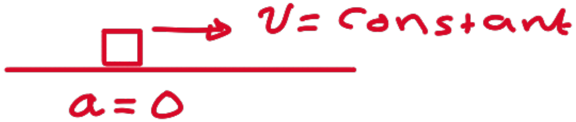
Chapter 1

Circular Motion-Part 1

الحركة الدائرية

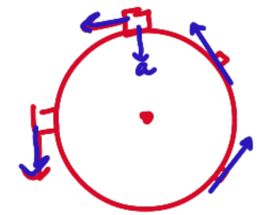
Chapter Outline

- 4.4 Uniform Circular Motion
- 4.5 Tangential and Radial Acceleration



الحركة الدائرية المنتظمة

4.4 - Uniform Circular Motion



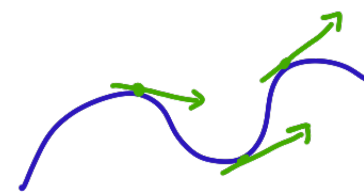
Definition

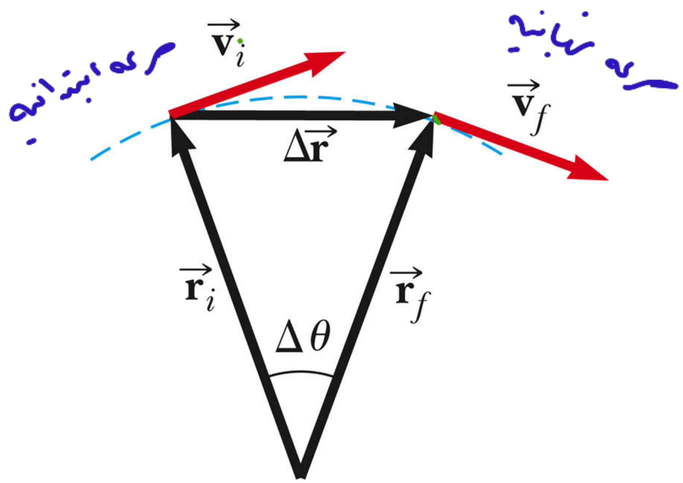
- Uniform circular motion occurs when an object moves in a circular path with a ***constant speed*** سرعة ثابتة

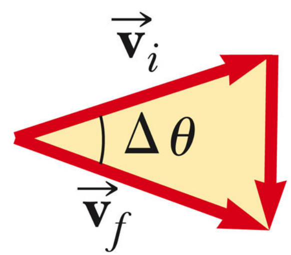
  اتجاه السرعة يختلف اذا هناك تسارع
- An acceleration exists since the direction of the motion is changing (the direction of velocity is changing)

  مماس الحركة = اتجاه السرعة
- The velocity vector is always ***tangent*** to the path of the object

الحركة الدائرية
لها تسارع لان
اتجاه السرعة ثابت







$\Delta\vec{v} = \bar{V}_f - \bar{V}_i$

4.4 - Uniform Circular Motion

التسارع المركزي

Centripetal Acceleration

اتجاه التسارع يكون عمودي على اتجاه الحركة
واتجاهه نحو المركز

- The acceleration vector in uniform circular motion is always ***perpendicular to the path*** and always points toward the center of the circle.

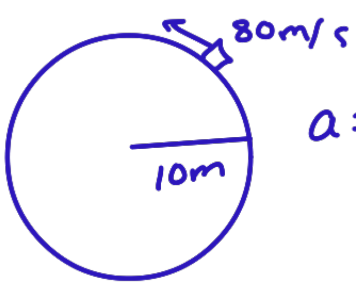
التسارع المركزي

- An acceleration of this nature is called a centripetal acceleration a_c (centripetal means center-seeking)

التسارع المركزي ليس ثابت لأنه اتجاهه متغير لكن مقداره ثابت

- The centripetal acceleration is not constant, since it ***changes direction*** as the object moves along the circular path.

