Section 1.2
Gaussian Elimination and Gauss-Jordan Elimination

determine the size of the matrix.

1. $\begin{bmatrix} 1 & 2 & -4 \\ 3 & -4 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ 3x3

2. $\begin{bmatrix} 2 & -1 & 4 & 2 \\ 1 & 0 & 2 & -6 \end{bmatrix}$ 2X4

4. $\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \end{bmatrix}$ 1X5

8. $\begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$ 4X1

In Exercises 9–14, determine whether the matrix is in row-echelon form. If it is, determine whether it is also in reduced row-echelon form.

11. $\begin{bmatrix} 2 & 0 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ *not* in row-echelon form

12. $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ row-echelon form

14. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Reduced row-echelon form

In Exercises 23–36, solve the system using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

24.
$$2x + 6y = 16$$
$$-2x - 6y = -16$$

The augmented matrix for this system is

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix}.$$

Use Gauss-Jordan elimination as follows.

$$\begin{bmatrix} 2 & 6 & 16 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow$$

R1/2→R1

$$\begin{bmatrix} 1 & 3 & 8 \\ -2 & -6 & -16 \end{bmatrix} \Rightarrow$$

2R1+R2→R2

$$\begin{bmatrix} 1 & 3 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

Converting back to system of linear equations

x +3y = 8.

<u>y= t</u> t is any real number x +3t = 8 then <u>x = 8− 3t</u>

29. $x_1 \quad - 3x_3 = -2$

$3x_1 + x_2 - 2x_3 = 5$

$2x_1 + 2x_2 + x_3 = 4$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{bmatrix} \Rightarrow$$

-3(R1)+R2→ R2

-2(R1)+R3→ R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{bmatrix} \Rightarrow$$

-2(R2)+R3→R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{bmatrix} \Rightarrow$$

(R3)/-7 →R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Back substitution now yields

$x_3 = 2$

$x_2 = 11 - 7x_3 = 11 - (7)2 = -3$

$x_1 = -2 + 3x_3 = -2 + 3(2) = 4.$

So, the solution is: $x_1 = 4$, $x_2 = -3$, and $x_3 = 2$.

31. $$\begin{aligned} x_1 + x_2 - 5x_3 &= 3 \\ x_1 \quad\quad - 2x_3 &= 1 \\ 2x_1 - x_2 - x_3 &= 0 \end{aligned}$$

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \\ 2 & -1 & -1 & 0 \end{bmatrix}.$$

- (R1)+R2→ R2

-2(R1)+R3→ R3

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & -1 & 3 & -2 \\ 0 & -3 & 9 & -6 \end{bmatrix}$$

- (R2) → R2

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & -3 & 9 & -6 \end{bmatrix}$$

3(R2)+R3→R3

-1(R2)+R1→R1

$$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 \quad - 2x_3 &= 1 \\ x_2 - 3x_3 &= 2. \end{aligned}$$

choosing $x_3=t$ t is any real number

$x_1 = 1+ 2\,t$ $x_2 = 2+ 3t$

34. $$\begin{aligned} x + 2y + z &= 8 \\ -3x - 6y - 3z &= -21 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ -3 & -6 & -3 & -21 \end{bmatrix}.$$

3 (R1)+R2→ R2

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$0 = 3$, there is no solution