Chapter 9

مركز الكتلة

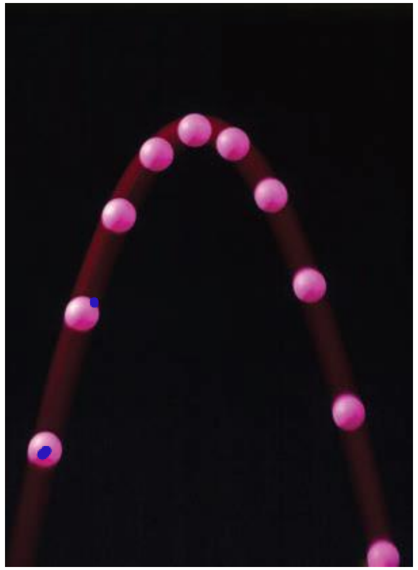
Center of Mass and Linear momentum

الزخم الخطي

The Center of Mass

- If you through a ball upward, you can fairly easily predict its motion.
- However, if you through a bat, for example, then the motion is more complicated
- To simplify analyzing the motion of any object, we use the concept of the **center of mass** (com)

لتمكين اعتبار نقطة انها مركز الكتلة



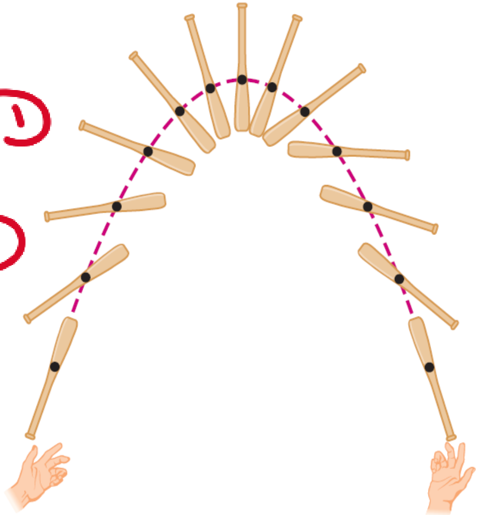
The center of mass of a system of particles is the point that moves as if:

1) all of the system's mass were concentrated there

① عندما تكون الكتلة مركزة فيه

2) all external forces were applied there.

② القوى الخارجية تؤثر فيه

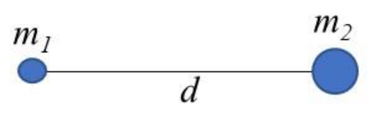


- We can determine the center of mass for one big object or for a system of small particles

Center of mass for a system of particles

Two particles:

If you have two particles with masses m_1 and m_2, separated by a distance d then the location of com (from m_1) is:



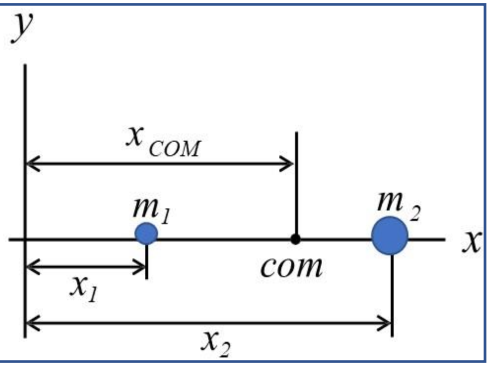
$$x_{com} = \frac{m_2}{m_1 + m_2} d$$

More generally, we can use the coordinate of each particle:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad \text{or} \qquad x_{com} = \frac{m_1 x_1 + m_2 x_2}{M}$$

where M is the total mass of the system ($M = m_1 + m_2$)

مجمع جميع الكتل

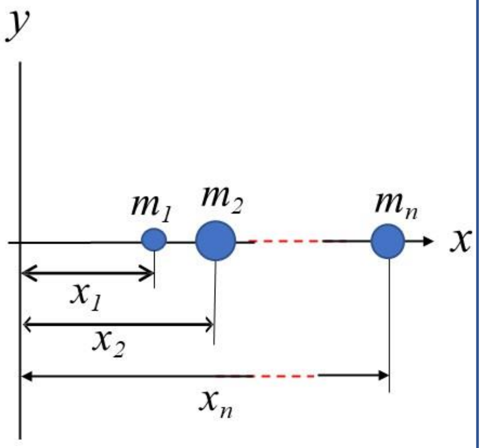


Many Particles:

If you have n particles along the x axis, then the total mass is $M = m_1 + m_2 + \cdot \cdot \cdot + m_n$, and the location of the center of mass is:

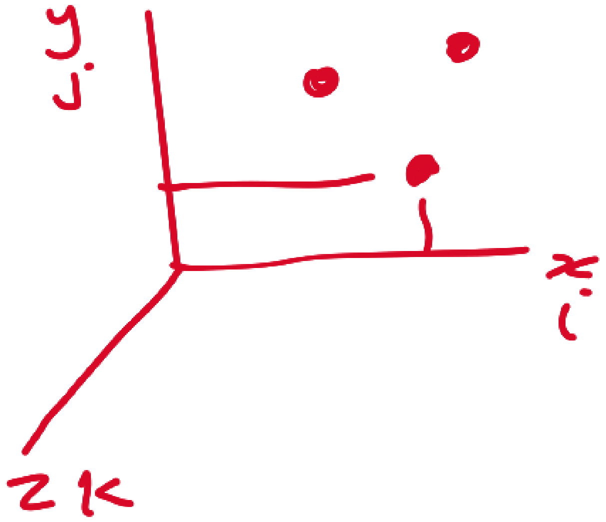
$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M}$$

$$x_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$



Three Dimensions: If the particles are distributed in three dimensions, the center of mass must be identified by three coordinates.

$Y_{com} = 3\hat{i}$



3

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

4

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots + m_n y_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

2

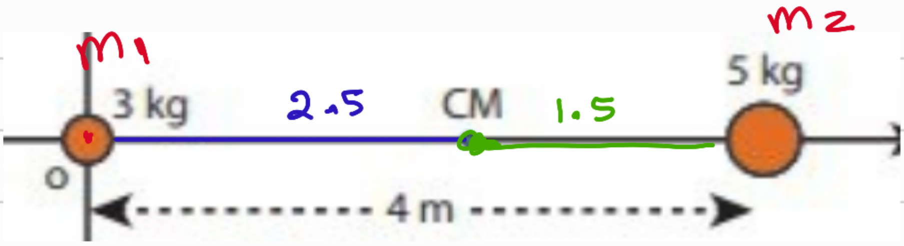
$$z_{com} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \cdots + m_n z_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

$\Longrightarrow$ the coordinates of com is (x_{com}, y_{com}, z_{com})

Or, we can write the position vector of com as:

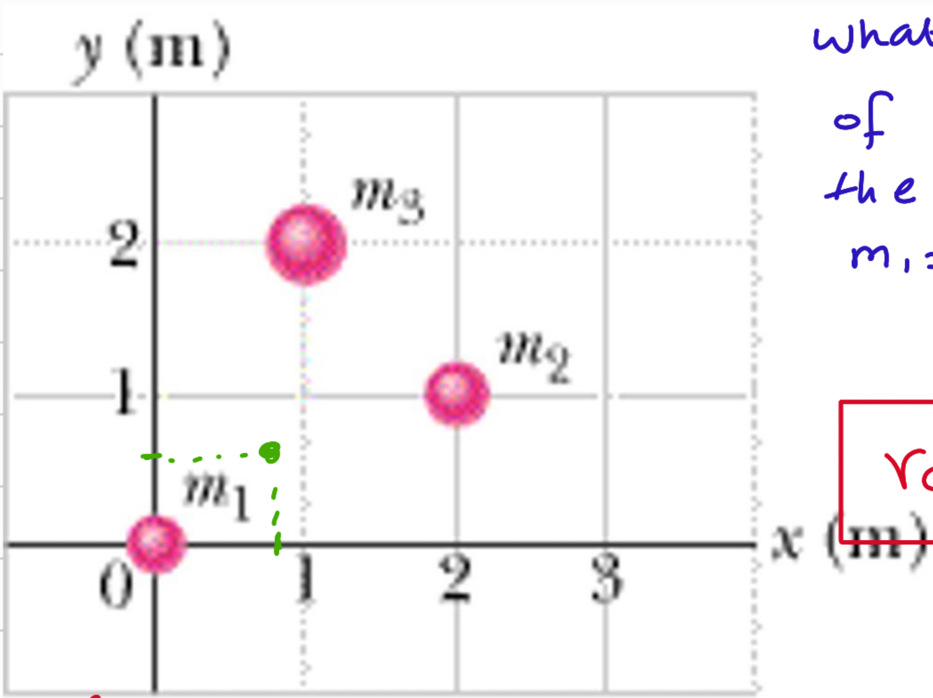
$$\vec{r}_{com} = x_{com}\hat{\imath} + y_{com}\hat{\jmath} + z_{com}\hat{k}$$

متجه مركز الكتلة



$$X_{com} = \frac{m_2}{m_1 + m_2} d = \frac{5}{3+5} \times 4$$

$$x_{com} = 2.5$$



what is the coordinates of the Com of the 3 prticles

$m_1 = 9$ kg $m_2 = 6$ kg

$m_3 = 2$ kg

$$\boxed{r_{cm} = 0.82\hat{i} + 0.588\hat{j}}$$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_2}{M}$$

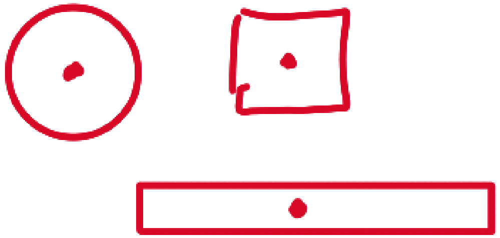
$$= \frac{9(0) + 6(2) + 2(1)}{17} = 0.82\ m$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M}$$

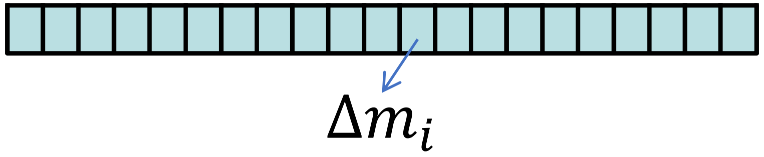
$$= \frac{9(0) + 6(1) + 2(2)}{17} = 0.588\ m$$

Solid Bodies:

If the object has a continuous distribution of matter, then we use integration to locate com.