- The ***magnitude*** of the centripetal acceleration is ***constant*** in a uniform circular motion, and it is given by



$$a = \frac{80^2}{10} = \frac{6400}{10} = 640\ m/s^2$$

$$a_c = \frac{v^2}{r}$$

التسارع المركزي = السرعة² / نصف القطر

4.4 - Uniform Circular Motion

الزمن الدوري

Period T

الزمن اللازم لاتمام دوره كاملة

- The period, $\boxed{T,}$ is the time required for one complete revolution.

- In one revolution, an object travels a distance equals to the circumference of the circular path. That is, $2\pi r$, where r is the radius of the circular path.

$\frac{2\pi r}{v}$

الزمن الدوري: مسافة دوره كاملة على الحركة

- Therefore, the period would be the distance traveled in one revolution divided by the speed,

$$T = \frac{2\pi r}{v}$$

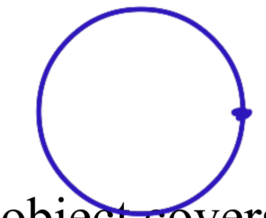
الزمن الدوري = $\frac{2 \times \pi \times \text{نصف القطر}}{\text{الحركة}}$

(S)

4.4 - Uniform Circular Motion

السرعة الزاوية

Angular Speed



- In one full revolution of the particle around a circle, the object covers an angle of 2π. If this angle is divided by the period, we get the angular speed,

$$\boxed{\omega = \frac{2\pi}{T}}$$

$\frac{2\pi}{\text{الزمن الدوري}}$ = السرعة الزاوية

$\text{rad/s} = \frac{1}{s} = s^{-1}$

- The unit for the angular speed is rad/s or s^{-1}.

- From the expression of the angular speed and the period, we can write:

السرعة = السرعة الزاوية × نصف القطر

$$v = r\omega$$

$v \rightarrow m/s \quad r \rightarrow m \quad \omega \rightarrow s^{-1}$

$$\omega = \frac{\cancel{2\pi}}{\frac{\cancel{2\pi} r}{v}} \Rightarrow \omega = \frac{v}{r}$$

$$\boxed{v = \omega r}$$

- This enable us to write the centripetal acceleration as

قانون التسارع المركزي بدلالة ω

$$\boxed{a_c = r\omega^2}$$

$$a_c = \frac{v^2}{r} = \frac{\omega^2 r^{\cancel{2}}}{\cancel{r}}$$

4.4 - Uniform Circular Motion

Quick Quiz: 1

Which of the following correctly describes the centripetal acceleration vector for a particle moving in a circular path?

a) Constant and always perpendicular to the velocity vector for the particle.

b) Constant and always parallel to the velocity vector for the particle.

c) Of constant magnitude and always perpendicular to the velocity vector for the particle.

d) Of constant magnitude and always parallel to the velocity vector for the particle.

4.4 - Uniform Circular Motion

Quick Quiz: 2

A particle moves in a circular path of radius r with speed v.
It then increases its speed to $2v$ while traveling along the same circular path. The centripetal acceleration of the particle has changed by what factor?

$$a_c = \frac{v^2}{r} = \frac{(2v)^2}{r} = 4\frac{v^2}{r}$$

a) 0.25

b) 0.5

c) 2

$\boxed{\text{d) } 4}$

From the same choices, by what factor has the period of the particle changed? 0.5

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{2v} = \boxed{\frac{1}{2}T}$$

4.4 - Uniform Circular Motion

Example 4.6 : The Centripetal Acceleration of The Earth

A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun. (The radius of Earth's orbit around the Sun is 1.496×10^{11}m) r

B) What is the angular speed of the Earth in its orbit around the Sun.

A) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi \times 1.496 \times 10^{11}}{365.25 \times 24 \times 60 \times 60} = 29785.68 \text{ m/s}$

$a_c = \dfrac{v^2}{r} = \dfrac{(29785.68)^2}{1.496 \times 10^{11}} = 0.00593$

$= 5.93 \times 10^{-3} \text{ m/s}^2$

B) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{365.25 \times 24 \times 60 \times 60} = 1.99 \times 10^{-7} \text{ rad/s}$

4.4 - Uniform Circular Motion

Example 4.6 : The Centripetal Acceleration of The Earth

Given

$$r = 1.496 \times 10^{11}\text{m}$$

We know that the Earth completes on revolution around the Sun in 365.25 days.

$$T = 365.25\text{days}\left(\frac{24\text{ hours}}{1\text{ day}}\right)\left(\frac{60\text{ min}}{1\text{hours}}\right)\left(\frac{60\text{s}}{1\text{min}}\right) = 3.156\times10^{7}\text{s}$$

Solution

A) $$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r^{\cancel{2}}}{\cancel{r}T^2} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(1.496 \times 10^{11})}{(3.156\times10^{7})^2} = 5.93\times10^{-3}\text{m/s}^2$$

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T}$$

B) $$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.156\times10^{7}} = 1.99\times10^{-7}\text{ rad/s}$$

المماسي الدائري التسارع

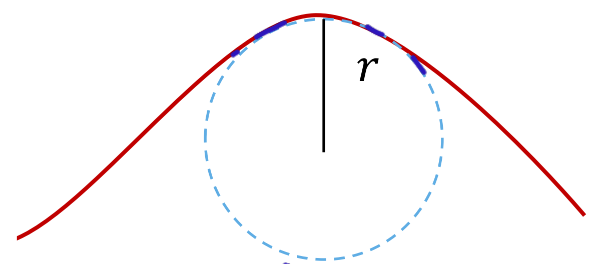
4.5 - Tangential and Radial Acceleration

a_t a_r

Description

سندرس في هذا الجزء حركة جسم في مسار منحني والحركة متغيرة

- In this section, we consider the motion of an object along a ***smooth curved path*** with the ***velocity changing*** in both magnitude and direction.



- To characterize the amount of curvature the path exhibits, a dashed circle is drawn in the point of interest. The radius of the dashed circle represents the ***radius of the curvature***. The smaller the radius, the greater the curvature.

نصف قطر الانحناء

4.5 - Tangential and Radial Acceleration

السرعه دائماً مماس لمسار الحركه

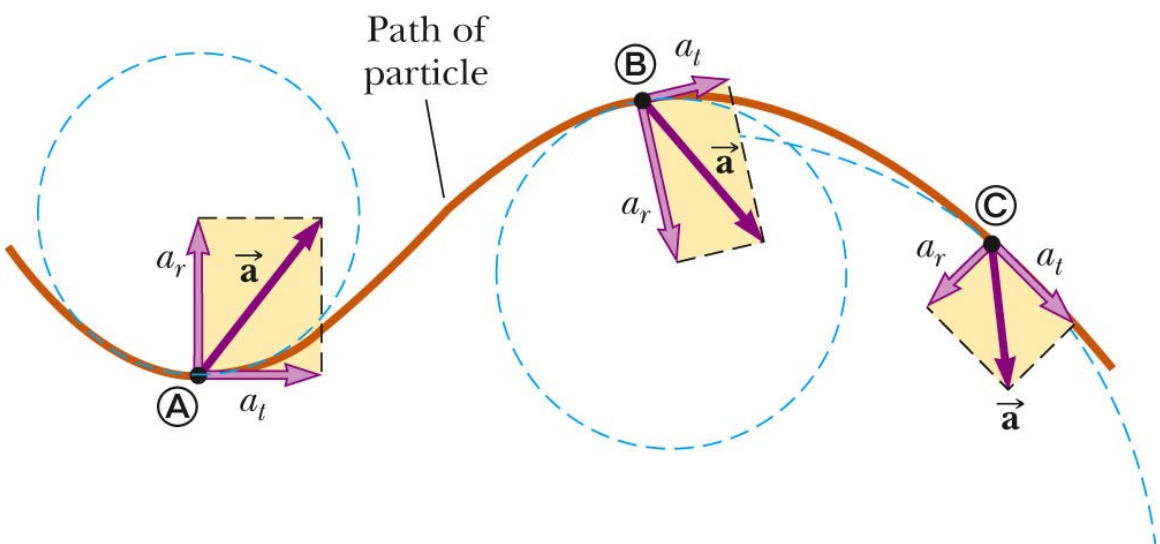
- The velocity vector is always ***tangent*** to the path.

