A. Choose the correct answers for the following (4) questions and fill the following table.

[4 Marks]

1	2	3	4
C	C	B	D

1. Let $A = \{x \mid x \in Z^{+} \text{ and } x^{2} \leq 10\}$, $B = \{1,2,3,4\}$ then:
   A. $A = B$
   B. $A \in B$
   C. $A \subset B$
   D. $B \subset A$

2. Let A and B be two sets, which set operation is defined by {x| x∈A and x∉B}
   A. A. A∩B
   B. B. A∪B
   C. C. A-B
   D. D. B-A

3. One of the following sets does **not** have a least element:
   A. Natural number
   B. Integers
   C. Positive Integer
   D. Positive Odd Integer

4. How many elements does the following set have. P({ 3, 4, {3, 4} })
   A. 3
   B. 4
   C. 6
   D. 8

B. Write True or False next to each statement. Any empty one will be considered as wrong answer. **[2 Marks]**

1. Let $P = T$, $R = F$, and $Q = F$, determine the truth value of $(P \to \neg Q) \wedge R$	False
2. What is the truth value of "4+3=7 XOR 5 is not prime"?	True
3. What is the truth value of "$\sqrt{2}$ *is rational if and only if* 2 *is rational*"	False
4. What is the truth value of $((p \to q) \wedge p) \to q$? where p, q are Unknown.	True

A. Choose the correct answers for the following (4) questions and fill the following table.

[4 Marks]

1	2	3	4
A	C	A	D

1. The negation of the statement:

$$\exists x\,(P(x) \rightarrow \exists y\, Q(y))$$

   A. $\forall x(P(x) \wedge \forall y\, \neg Q(y))$
   B. $\forall x(P(x) \wedge \exists y\, \neg Q(y))$
   C. $\forall x(P(x) \vee \forall y\, \neg Q(y))$
   D. None of the above

2. Consider the following statement:
   $\exists x \forall y(R(x, y) \rightarrow Q(y))$, where $R(x, y)$ denotes *"x trusts y"* and $Q(y)$ denotes *"y is honest."*
   Which of the following statements is equivalent to this nested quantifier?
   A. "There exists someone whom everyone trusts, and that person is honest."
   B. Everyone trusts at least one honest person.
   C. There exists an honest person whom everyone trusts.
   D. Everyone trusts everyone, and everyone is honest.

3. Let *P(x,y)* means *"x and y are real numbers such that x + 2y = 5"*. Determine the truth value of the statement: $\forall x \exists y P(x,y)$.
   A. True
   B. False

4. Suppose that P(x) is "if $x > 2$, then $x^2 > 4$". Find the negative of the statement $\exists x\, p(x)$:
   A. $\exists x$, if $x\ \ 2$, then $x^2 \le 4$
   B. $\forall x$, $x > 2$ *or* $x^2 \le 4$
   C. $\forall x$, $x > 2$ *and* $x^2 > 4$
   D. $\forall x$, $x > 2$ and $x^2 \le 4$

Step	**Reason**
1. w	Hypothesis
2. $w \rightarrow d$	Hypothesis
3. d	?
4. $d \rightarrow \neg j$	Hypothesis
5. $\neg j$	Modus ponens using (3) and (4)

Modus ponens using (2) and (3)

A. Choose the correct answers for the following (6) questions and fill the following table.

[6 Marks]

1	2	3	4	5	6
B	C	D	A	D	B

1. We define the following propositions: p: *"you study well "*, q: *"you solve extra tutorial "*, and r: *"you will pass this exam"* Determine the **negation** of the following sentence: " if you study well and solve extra tutorial, you will pass this exam "
   A. $p \wedge q \rightarrow r$
   B. $p \wedge q \wedge \neg r$
   C. $p \vee q \rightarrow r$
   D. $p \vee q \wedge \neg r$

2. The statement $(p \wedge q) \rightarrow (p \vee q)$ is a:
   A. Contingency.
   B. Contradiction.
   C. Tautology.
   D. None of the above.