- **In one dimension**:



$$\Rightarrow \boxed{x_{com} = \frac{1}{M}\int x\, dm}$$

where M is the total mass of the object, and $dm = \Delta m$ when ($\Delta m \to 0$)

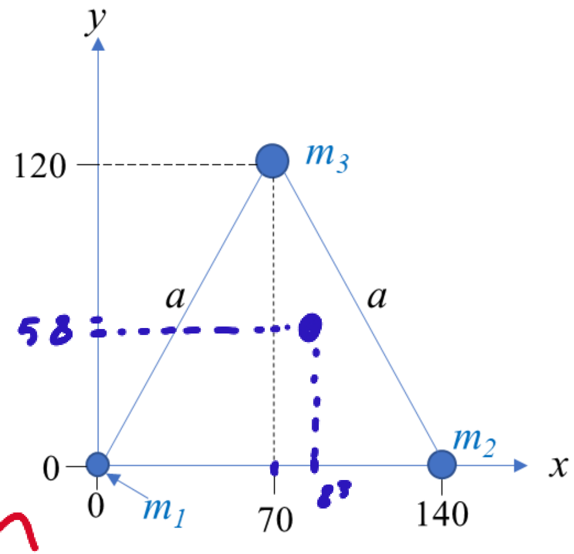
- **In three dimensions**:

The coordinates of com are:

$$\boxed{\begin{aligned} x_{com} &= \frac{1}{M}\int x\, dm \\ y_{com} &= \frac{1}{M}\int y\, dm \\ z_{com} &= \frac{1}{M}\int z\, dm \end{aligned}}$$

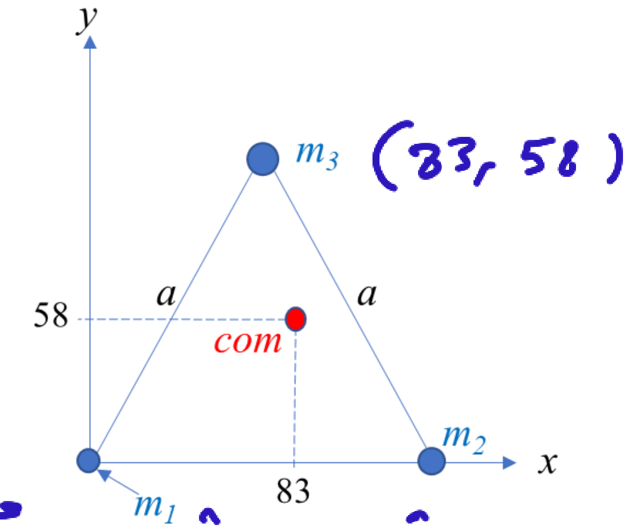
Again, com position vector will be: $\vec{r}_{com} = x_{com}\hat{\imath} + y_{com}\hat{\jmath} + z_{com}\hat{k}$

Example: Three particles of masses m_1 = 1.2 kg, m_2 = 2.5 kg, and m_3 = 3.4 kg form an equilateral triangle of edge length a = 140 cm. Where is the center of mass of this system? (the coordinates of each mass is shown in the figure).



$$x_{com} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{M}$$

$$= \frac{0(1.2) + 140(2.5) + 70(3.4)}{1.2 + 2.5 + 3.4} = 83 \text{ cm}$$



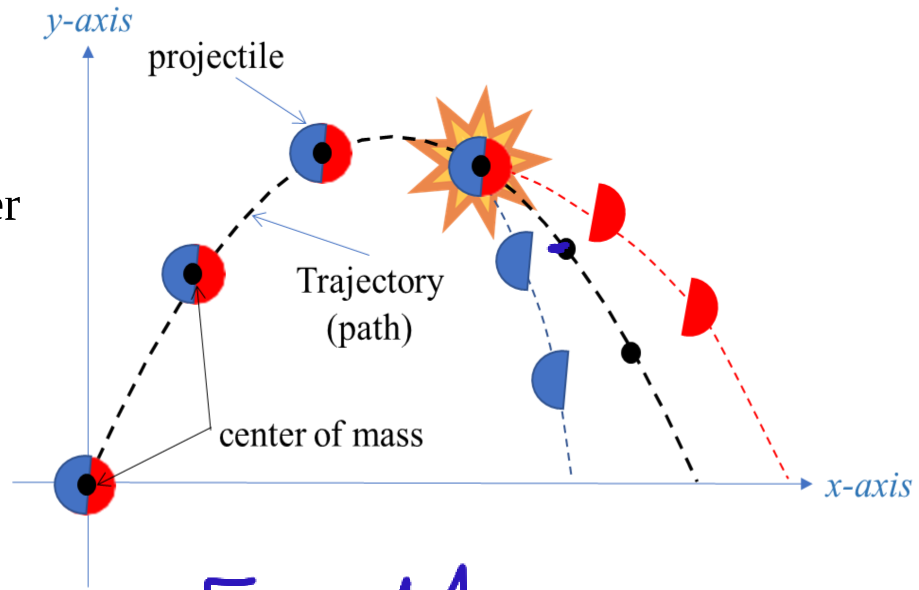
$$y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}$$

$$= \frac{0(1.2) + 0(2.5) + 120(3.4)}{1.2 + 2.5 + 3.4} = 58 \text{ cm}$$

$$\vec{r} = 83\hat{i} + 58\hat{j}$$

Newton's Second Law for a System of Particles

- Assume that a projectile is moving as shown in the figure
- If this projectile breaks in the air, the center of mass of all its parts will continue falling in the same trajectory of the projectile itself
- We can neglect the individual motions of these broken parts, and consider *only* in the motion of the center of mass (com) of the system.
- In this case, we assume that com has the total mass of the system.
- We apply all physics laws to this com instead of the whole system
- We can apply Newton's second law as:



$$\boxed{\vec{F}_{net} = M\vec{a}_{com}}$$

$$F_{net} = M a_{com}$$

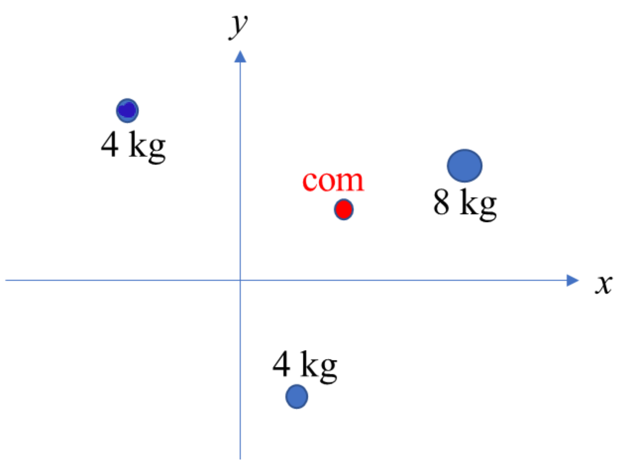
where,
$\vec{F}_{net}$ is the net force of *all external forces* that act on the system
M is the total mass of the system
$\vec{a}_{com}$ is the acceleration of the *center of mass* of the system (not the acceleration of any individual point of the system)

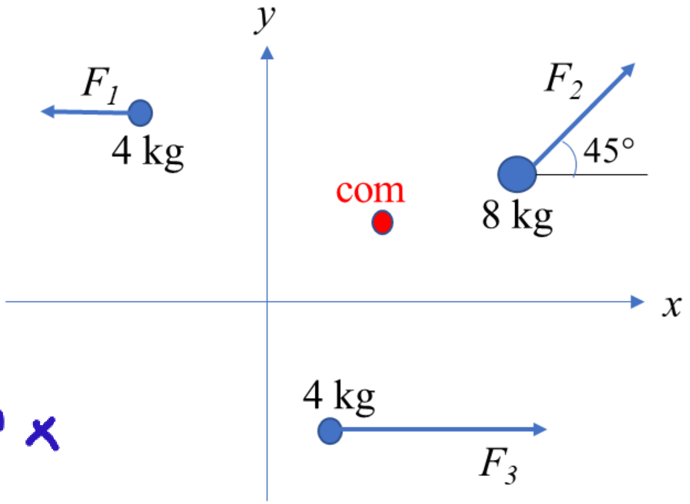
In three dimensions:

We can write the components of $\vec{F}_{net}$ and $\vec{a}_{com}$ as:

$$\vec{F}_{net} = M\vec{a}_{com} \quad \left\{ \begin{array}{l} \vec{F}_{net,x} = M\vec{a}_{com,x} \\ \vec{F}_{net,y} = M\vec{a}_{com,y} \\ \vec{F}_{net,z} = M\vec{a}_{com,z} \end{array} \right.$$

Example: The three particles are initially at rest. Each particle experiences an *external* force as shown in the figure. The directions are indicated, and the magnitudes are F_1 = 6 N, F_2 = 12 N, and F_3 = 14 N. What is the acceleration of the center of mass (com) of the system, and in what direction does it move?



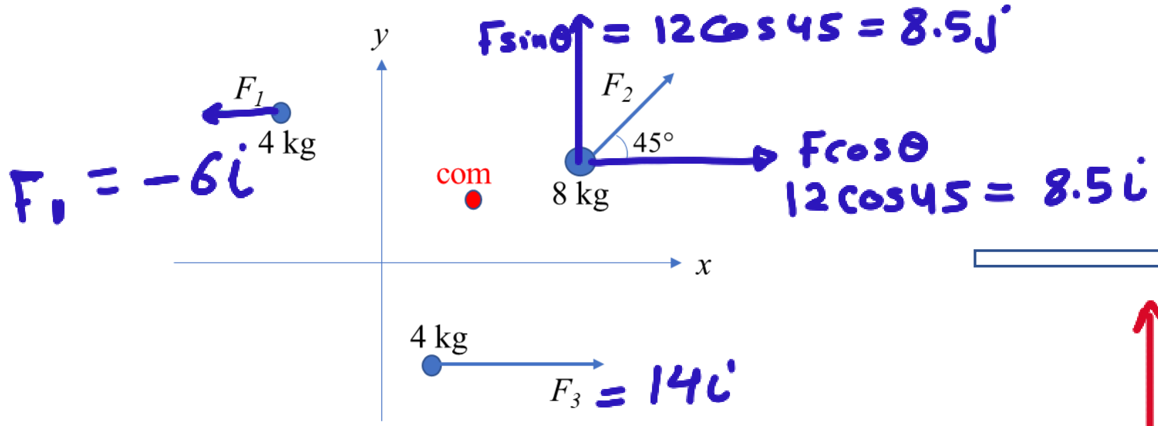


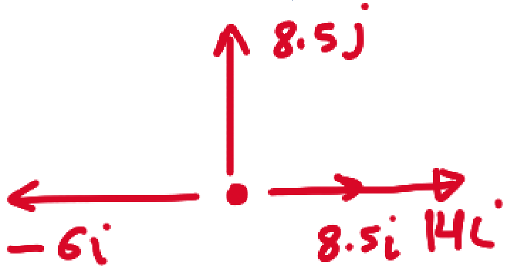
$$F_{net_x} = M a_{com\,x}$$

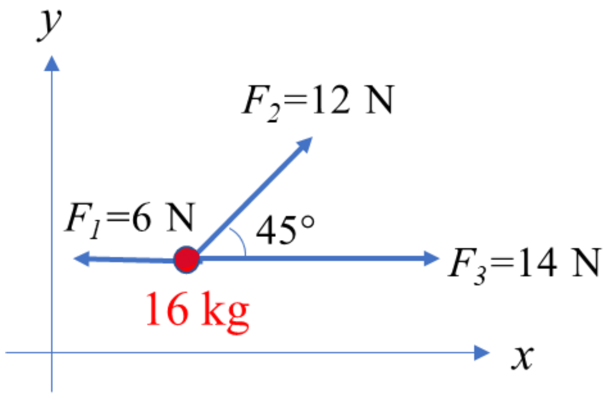
$$a_{com\,x} = \frac{F_x}{M} \qquad a_{com\,y} = \frac{F_y}{M}$$

Solution:
We deal with this problem as if all the masses (total of *M*) are concentrated at the com, and all forces act on this single point.

