. yes F

d) 4 + x = 5. No X

e) The moon is made of green cheese. yes F

f) $2^n \geq 100$. No x

Exercise 5-7 pp 13 @[KR 8ed]

5-7. What is the **negation** of each of these propositions?

5b) There is no pollution in New Jersey.

6a) Jennifer and Teja are friends.

6b) There are 13 items in a baker's dozen.

6c) Abby sent more than 100 text messages yesterday.

7c) $7 \cdot 11 \cdot 13 \neq 999$.

7d) Diane rode her bicycle 100 miles on Sunday.

Exercise 14 pp 16 @[KR 8ed]

↔ If and only If

14. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of these propositions as an **English sentence**.

b) ¬q ↔ r

c) q → ¬r

e) (p → ¬r) ∨ (q → ¬r)

b) ¬q ↔ r: You won't miss the final examination if and only if you pass the course.(Bi-conditional statement)

c) q → ¬r: If you miss the examination then you will be failing the course.(Implication Statement)

e) (p → ¬r) ∨ (q → ¬r): If you have the flu then you'll not pass the course OR If you miss the final examination then you'll fail the course.((P implies not R) OR (Q implies not R))

Exercise 16 pp 14 @[KR 8ed]

16. Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Express the following propositions using **propositional algebra**.

a) You get an A in this class, but you do not do every exercise in this book.

c) To get an A in this class, it is necessary for you to get an A on the final.

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Exercise 18-20 pp 14 @[KR 8ed]

18-20. **Determine** whether each of these statements is **true or false**.

18a) 2 + 2 = 4 if and only if 1 + 1 = 2. T ↔ T = T

18d) 0 > 1 if and only if 2 > 1. F ↔ T = F

19c) If 1 + 1 = 3, then 2 + 2 = 5. F → F = T

20d) If 1 + 1 = 2, then dogs can fly. T → F = F

Exercise 29-30 pp 16 @[KR 8ed]

$p \to q$ $q \to p$ $\neg q \to \neg p$ $\neg p \to \neg q$

29-30. **State** the **converse**, **contrapositive**, and **inverse** of each of these conditional statements.

29a) If it snows today, I will ski tomorrow.

30b) I go to the beach whenever it is a sunny summer day.

30c) When I stay up late, it is necessary that I sleep until noon.

Exercise 29-30 pp 16 @[KR 8ed]

29-30. **State** the **converse**, **contrapositive**, and **inverse** of each of these conditional statements.

29a) 1.If it snows tonight, then I will stay at home.

30b) I go to the beach whenever it is a sunny summer day.

30c) When I stay up late, it is necessary that I sleep until noon.

1.If it snows tonight, then I will stay at home.

converse =If I will stay at home then it snows tonight.

contrapositive, =If I will not stay at home then it does not snow tonight.

inverse =If it does not snow tonight, then I will not stay at home.

2.I go to the beach whenever it is a sunny summer day.

converse = It is a sunny summer day whenever I go to the beach.

contrapositive, =it is not a sunny summer day whenever I do not go to the beach.

inverse =I do not go to the beach whenever it is not a sunny summer day.

3. we can re write the statement like if i stay up late then i sleep until noon .

converse : if i sleep until noon then i stay up late .

contrapositive : if i dont sleep until noon then i did not stay up late

inverse : if i didnt stay up late then i dont sleep until noon

Exercise 31-32 pp 16 @[KR 8ed]

عدد الصفوف

31-32. **How many rows** appear in a truth table for each of these compound propositions?

31a) $p \rightarrow \neg p$. $2^n = 2^1 = 2$

32c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$. $2^6 = 64$

Exercise 33-38 pp 16 @[KR 8ed]

33-38. **Construct** a **truth table** for each of these compound propositions.

34c) $p \oplus (p \vee q)$.

36c) $p \oplus \neg p$. q

38e) $(p \vee q) \wedge \neg r$

$p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

$p \oplus \neg q$

p	q	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

$p \oplus \neg p$

p	$\neg p$	$p \oplus \neg p$
T	F	T
F	T	T

$(p \vee q) \wedge \neg r$

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \wedge \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Exercise 6 pp 38 @[KR 8ed]

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6. **Use** a **truth table** to verify the first De Morgan law.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q.$$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Exercise 8 pp 38 @[KR 8ed]

8. **Use De Morgan's laws** to **find** the **negation** of each of the following statements.

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b) Yoshiko knows Java and calculus. $\neg (J \wedge C) = \neg J \vee \neg C$

d) Rita will move to Oregon or Washington. $\neg (O \vee W) = \neg O \wedge \neg W$

Yoshiko does not know Java or does not know calculus.

Rita will not move to Oregon and will not move to Washington.

Exercise 13 pp 38 @[KR 8ed]

13. **Show** that each of these conditional statements is a tautology دائما صحيحة **without using truth tables**.

b) $p \rightarrow (p \vee q)$.

f) $\neg(p \rightarrow q) \rightarrow \neg q$.

b) $p \rightarrow (p \vee q)$

$= \neg p \vee p \vee q$

$= T \vee q$

$= T$

F) $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg\neg(p \rightarrow q) \vee \neg q$

$(p \rightarrow q) \vee \neg q$

$\neg p \vee q \vee \neg q$

$\neg p \vee T$

$= T$

Exercise 26 pp 38 @[KR 8ed]

26. **Show** that (p → q) ∧ (p → r) and p → (q ∧ r) are **logically equivalent**.

$$(p \to q) \wedge (p \to r)$$

$$(\neg p \vee q) \wedge (\neg p \vee r)$$

$$\neg p \vee (q \wedge r)$$

$$p \longrightarrow (q \wedge r)$$

Exercise 32 pp 38 @[KR 8ed]

32. **Show** that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are **logically equivalent**.

We know that $p \leftrightarrow q$ is true precisely when p and q have the same truth value. But this happens precisely when $\neg p$ and $\neg q$ have the same truth value, that is, $\neg p \leftrightarrow \neg q$.

p	q	-p	-q	p↔q	-p↔-q
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Exercise 66 pp 40 @[KR 8ed]

66. **Determine** whether each of these compound propositions is **satisfiable**.

a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$.

p	q	r	s	
T	T	T	T	✗
T	T	T	F	

$(T \vee T \vee F) \wedge (T \vee F \vee F) \wedge (T \vee F \vee F)$

$(F \vee F \vee F) \wedge (T \vee T \vee F) = T \wedge T \wedge T \wedge F \wedge T = F$

$(T \vee T \vee F) \wedge (T \vee F \vee T) \wedge (T \vee F \vee T) \wedge (F \vee F \vee T) \wedge (T \vee T \vee T)$

$T \wedge T \wedge T \wedge T \wedge T = T$

the Preposition is Statisfiable

when

P:T q:T r:T S:F