3. The result of (1111 0100) **XOR** (0101 1111) is:
   A. 0101 0100
   B. 1111 1111
   C. 0101 0010
   D. 1010 1011

4. What is the **converse** of the statement "*I am happy whenever I am out of the city*".
   A. If I am happy then I am out of the city.
   B. If I am out of the city, then I am happy.
   C. If I am not happy, then I am not out of the city.
   D. If I am not out of the city, then I am not happy.

5. Consider the statement $p \rightarrow (\neg q \rightarrow \neg r)$ what is its **contrapositive**?
   A. $(\neg q \vee r) \rightarrow \neg p$
   B. $\neg p \rightarrow (q \rightarrow r)$
   C. $(\neg q \rightarrow \neg r) \rightarrow p$
   D. $(\neg q \wedge r) \rightarrow \neg p$

6. The **Inverse** of "*I will find a good job when I graduate from CCIS*":
   A. If I will not find a good job then I don't graduate from CCIS.
   B. If I don't graduate from CCIS then I will not find a good job.
   C. If I graduate from CCIS then I will find a good job.
   D. I will graduate from CCIS and I will find a good job.

What rule of inference is used in each of these arguments?

1) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major. This is an addition rule, with form $p \rightarrow (p \vee q)$
2) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major. This is a simplification rule. with form $(p \wedge q) \rightarrow p$
3) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed. This is modes ponens rule. with form $((p \rightarrow q) \wedge p) \rightarrow q$
4) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today. This is modes tollens rule. With form $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
5) If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn. This is a hypothetical syllogism rule. With form $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Step	Reason
1. $\neg t$	Hypothesis
2. $s \rightarrow t$	Hypothesis
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
5 ? ______	Contrapositive of (4)
6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition, using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

$$(\neg(s \wedge l)) \rightarrow \neg(\neg r \vee \neg f)$$

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all $\boxed{\text{integers.}}$

1) $\forall x(x^2 \geq x)$ This is true, so there is no counterexample.
2) $\forall x(x>0 \lor x<0)$ 0 is counter example
3) $\forall x(x=1)$ any integer other than 1 is a counterexample

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all $\boxed{\text{real numbers.}}$

1) $\forall x(x^2 \neq x)$ Since $1^2 = 1$, this statement is false; $x = 1$ is a counterexample. So is $x = 0$ (these are the only two counterexamples).
2) $\forall x(x^2 \neq 2)$ There are two counterexamples: $x = \sqrt{2}$ and $x = \sqrt{-2}$.

Let Q(x) be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

1) $Q(0)$ T
2) $Q(-1)$ T
3) $Q(1)$ F
4) $\exists x\, Q(x)$ From part (1) we know that there is at least one x that makes Q(x) true, so $\exists x\, Q(x)$ is true.
5) $\forall x\, Q(x)$ From part (3) we know that there is at least one x that makes Q(x) false, so $\forall x\, Q(x)$ is false.
6) $\exists x\, \neg Q(x)$ From part (3) we know that there is at least one x that makes Q(x) false, so $\exists x\, \neg Q(x)$ is true.
7) $\forall x\, \neg Q(x)$ From part (1) we know that there is at least one x that makes Q(x) true, so $\forall x$

1. Select the value of $\lceil -4.3 \rceil$.

- a. -4
- b. -5
- c. 4
- d. 5

2. Select the value of x such that $\lceil 2x \rceil \neq 2\lceil x \rceil$.

- a. 0
- b. 1.2
- c. -1.2
- d. -3

3. Select the number that is equal to $\lfloor \sqrt{40} \rfloor$.

- a. 5
- b. 6
- c. 7
- d. 40

4. Given $A = \{p, q, r, s\}$ and $X = \{1, 2, 3, 4\}$, select the function that has a well-defined inverse.

- a. $f = \{(p, 2), (q, 3), (r, 2), (s, 4)\}$
- b. $f = \{(p, 3), (q, 3), (r, 3), (s, 3)\}$
- c. $f = \{(p, 1), (q, 2), (r, 3), (s, 4)\}$
- d. $f = \{(p, 2), (q, 4), (r, 1), (s, 4)\}$