① بكفوه ولأ 3 حساب F_x ، F_y







$\theta = \tan^{-1}\dfrac{0.53}{1.03}$

$\theta = 27$

$F_{net}\,x = 14 + 8.5 - 6 = 16.5$

$a_{x\,com} = \dfrac{F_{net\,x}}{M} = \dfrac{16.5}{4+8+4}$

$a_{x\,com} = 1.03\ m/s^2$

$F_{y\,net} = 8.5$

$a_{com\,y} = \dfrac{F_{y\,net}}{M}$

$a_{com\,y} = \dfrac{8.5}{4+4+8} = 0.53$

$a_{com} = 1.03\hat{i} + 0.53\hat{j}$

$a = \sqrt{1.03^2 + 0.53^2} = 1.2\ m/s^2$

الزخم الخطي $\vec{P} = m\vec{v}$

5kg → 2m/s $\quad P = 2 \times 5 = 10$ $\boxed{\text{kg m/s}}$

Linear Momentum

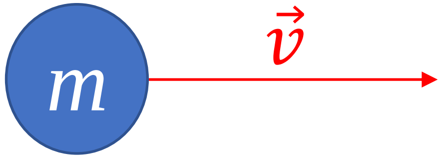
- If an abject of mass m moves with a velocity $\vec{v}$, then we define its **linear momentum** $\vec{\boldsymbol{p}}$ as:

الزخم كمية متجهة

الوحدة kg m/s

اتجاهها نفس اتجاه الحركة

$$\boxed{\vec{p} = m\vec{v}}$$



- $\vec{p}$ is a vector quantity, and it has the same direction of $\vec{v}$ (the linear momentum has the same direction as the velocity)

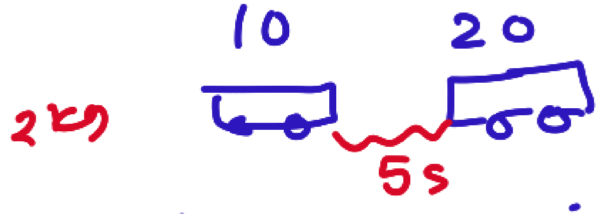
- The SI unit of $\vec{p}$ is kg.m/s

- Note that this equation for a single particle only

العلاقة بين القوه والزخم

Force and Momentum:

- If an object moves with a momentum of $\vec{p}$, then we can calculate the net force acting on this object from:



القوه, كونها تساوي معدل التغير في الزخم

$$\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$$ instantaneous net force — قوه لحظيه

$$\boxed{\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}}$$ average net force — قوه متوسطه

$$\Delta P = P_2 - P_1 = mv_2 - mv_1 = 2(20) - 2(10) = 40 - 20 = 20\ \text{kg m/s}$$

- The time rate of change of the momentum $\Delta\vec{p}$ of a particle is equal to the net force $\vec{F}_{net}$ acting on the particle
- The direction of $\Delta\vec{p}$ is in the direction of the net force $\vec{F}_{net}$.
- $\therefore$ If there is no net external force, $\vec{p}$ of the object *cannot* change.
- Note that this equation for a single particle only

(متوسطه) $F = \frac{\Delta P}{\Delta t} = \frac{20}{5}$

$$\boxed{F = 4\ N}$$

$$F = \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$

- This equation $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ is similar to Newton's second law because:

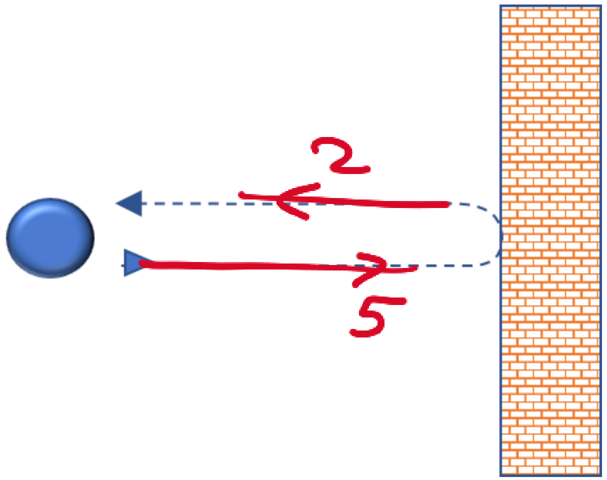
بشكل اخر من قانون نيوتن الثاني

$$\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}} \Rightarrow \vec{F}_{net} = \frac{d}{dt}(m\vec{v}) \Rightarrow \vec{F}_{net} = m\frac{d\vec{v}}{dt} \Rightarrow \boxed{\vec{F}_{net} = m\vec{a}}$$

$m = 0.7$ $\quad v_i = 5\,m/s$

$v_f = -2\,m/s$

Example: A 0.7 kg ball moving horizontally at 5 m/s strikes a vertical wall and rebounds with speed 2 m/s. What is the change in its linear momentum?



Solution:

Remember that velocity and linear momentum are vector quantities:

$$v_i = +\,5 \text{ m/s} \Longrightarrow p_i = m\, v_i = 0.7 \times 5 = 3.5 \text{ kg.m/s}$$

$$v_f = -\,2 \text{ m/s} \Longrightarrow p_f = m\, v_f = 0.7 \times (-\,2) = -\,1.4 \text{ kg.m/s}$$

$$\therefore \Delta p = p_f - p_i = -\,1.4 - 3.5 = -\,4.9 \text{ kg.m/s}$$

ما مقدار التغير في الزخم

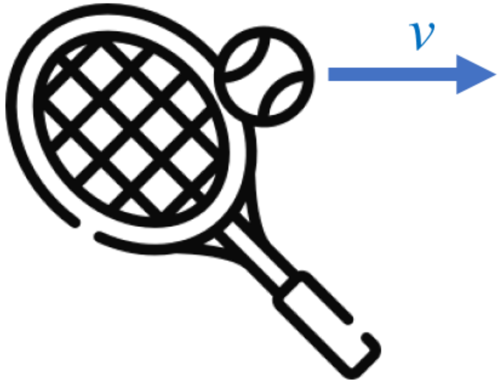
ΔP

$$\Delta P = P_f - P_i = m v_f - m v_i$$

$$= m(v_f - v_i) = 0.7(-2-5)$$

$$= 0.7\,(-7) = -4.9 \text{ kgm/s}$$

m



Example: A tennis player hits the ball of mass = 0.057 kg reaching a speed of 58 m/s. What is the average force exerted on the ball if it remained in contact with the racquet for 5 ms (milliseconds)?
Assume that the ball's speed just after impact is 58 m/s and that the initial horizontal component of the velocity before impact is negligible.

السرعة الابتدائية مهملة

Solution:

المطلوب حساب متوسط القوة

$m = 0.057$ kg
$t = 5 \times 10^{-3}$ s
$v_f = 58$ m/s
$v_i = 0$ m/s

$$v_i = 0 \text{ m/s} \Longrightarrow p_i = m\, v_i = 0.057 \times 0 = 0 \text{ kg.m/s}$$

$$v_f = 58 \text{ m/s} \Longrightarrow p_f = m\, v_f = 0.057 \times 58 = 3.3 \text{ kg.m/s}$$

$$\therefore \Delta p = p_f - p_i = 3.3 - 0 = 3.3 \text{ kg.m/s}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} = \frac{m v_f - m v_i}{\Delta t}$$

$$\vec{F} = \frac{3.3}{5 \times 10^{-3}} = 661.2 \text{ N}$$

$$F = \frac{0.057 \times 58 - \cancel{0.057 \times 0}}{5 \times 10^{-3}} = 661.2 \text{ N}$$

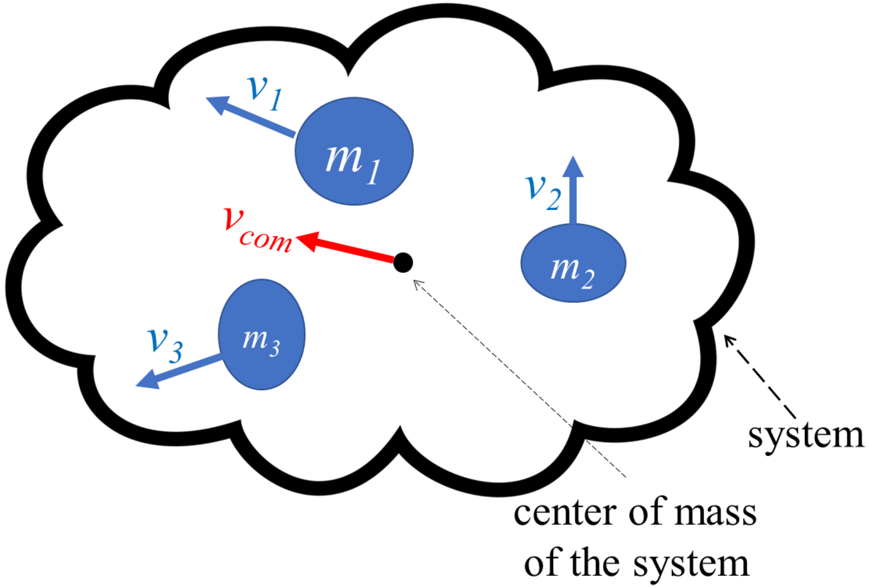
The net momentum and the net force are in the same direction (that is the positive direction of x-axis)

The Linear Momentum of a System of Particles

- If you have a system of n particles, each particle with its own mass and velocity.
- Each mass has its own momentum
- The total linear momentum of the whole system $\vec{P}$ is:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$



$$\boxed{\vec{P} = M\vec{v}_{com}}$$

where,

الزخم الكلي

$\vec{P}$ is the total linear momentum of the system

M is the total mass of the system الكتلة الكلية

$\vec{v}_{com}$ is the velocity of the center of mass of the system

سرعة مركز الكتلة

$$\vec{P} = m_1v_1 + m_2v_2 + m_3v_3$$

$$\vec{P}_{total} = M\,V_{com}$$

- $\therefore$ The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass $\vec{v}_{com}$.

Force and Momentum:

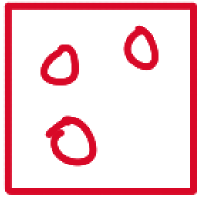
- If you have a **system of *n* particles**, each particle with its own mass, velocity and linear momentum, then the total linear momentum of the whole system $\vec{P}$
- The net force acting on this **system** is:

القوة اللحظية

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$ instantaneous net force

القوة المتوسطة

$$\vec{F}_{net} = \frac{\Delta\vec{P}}{\Delta t}$$ average net force



- The time rate of change of the momentum of a system is equal to the net force acting on the system, and is in the direction of that force.

اذا كانت القوة الخارجية = صفر

التغير في الزخم الكلي = صفر

- $\therefore$ If there is no net external force, $\vec{P}$ of the system *cannot* change.
- This equation $\vec{F}_{net} = \frac{d\vec{P}}{dt}$ is similar to Newton's second law for the system of particles because:

هو شكل آخر لقانون نيوتن الثاني

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}} \Rightarrow \vec{F}_{net} = \frac{d}{dt}(M\vec{v}_{com}) \Rightarrow \vec{F}_{net} = M\frac{d\vec{v}_{com}}{dt} \Rightarrow \vec{F}_{net} = \boxed{Ma_{com}}$$

Collision and Impulse

التصادم و الدفع

We will understand the concepts of impulse and momentum through considering collisions between objects.

Single Collision

- Assume that you are hitting a ball with bat. Your force $F(t)$ will change the linear momentum $d\vec{p}$ of the ball
- If your force acts on the ball for some certain period of time t (your force starts at time t_i and finishes at time t_f), then the momentum of the ball will change from p_i to p_f.
- We know that

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow d\vec{p} = \vec{F}(t)dt$$

$$\Rightarrow \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}(t)dt$$

$$\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

$\int_i^f dx = x_f - x_i$

$F = \frac{\Delta P}{\Delta t}$

Implus = J

$\boxed{J = \Delta P}$

$= P_f - P_i$

$= mV_f - mV_i$

$\boxed{J = F\Delta t}$

$\boxed{J = \int_{t_i}^{t_f} F\,dt}$

- Now we define a new quantity called **impulse ($\vec{J}$)**, where

الدفع كمية متجهة لها نفس اتجاه القوة

$$\boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt}$$

- $\vec{J}$ is a vector, and has the same direction as $\vec{F}$
- Unit of $\vec{J}$ is kg.m/s

$$\therefore \boxed{\Delta\vec{p} = \vec{J}}$$

$J = \Delta P = P_f - P_i =$

$J = F\Delta t = \int_{t_i}^{t_f} F\,dt$

So we learned 2 new results:

1) force × time is called impulse ⟹ (special case, if the **force is constant**): $\boxed{J = F\,t}$

2) The equation $\Delta\vec{p} = \vec{J}$ means that the change in linear momentum of the object equals to the impulse (force × time)

J $\qquad$ $F\Delta t$

- However, when you hit the ball by the bat, your force is not constant
- Your force changes during the time Δt
- We can calculate your average force F_{avg} by using impulse:

we know that

$$\boxed{J = F_{avg}\Delta t}$$

$$\Rightarrow \boxed{F_{avg} = \frac{J}{\Delta t}}$$

impulse of your force

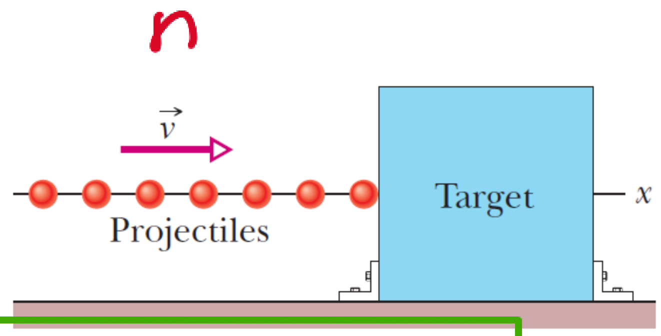
your average force

time

Series of Collisions سلسلة من التصادمات

- Now let's consider the force on a fixed body when it is hit by n of identical projectiles
  - Each projectile has a mass m and velocity $v \Longrightarrow$ each projectile has initial momentum

$$p = mv$$



- Each projectile undergoes a change Δp in linear momentum because of the collision.
- The total change in linear momentum for n projectiles during interval Δt (the total impulse on all projectiles) is:

قذيفة

$$\boxed{J = n\,\Delta p}$$

القذائف $J = n\Delta p$

الهدف $J = -n\Delta p$

and the total impulse on the target is (it will be on the opposite direction):

هدف

$$\boxed{J = -n\,\Delta p}$$

$\therefore$ the average force F_{avg} acting on the target during the collisions:

$$\boxed{F_{avg} = \frac{J}{\Delta t} \Rightarrow F_{avg} = -\frac{n}{\Delta t}\Delta p \Rightarrow F_{avg} = -\frac{n}{\Delta t}m\Delta v}$$

where Δv is the change in the velocity of the projectiles

$$F = \frac{n\Delta P}{\Delta t} = \frac{n(P_f - P_i)}{\Delta t} \qquad \boxed{F_{avg} = -\frac{nm}{\Delta t}\Delta v} = -\frac{nm.\Delta V}{\Delta t}$$

Remember that $\Delta v = v_f - v_i$

1) If the projectiles move with velocity v but then stop upon impact, then $v_f = 0$

$\Delta v = v_f - v_i = 0 - v = -v$

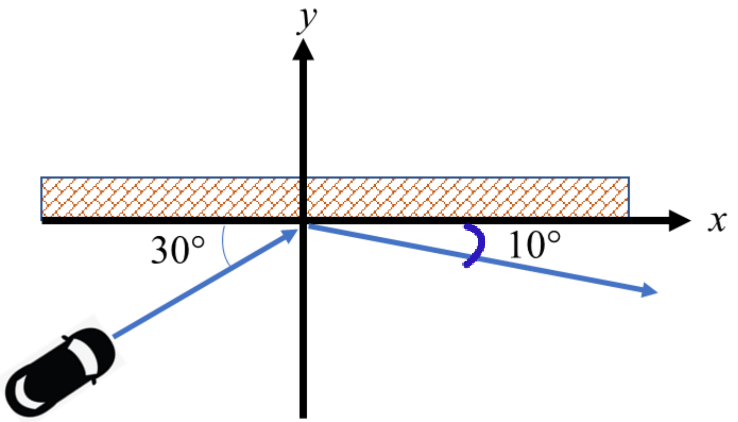
2) If the projectiles bounce (rebound) directly backward from the target with no change in speed, then $v_f = -v$

$\Delta v = v_f - v_i = -v - v = -2v$

$v \rightarrow$ $\Delta V = -V_i$

$v+ \rightarrow$ $-v \leftarrow$ $\Delta V = -V - V = -2V$

Example: a car collides with a wall as shown in the figure. Before the collision the speed was v_i = 70 m/s at 30° and it became v_f = 50 m/s at 10° after the collision. If the driver mass m is 80 kg, (a) what is the impulse $\vec{J}$ on the driver due to the collision? (b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?

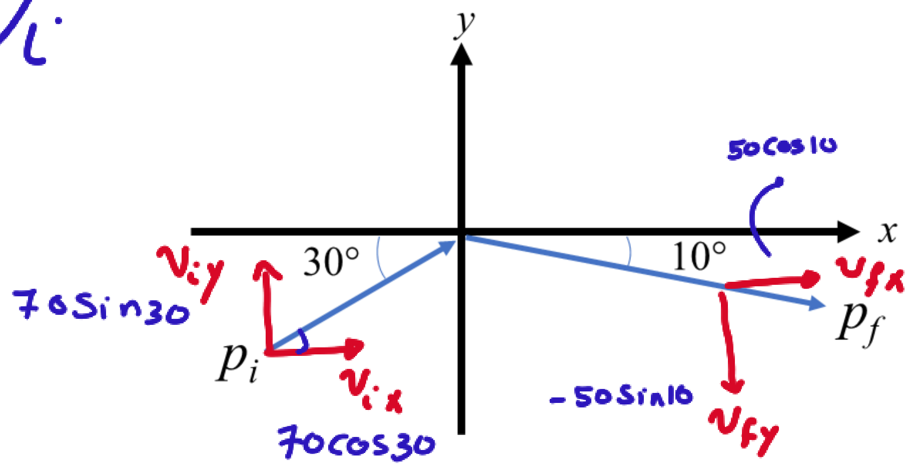


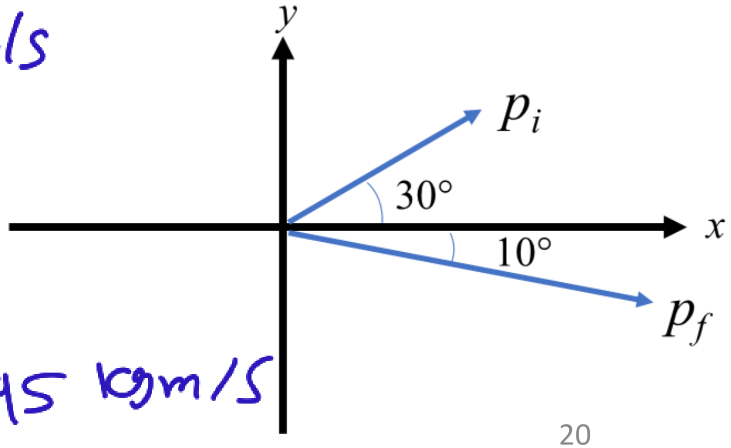
a) $J = \Delta P = P_f - P_i = m v_f - m v_i$

$J_x = m(v_{fx} - v_{ix})$

$J_x = 80(50\cos 10 - 70\cos 30)$

$80(-11.38) = -910.5$ kgm/s





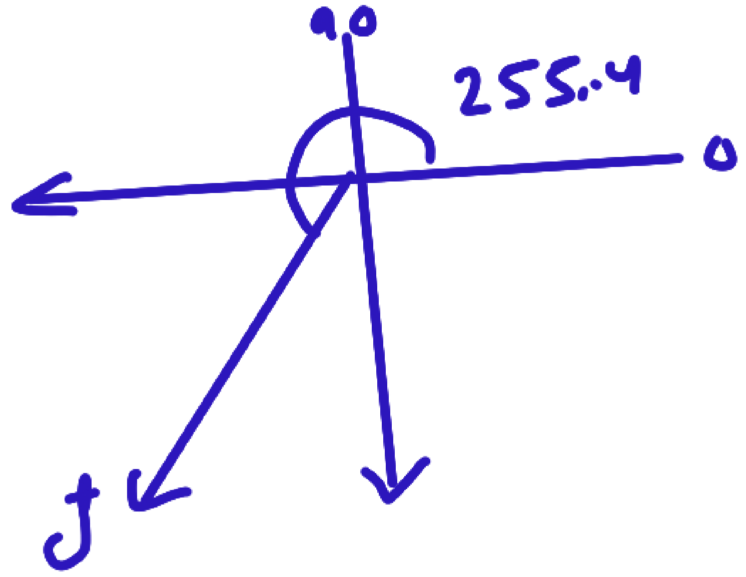
$J_y = m(v_{fy} - v_{iy})$

$= 80(-50\sin 10 - 70\sin 30) = -3495$ kgm/s

$$J = -910.5\,\hat{i} - 3495\,\hat{j}$$

$$J = \sqrt{(-910.5)^2 + (-3495)^2} = 3612 \text{ kg m/s}$$

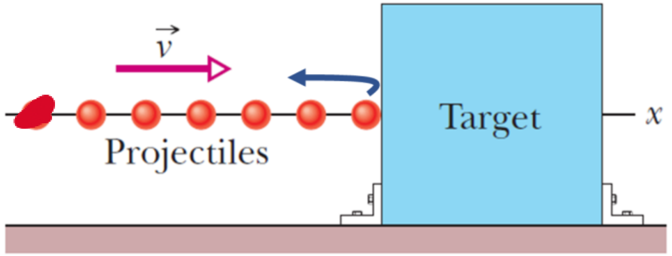
$$\theta = \tan^{-1}\left(\frac{-3495}{-910}\right) = 75.4 + 180 = 255.4^\circ$$