7. Select the function that does not have a well-defined inverse.

- a. $f : \mathbb{Z} \to \mathbb{Z}, f(x) = \lfloor x + 3 \rfloor$
- b. $f : \mathbb{R} \to \mathbb{R}, f(x) = -3x + 7$
- c. $f : \mathbb{R} \to \mathbb{Z}, f(x) = \lfloor x \rfloor$
- d. $f : \mathbb{R} \to \mathbb{R}, f(x) = 2x + 8$

6. Given the functions $f(x) = |x|$ and $g(x) = 4x - 1$, select the correct value for $(g \circ f)(3)$.

- a. 11
- b. -11
- c. 13
- d. -13

1) -9 is an odd number because $-9 = 2k + 1$ for some integer k.

42 is an even number because $42 = 2j$ for some integer j.

Select the correct values for k and j.

a. $k = -5, j = 21$
b. $k = -5, j = 21$
c. $k = -4, j = 21$
d. $k = -5, j = -21$

4) Select the value for x that is a counterexample to the following statement:

For every integer x, $x < x^2$.

a. $x = 0$
b. $x = -1$
c. $x = 1$
d. $x = 2$

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5) Theorem: If x is an even integer between -5 and 5, then $x^3 \leq 2x^2$.

Which set of facts must be proven in a proof by exhaustion of the theorem?

$\{-4, -2, 0, 2, 4\}$

3) Select the number that has the same parity as 2.

a. 1
b. 10
c. 17
d. 21

6) Select the value for x that is a counterexample to the following statement:

If x is an integer greater than 15 and is composite, then x is divisible by 5.

a. $x = 10$
b. $x = 49$
c. $x = 35$
d. $x = 25$

13) Theorem: If x and y are both rational numbers, then $x + y$ is also rational.

Which facts are assumed and which facts are proven in a proof by contrapositive of the theorem?

a. Assumed: x is rational or y is rational
b. Assumed: x is rational and y is rational
Proven: $x + y$ is rational
c. Assumed: $x + y$ is irrational
Proven: x is irrational or y is irrational
d. Assumed: $x + y$ is irrational
Proven: x is irrational and y is irrational

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14) Theorem: There is no smallest positive rational number.

A proof by contradiction of the theorem starts by assuming which fact?

a. Let r be an arbitrary positive rational number.
b. Let r be the smallest rational number.
c. Let r be the largest positive rational number.

Write the nth term formula: $5 + 11 + 17 + 23 + \ldots + 35$

$n = 1, 2, 3$

a) $t(n) = 5(6)^{n-1}$

b) $t(n) = 5 + 6(n - 1)$

c) $t(n) = -1 + 6(n - 1)$

d) $t(n) = 5 + 6n$

$$\sum_{j=1}^{4} 2j + 3$$

Find the sum of this series:

a) 8

b) 32

c) 22

d) 20

Identify the common ratio.

3, 9,27,81, ...

a) 5

b) 3

c) 4

d) 6

7. **Consider the following double summation:**

$$\sum_{i=1}^{n}\sum_{j=1}^{n} ij$$

Which of the following is the correct expression for this sum?

a. $\left(\frac{n(n+1)}{2}\right)^2$

b. $\frac{n(n+1)(2n+1)}{6}$

c. $\frac{n(n+1)}{2}$

d. $n^2(n+1)$

8. Evaluate the following double summation for $n = 3$:

$$\sum_{i=1}^{3} \sum_{j=1}^{3} (i + j)$$

$\boxed{36}$

What is the length of the string "This book"?

a. 8

b. 9

c. 7

d. 10

If $h(1) = 3$ and $h(n) = h(n-1) + n^2$ for $n \geq 2$, what is the value of $h(3)$?

$h(2) = h(1) + 4$

$= 7$

$h(3) = h(2) + 3^2$

$7 + 9 = 16$