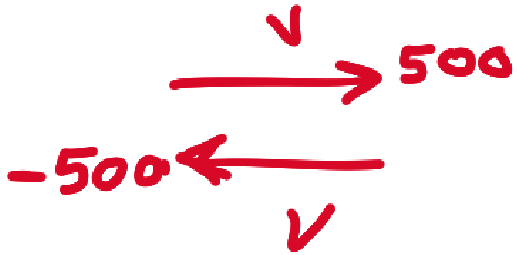


$$F_{avg} = \frac{J}{\Delta t} = \frac{3612}{14 \times 10^{-3}}$$

$$= 2.58 \times 10^{5} \text{ N}$$

Example: A series of 3 g balls hit a fixed box at the rate of 100 balls/min, and the speed of each ball is 500 m/s. Suppose too that the balls rebound straight back with no change in speed. (a) What is the change of linear momentum of 1 ball? (b) What is the impulse on 100 balls? (c) What is the magnitude of the average force on box?





a) $\Delta P = P_f - P_i = m(v_f - v_i)$

$= 3\times10^{-3}(-500 - 500) = -3\ kgm/s$

b) $J = n\Delta P = 100(-3) = -300\ kgm/s$

c) $n = 100$ $\quad t = 1\,min = 60\,s$

$F_{avg} = -\frac{J}{\Delta t} = -n\frac{\Delta P}{\Delta t} = -\frac{100(-3)}{60} = 5N$

Conservation of Linear Momentum

حفظ الزخم الخطي

- If there is a system which is:

معزول 1) Isolated: no net external force acts on the system

النظام المعزول لا تؤثر عليه أي قوة

مغلق 2) Closed: no mass enters or leaves the system

النظام المغلق: لا تدخل، تخرج كتلة

then the total **linear momentum of this system is constant.**

الزخم الكلي للنظام ثابت

- $\boxed{\vec{P} = \text{constant}}$ ($\vec{P}$ has the same value at all time)
- This means that:

(total linear momentum at some initial time t_i) = (total linear momentum at some later time t_f)

الزخم الابتدائي $\boxed{\vec{P}_i = \vec{P}_f}$ الزخم النهائي

This is called the **law of conservation of linear momentum**

قانون حفظ الزخم

- $\because$ $\vec{P}$ does not change with time t (*for isolated and closed systems*), then we can write:

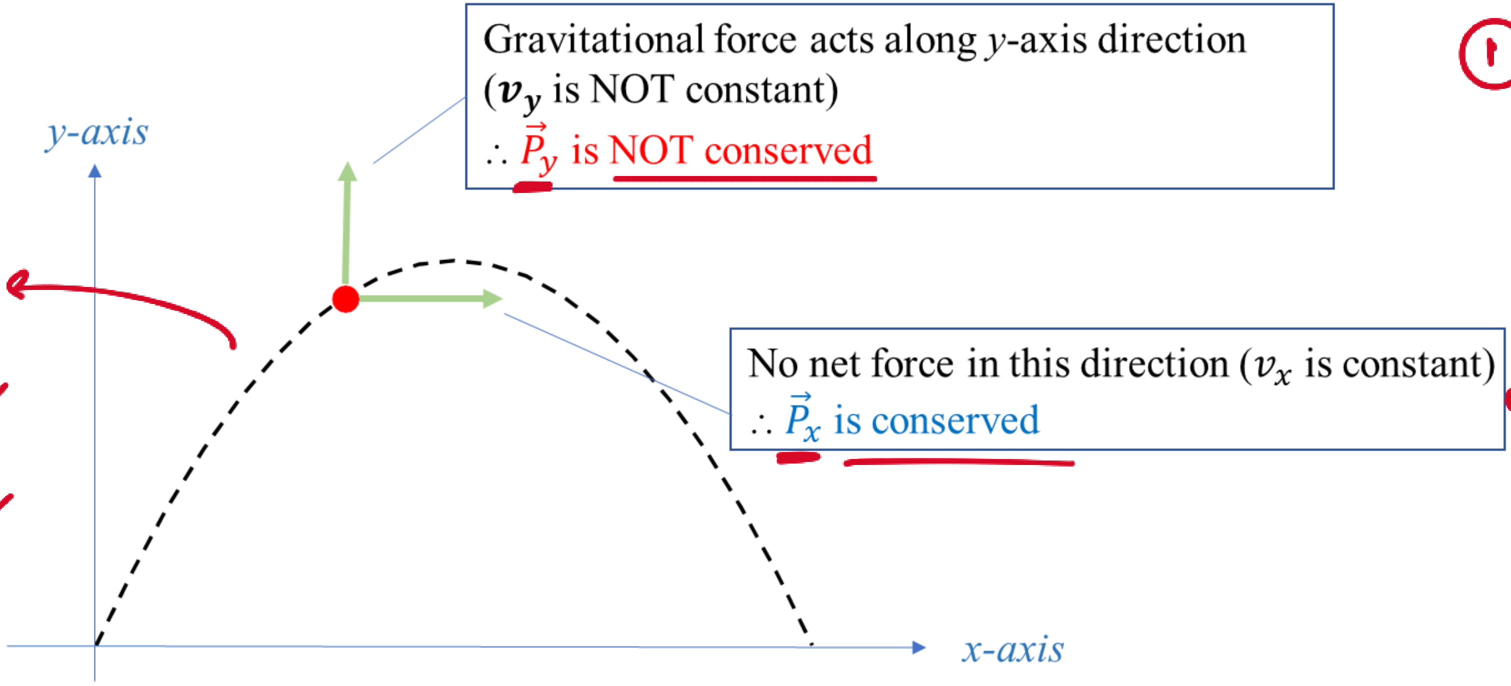
$$\boxed{\frac{d\vec{P}}{dt} = 0} = F$$

In 3 dimensions, we can apply the law of conservation of linear momentum as following:

- If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

- For example, in the **projectile motion**, the only force acting on the projectile is gravity (downward), which is in the *y*-axis direction. There is no force in the *x*-axis direction $\Longrightarrow$ linear momentum in the *x*-axis direction $\vec{P}_x$ is conserved (but $\vec{P}_y$ is not conserved)

الزخم محفوظ
باتجاه X لأنه
لاتوجد قوه في
X

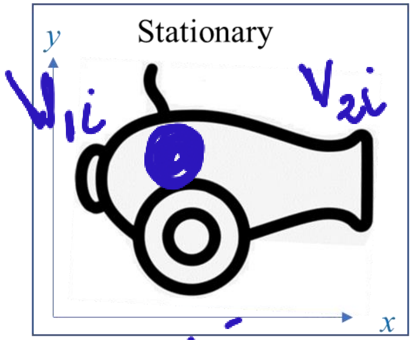


15 10
① → ← ②
$P_i = 25$
∞
5 20
← ① ② →
$P_f = 25$

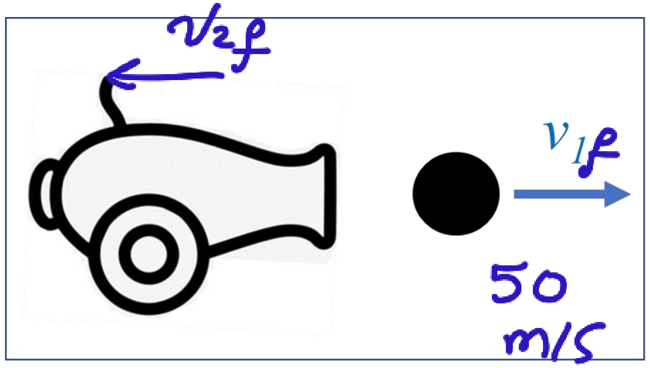
ساكن

Example: a canon is at stationary before it fires a ball of mass m_1 = 2 kg with a velocity v_1 = 50 m/s as shown. What are the magnitude and direction of the canon's velocity if its mass m_2 = 110 kg?

Zero



قبل



Solution:

∵ there is no external force

∴ linear momentum is conserved

الزخم قبل = صفر

الزخم بعد = صفر

$$\vec{P}_i = \vec{P}_f$$

In the *y*-axis direction:

There is no motion ⟹ $v_{1y} = v_{2y} = 0$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = 2 \times 50 + 110 \times v_{2f}$$

In the *x*-axis direction:

$$\vec{P}_{ix} = \vec{P}_{fx}$$

$$100 + 110\, v_{2f} = 0$$

$$0 = m_1 v_{x1} + m_2 v_{x2}$$

$$110\, v_{2f} = -100$$

$$m_2 v_{x2} = -m_1 v_{x1}$$

(m_2 and v_2 are for the canon after firing)

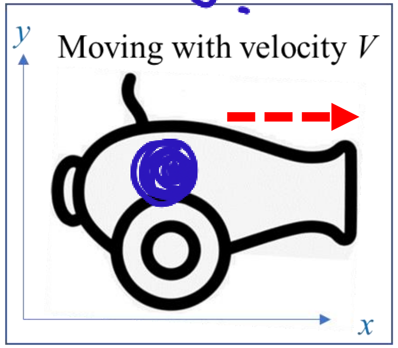
$$v_{x2} = \frac{-m_1 v_1}{m_2} = \frac{-2\times 50}{110} = \boxed{-0.9\ \mathrm{m/s}}$$

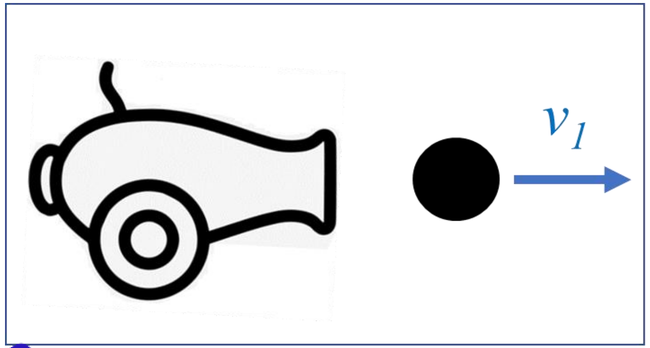
$$v_{2f} = \frac{-100}{110} = -0.9\ \mathrm{m/s}$$

∴ the canon will move with a velocity of 0.9 m/s in the negative direction of the *x*-axis

Example: repeat the last example if the canon is moving with a velocity V = 0.9 m/s in the positive x-axis direction before it fires.

قبل





$P_i = P_f$

Solution:

∵ there is no external force

∴ linear momentum is conserved

$$\vec{P}_i = \vec{P}_f$$

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\frac{m_1 V_{1i} + m_2 V_{2i} - m_1 V_{1f}}{m_2} = V_{2f}$$

In the y-axis direction:

There is no motion $\Longrightarrow v_{y1} = v_{y2} = 0$

In the x-axis direction:

Note that the mass of the system before firing is M = mass of canon + mass of ball = 110 + 2 = 112 kg

$$\vec{P}_{ix} = \vec{P}_{fx}$$

$$MV = m_1 v_{x1} + m_2 v_{x2}$$

$$V_{f2} = \frac{2\times 0.9 + 110\times 0.9 - 2\times 50}{110}$$

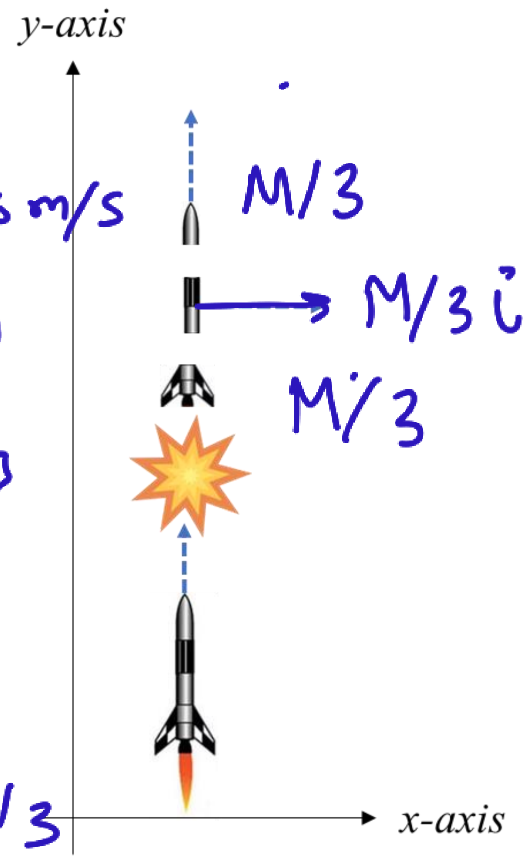
$$m_2 v_{x2} = MV - m_1 v_{x1}$$

$$v_{x2} = \frac{MV - m_1 v_{x1}}{m_2} = \frac{(112\times 0.9) - (2\times 50)}{110} = \frac{100.8 - 100}{110} = 7.3\times 10^{-3}\ \text{m/s}$$

$= 0.0073$ m/s

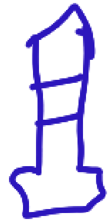
∴ the canon will continue moving in the positive directing of the x-axis but with a velocity of 7.3×10^{-3} m/s

Example: A rocket is fired vertically upward. When it reaches a speed of 300 m/s, it explodes into three fragments having equal mass. One fragment continues to move upward with a speed of 450 m/s following the explosion. The second fragment has a speed of 240 m/s and is moving east right after the explosion. What is the velocity of the third fragment right after the explosion?



$V_i = 300$ m/s

M



Solution:

Let us call the total mass of the rocket $M \Longrightarrow$ the mass of each fragment is $M/3$.

$\because$ the system is isolated and closed $\Longrightarrow \vec{P}_i = \vec{P}_f$

$P_i = P_f$

$\vec{P}_i = M v_i = M(300\hat{j})$

$\cancel{M} V_i = \frac{\cancel{M}}{3} V_1 + \frac{\cancel{M}}{3} V_2 + \frac{\cancel{M}}{3} . V_3$

$\vec{P}_f = \frac{M}{3}(450\hat{j}) + \frac{M}{3}(240\hat{i}) + \frac{M}{3} v_f$ (where v_f is the unknown velocity of the third fragment)

$V_i = \frac{1}{3}(V_1 + V_2 + V_3)$

$\therefore M(300\hat{j}) = \frac{M}{3}(450\hat{j}) + \frac{M}{3}(240\hat{i}) + \frac{M}{3} v_f$

$300\hat{j} = \frac{1}{3}(450j + 240i + V_3)$

$(300\hat{j}) = \frac{1}{3}(450\hat{j}) + \frac{1}{3}(240\hat{i}) + \frac{1}{3} v_f$

$900\hat{j} = 450j + 240i + V_3$

$$(300\hat{j}) = (150\hat{j}) + (80\hat{\imath}) + \frac{1}{3} v_f$$

$$\frac{1}{3} v_f = -(80\hat{\imath}) + (300\hat{j}) - (150\hat{j})$$

$$\frac{1}{3} v_f = -(80\hat{\imath}) + (150\hat{j})$$

$$v_f = -3(80\hat{\imath}) + 3(150\hat{j})$$

$$v_f = (-240\hat{\imath}) + (450\hat{j})$$

$900j - 450j - 240i = V_3$

$450j - 240i = V_3$

$-240i + 450j = V_3$

$|V_3| = \sqrt{240^2 + 450^2} = 510$ m/s

∴ the third fragment will move with a velocity component of 240 m/s in the negative direction of *x*-axis (west) and another velocity component of 450 m/s in the positive direction of *y*-axis (north):

الاكبه

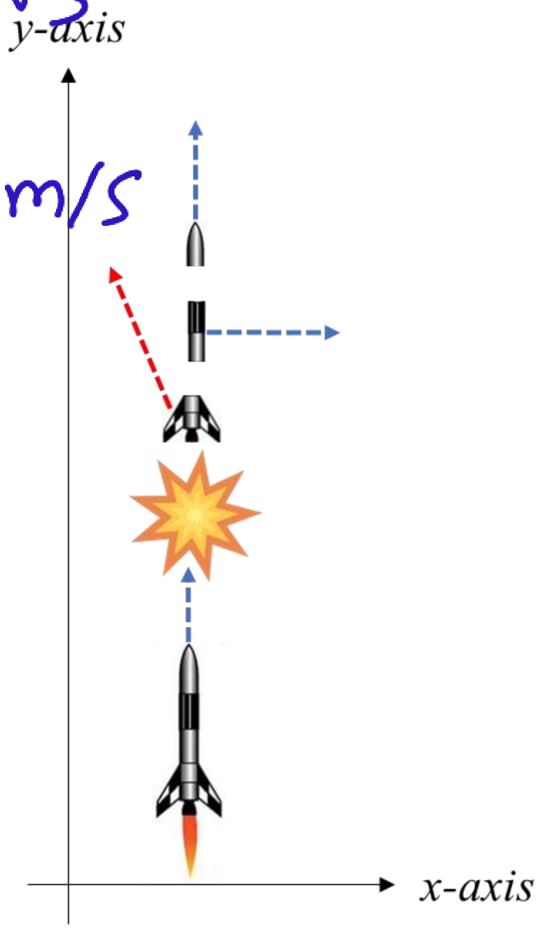
$$\theta = tan^{-1} \frac{v_y}{v_x} = tan^{-1} \frac{450}{-240} = -61.9°$$

$$\theta = -61.9° + 180° = 118.1°$$

$\theta = tan^{-1}\left(\frac{450}{-240}\right)$

$-61.9 + 180$

$\theta = 118.1°$



Momentum and Kinetic Energy in Collisions

When objects collide, there are **2 types of collisions**

تصادم مرن

تصادم غير مرن

elastic collision
No loss in kinetic energy
Kinetic energy is conserved (it is the same before and after the collision)
Linear momentum is conserved (it is the same before and after the collision)

الطاقة محفوظة لا تضيع

inelastic collision
There is some loss in kinetic energy
Kinetic energy is NOT conserved (it is NOT the same before and after the collision)
Linear momentum is conserved (it is the same before and after the collision)

هناك ضياع بالطاقة - الطاقة غير محفوظة

$$P_i = P_f$$

- Usually, there is some loss in kinetic energy in every collision in real life. However, if the loss is small, we can consider this collision as an elastic collision.

- In the inelastic collision, if the **bodies stick together**, the collision is called a **completely inelastic collision.**

تصادم عديم المرونة: الاجسام تلتحم مع بعض بعد التصادم

Inelastic Collisions in One Dimension

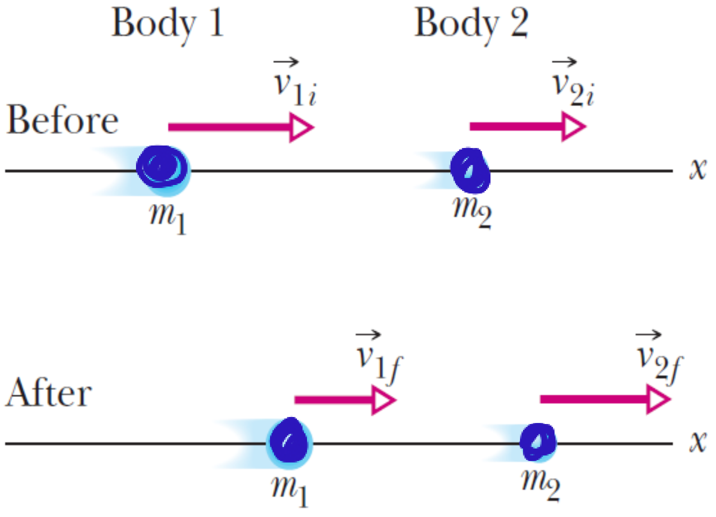
Remember: in the inelastic collision: some kinetic energy is lost (K is not conserved).

However, linear momentum is conserved.

تصادم غير مرن

One-Dimensional Inelastic Collision

قبل



If there are 2 bodies colliding:

بعد

Body 1 with mass m_1 with velocity v_{1i} before collision and v_{1f} after collision

Body 2 with mass m_2 with velocity v_{2i} before collision and v_{2f} after collision,

Then we can write the law of conservation of linear momentum for this ***system*** as:

(total momentum $\vec{P}_i$ before the collision) = (total momentum $\vec{P}_f$ after the collision)

$\therefore$ considering each body in this system we can write:

$$\boxed{\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + \vec{p}_{f2}}$$

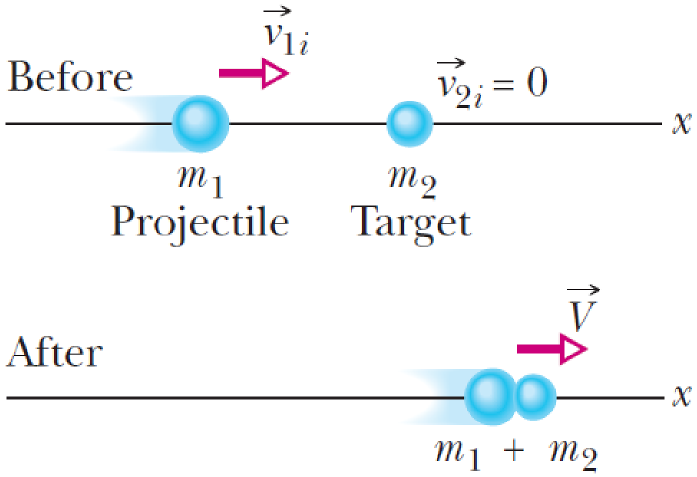
الزخم قبل = الزخم بعد

we now that $p = mv$,

$$\Longrightarrow \boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

تصادم عديم المرونة

One-Dimensional Completely Inelastic Collision



Completely inelastic collision means that both bodies stick together after collision.

Assume that Body 2 was at rest before collision ($\Longrightarrow v_{2i} = 0$), and from the law of conservation of linear momentum we can write:

$$m_1 v_{1i} + \cancel{m_2 v_{2i}} = m_1 V + m_2 V$$

V is the velocity of the stuck-together bodies after collision

$$m_1 v_{1i} = (m_1 + m_2) V$$

اذا كان الهدف ساكن نستخدم هذا القانون

$$\boxed{V = \frac{m_1}{m_1 + m_2} v_{1i}}$$

$$\boxed{\frac{m_1 v_{i1} + m_2 v_{2i}}{m_1 + m_2} = V}$$

قانون خاص لسرعة جسم بعد التصادم عديم المرونة .

Velocity of the Center of Mass

في النظام المعزول سرعة مركز الكتلة ثابتة

If the system is closed and isolated $\Longrightarrow$ no net external force ats on the system $\Longrightarrow$

the velocity of the center of mass $\vec{v}_{com}$ cannot be changed ($\vec{v}_{com}$ is constant),

We know that:

$$\vec{P} = M\vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2}$$

but $\vec{P} = \vec{p}_{i1} + \vec{p}_{i2}$

$$\therefore \boxed{\vec{v}_{com} = \frac{\vec{p}_{i1} + \vec{p}_{i2}}{m_1 + m_2}}$$

$\vec{v}_{com}$ is the velocity of com of the system

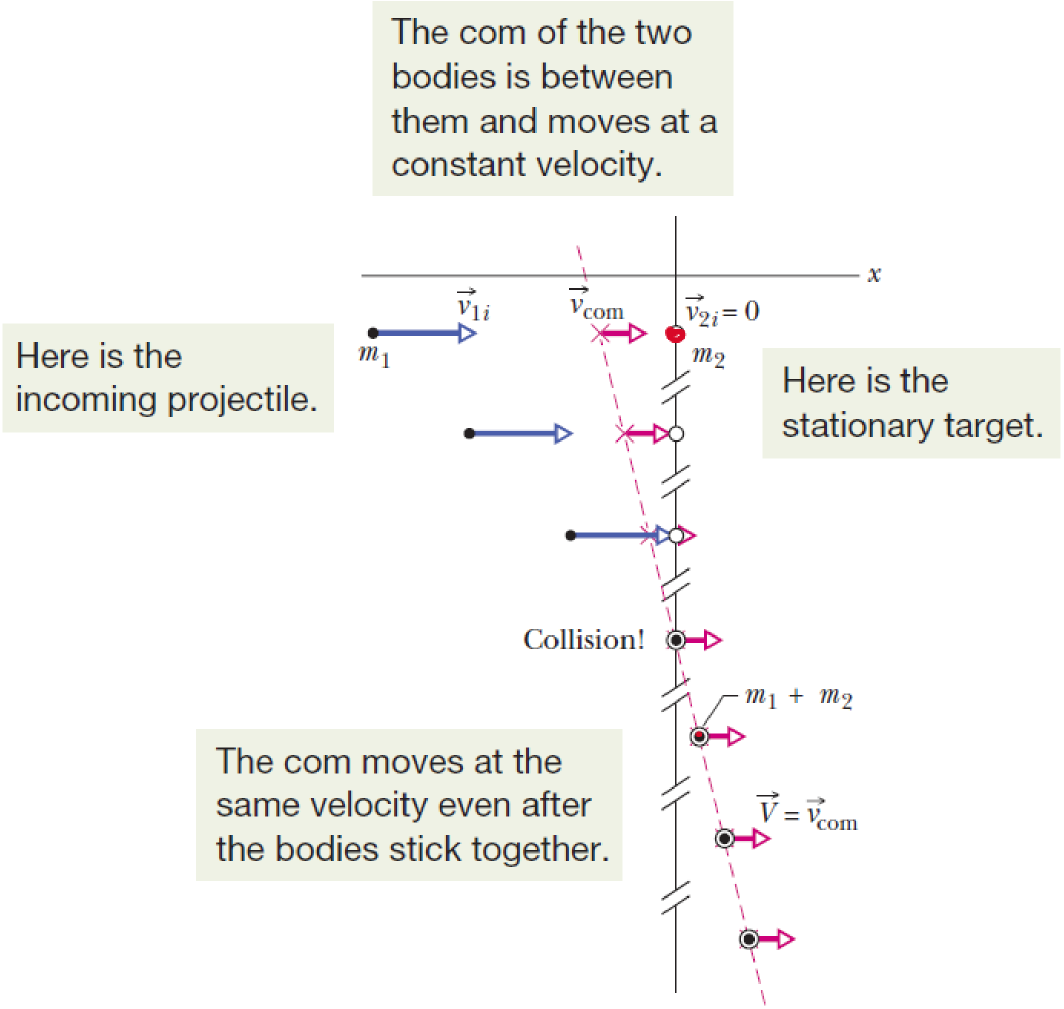
$\vec{p}_{i1}$ is initial momentum for Body 1

$\vec{p}_{i2}$ is initial momentum for Body 2

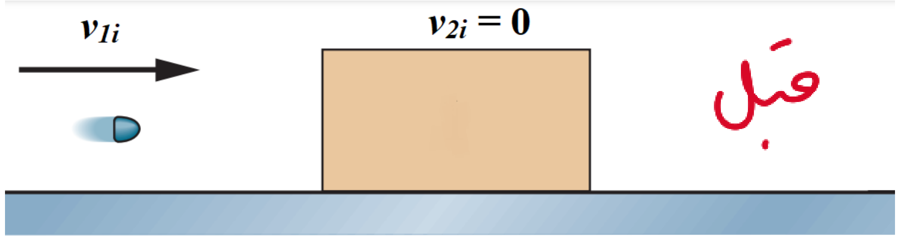
m_1 is mass of Body 1

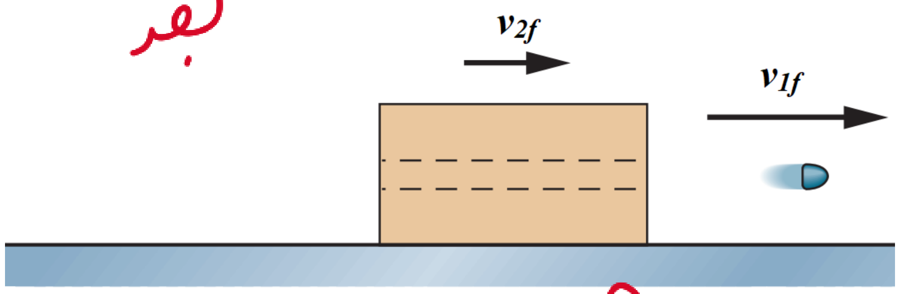
m_2 is mass of Body 2

قانون لحساب سرعة مركز الكتلة باستخدام الزخم



Example: A 3.5 g bullet is fired horizontally with a speed v_{1i} at a block at rest on a frictionless table. The bullet passes through block (mass 1.2 kg) and leaves with a speed v_{1f} = 720 m/s . The block ends up with speeds v_{2f} = 0.63 m/s. Find the speed v_{1i} of the bullet as it enters the block.





Solution:

We apply the momentum conservation because the system is closed and isolated:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$ (because the block was stationary before collision)

$$v_{1i} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1} = \frac{(3.5 \times 10^{-3} \times 720) + (1.2 \times 0.63)}{3.5 \times 10^{-3}} = 936 \text{ m/s}$$

$$P_i = P_f$$

$$m_1 V_{1i} + \cancel{m_2 V_{2i}} = m_1 V_{1f} + m_2 V_{2f}$$

$$V_{1i} = \frac{m_1 V_{1f} + m_2 V_{2f}}{m_1}$$

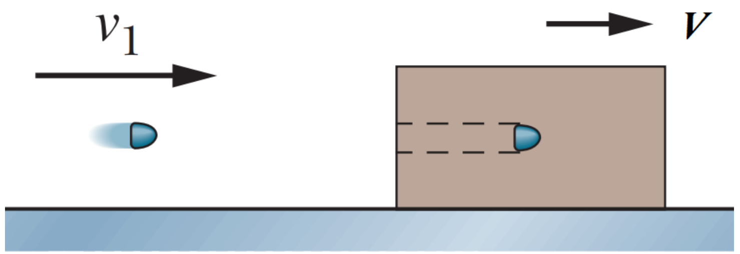
$$V_{1i} = \frac{3.5 \times 10^{-3} \times 720 + 1.2 \times 0.63}{3.5 \times 10^{-3}}$$

$$V_{i1} = 936 \text{ m/s}$$

Example: A 3.5 g bullet is fired horizontally at a block at rest on a frictionless table. The bullet embeds itself in the block (mass 1.8 kg). The block ends up with speeds V = 1.4 m/s. Find the speed v_1 of the bullet as it enters the block.

حالة تصادم عديم مرونة

Solution:



Both the bullet and the block are stuck together

$\therefore$ we can use the equation:

$$V = \frac{m_1}{m_1+m_2} v_1$$

where, m_1 is the bullet mass

m_2 is the block mass

$$v_1 = \frac{m_1+m_2}{m_1} V$$

$$v_1 = \frac{3.5 \times 10^{-3}+1.8}{3.5 \times 10^{-3}} \times 1.4 = 720 \text{ m/s}$$

$$V = \frac{m_1 \boxed{v_{1i}} + \cancel{m_2 v_{2i}}}{m_1 + m_2}$$

$$\frac{V(m_1+m_2)}{m_1} = v_{1i}$$

$$v_{1i} = \frac{1.4(3.5 \times 10^{-3}+1.8)}{3.5 \times 10^{-3}} = 720 \text{ m/s}$$

التصادم المرن

Elastic Collisions in One Dimension

- Remember: in the elastic collision, total kinetic energy of the system is conserved (does NOT change)
- Note: In the elastic collision, the kinetic energy of each body may change, but the **total** kinetic energy of the system does not change.

الطاقة الحركية الكلية ثابتة

قبل 1 → 2 →

بعد 1 → 2 →

(total kinetic energy before the collision) = (total kinetic energy after the collision)

$$\boxed{K_{1i} + K_{2i} = K_{1f} + K_{2f}}$$

$$\boxed{\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2}$$

الطاقة قبل = الطاقة بعد

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

We will study 2 cases:

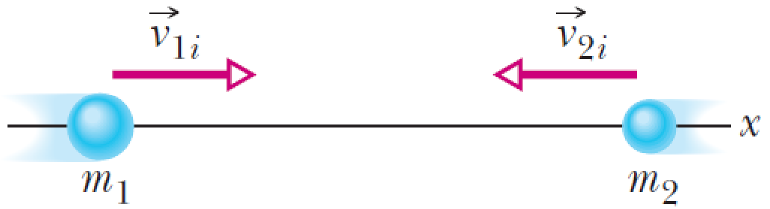
هدف ساكن

1) Stationary Target

2) Moving Target

هدف متحرك

Moving Target



Conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

قانون حفظ الزخم

Conservation of kinetic energy:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

حفظ الطاقة

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2\, m_2}{m_1 + m_2} v_{2i}}$$

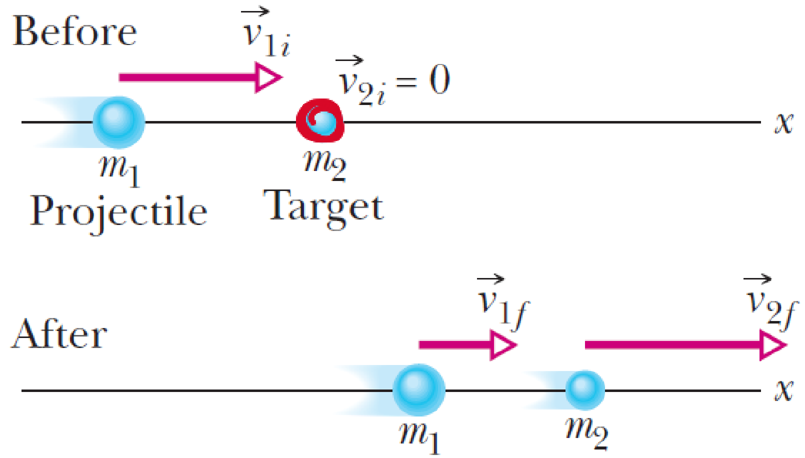
$$\boxed{v_{2f} = \frac{2\, m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}}$$

قوانين خاصة لحساب سرعة لجسمين بعد التصادم (التصادم ع ف)

From these equations we can calculate the final velocities of each body (after collision)

(أحد الجسمين ساكن) الهدف ساكن

Stationary Target



If the target is stationary (at rest) $\Longrightarrow v_{2i} = 0$

We can use the same equations from the previous slide but with $v_{2i} = 0$:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2\,m_1}{m_1 + m_2} \times 0$$

$$v_{2f} = \frac{2\,m_1}{m_1 + m_2} v_{1i} + \frac{m_1 - m_2}{m_1 + m_2} \times 0$$

$$\boxed{\begin{aligned} \boldsymbol{v_{1f}} &= \boldsymbol{\frac{m_1 - m_2}{m_1 + m_2} v_{1i}} \\ \boldsymbol{v_{2f}} &= \boldsymbol{\frac{2\,m_1}{m_1 + m_2} v_{1i}} \end{aligned}}$$

قانون خاص بسرعة الجسمين بعد التصادم

From these equations we can calculate the final velocities of each body (after collision)

Example: A ball (A) of mass 6 kg is moving at a speed 3.5 m/s, collides with another motionless ball (B) of mass 8 kg. What are the velocities of the two balls after the collision? Suppose that the collision is perfectly elastic.

m_1 V_{1i} m_2

$\vec{v}_{Ai} = 3.5\ \mathrm{m/s}$ $\vec{v}_{Bi} = 0$

$V_{2i} = 0$

Solution:

∵ The collision is elastic, and the ball (B) was at rest before collision (stationary target)

∴ we can use the equations:

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} \quad \text{and} \quad v_{Bf} = \frac{2\, m_A}{m_A + m_B} v_{Ai}$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

$$V_{1f} = \frac{6-8}{6+8} \times 3.5 = -0.5\ \mathrm{m/s}$$

- Velocity of the ball (A) after the collision: $v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} = \frac{6-8}{6+8} \times 3.5 = -0.5\ \mathrm{m/s}$
- Velocity of the ball (B) after the collision: $v_{Bf} = \frac{2\, m_A}{m_A + m_B} v_{Ai} = \frac{2 \times 6}{6+8} \times 3.5 = 3\ \mathrm{m/s}$

$$V_{2f} = \frac{2 m_1}{m_1 + m_2} m_{1i}$$

This means that ball (A) will recoil and move the negative direction of x-axis after collision with a speed of 0.5 m/s, while ball (B) will gain a speed of 3 m/s in the positive direction of x-axis after collision

$$= \frac{2 \times 6}{6+8} \times 3.5$$

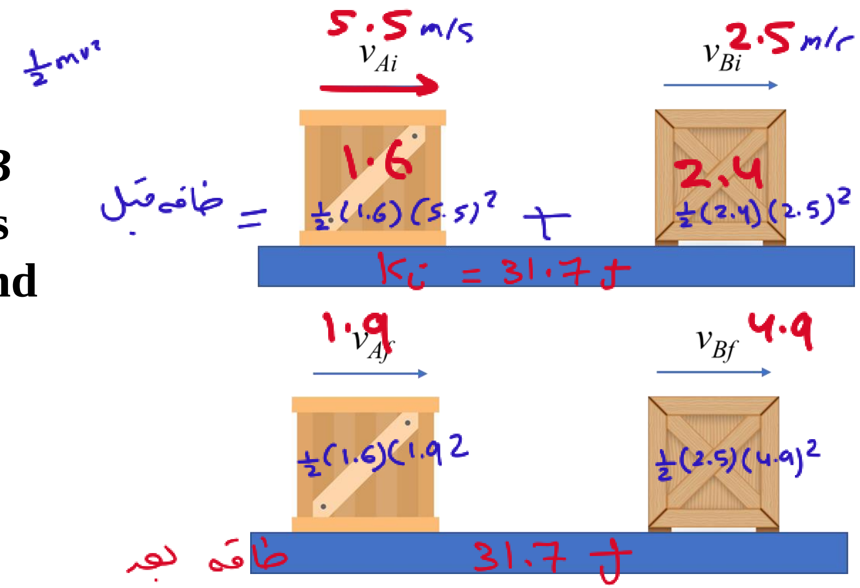
$\vec{v}_{Ai} = -0.5\ \mathrm{m/s}$ $\vec{v}_{Bi} = 3\ \mathrm{m/s}$

$$V_{2f} = 3\ \mathrm{m/s}$$

V_{2i} V_{1i}

Example: ***Box A*** **(mass 1.6 kg) slides at a velocity of 5.5 m/s into** ***Box B*** **(mass 2.4 kg) that moves with a velocity of 2.5 m/s, along a frictionless surface as shown. After collision, velocity of** ***Box A*** **becomes 1.9 m/s and velocity of** ***Box B*** **becomes 4.9 m/s. Is the collision elastic?**

هل هذا التصادم مرن



Solution:

To see whether the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision:

Before the collision: $K_i = \frac{1}{2}m_1 v_{Ai}^2 + \frac{1}{2}m_2 v_{Bi}^2 = \frac{1}{2} \times 1.6 \times 5.5^2 + \frac{1}{2} \times 2.4 \times 2.5^2 = 31.7\ \text{J}$

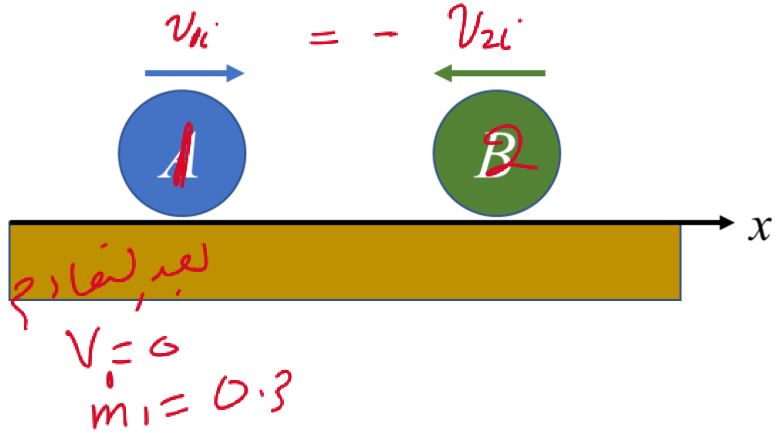
after the collision: $K_f = \frac{1}{2}m_1 v_{Af}^2 + \frac{1}{2}m_2 v_{Bf}^2 = \frac{1}{2} \times 1.6 \times 1.9^2 + \frac{1}{2} \times 2.4 \times 4.9^2 = 31.7\ \text{J}$

$\because$ the kinetic energy before collision = the kinetic energy after the collision ($K_i = K_f$),

$\therefore$ the collision is elastic.

لهذا التصادم مرن

Example: Two spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 0.3 kg, remains at rest. (a) What is the mass of the other sphere? (b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2 m/s?



Solution:

Before collision, Sphere *A* was moving in the positive direction while sphere *B* was moving in the negative direction, and both spheres had the same speed *v*:

$$v_{Ai} = v$$
$$v_{Bi} = -v$$

a)

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} + \frac{2\, m_B}{m_A + m_B} v_{Bi}$$

But we know that sphere A stopped after collision:

$$v_{Af} = 0$$

$$0 = \frac{m_A - m_B}{m_A + m_B} v + \frac{2\, m_B}{m_A + m_B}(-v)$$

$$0 = \frac{m_A - m_B}{m_A + m_B} v - \frac{2\, m_B}{m_A + m_B} v$$

$$0 = \frac{m_A - m_B}{m_A + m_B} - \frac{2\, m_B}{m_A + m_B}$$

$$0 = \frac{m_A - 3\, m_B}{m_A + m_B}$$

$$0 = m_A - 3\, m_B$$

$$0 = 0.3 - 3\, m_B$$

$$m_B = \frac{0.3}{3} = 0.1 \text{ kg}$$

b)

$$\vec{v}_{com} = \frac{\vec{p}_{Ai}+\vec{p}_{Bi}}{m_A+m_B}$$

$$\vec{v}_{com} = \frac{m_A v_{Ai}+m_B v_{Bi}}{m_A+m_B}$$

Remember that:

$v_{Ai} = v$

$v_{Bi} = -v$

$\therefore v_{Ai} = 2\ \text{m/s}$

$v_{Bi} = -2\ \text{m/s}$

$$\vec{v}_{com} = \frac{m_A v_{Ai}+m_B v_{Bi}}{m_A+m_B}$$

$$= \frac{0.3\times 2+0.1\times(-2)}{0.3+0.1}$$

$$= \frac{0.3\times 2-0.1\times 2}{0.3+0.1}$$

$$= 1\ \text{m/s}$$

$$v_{1f} = 0 = \frac{m_1-m_2}{m_1+m_2} v - \frac{2m_2}{m_1+m_2} v$$

$$\frac{m_1-m_2}{m_1+\cancel{m_2}} \cancel{v} = \frac{2m_2 \cancel{v}}{m_1+\cancel{m_2}}$$

$$m_1 - m_2 = 2m_2$$

$$m_1 = 3m_2$$

$$0.3 = 3m_2$$

$$\boxed{m_2 = \frac{0.3}{3} = 0.1}$$

$$V_{com} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$= \frac{m_1 v - m_2 v}{m_1 + m_2}$$

$$= \frac{0.3(2) - (0.1)(2)}{0.3 + 0.1}$$

$$= 1 \text{ m/s}$$