التسارع يكون مركبتين

- The acceleration vector is at some angle to the path . The acceleration vector can be resolved into two components: a $\boxed{\text{radial component } a_r}$ and a $\boxed{\text{tangential component } a_t}$.

تسارع دائري

تسارع مماسي



4.5 - Tangential and Radial Acceleration

Tangential Acceleration

مركبة مسؤولة عن تغيير مقدار سرعة الجسم وتمثل التغير في السرعة اللحظية

- The ***tangential acceleration*** component causes the ***change in the speed*** of the particle. This component is parallel to the instantaneous velocity, and is given by

25

50 60

$$a_t = \left|\frac{dv}{dt}\right|$$

التسارع المماسي = مشتقة السرعة

$a_t = \frac{60-50}{2} = 5 m/s^2$

اتجاه التسارع المماسي a_t

- The direction of the tangential component vector ($\vec{a}_t$):

اذا زادت السرعة التسارع مع اتجاه الحركة

v increasing

Same direction as $\vec{v}$

v, a_t

اذا كانت السرعة متناقصة اتجاه التسارع عكس اتجاه الحركة

v decreasing

Opposite $\vec{v}$

v, a_t

4.5 - Tangential and Radial Acceleration

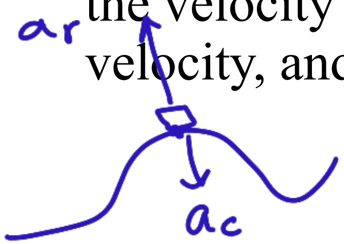
التسارع الدائري

Radial Acceleration

التسارع الدائري تكونه بسبب تغير اتجاه الحركة

- The ***radial acceleration*** component arises from the ***change in direction*** of the velocity vector. This component is perpendicular to the instantaneous velocity, and is given by

اتجاهه عمودي على اتجاه الحركة



$$a_r = -a_c = -\frac{v^2}{r}$$

التسارع الدائري = ـ التسارع المركزي

where r is the radius of curvature.

- At a given speed, a_r is large when the radius of curvature is small and a_r is small when the radius of curvature is large.

كلما زادت r قلت ar
كلما قلت r زادت ar

- The minus sign indicates that the component points opposite the radial unit vector $\hat{\mathbf{r}}$ which points radially outward from the center of the circle (more on this in the next slides).

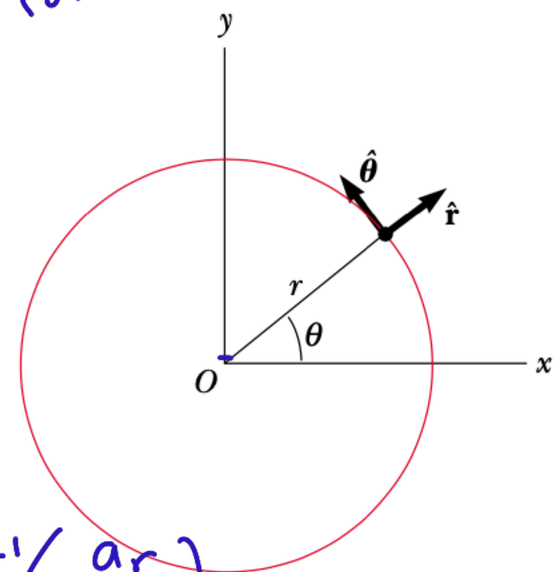
الاشارة السالبة تدل على ان اتجاه التسارع الدائري
عكس اتجاه التسارع المركزي (اتجاهه دائما خارج مركز الدائرة (

4.5 - Tangential and Radial Acceleration

Total Acceleration

التسارع الكلي = a مجموع مركبتي التسارع

$\bar{a}_r + \bar{a}_t$



- The total acceleration in terms of the components vector is:

$$\vec{a} = \vec{a}_t + \vec{a}_r$$

- The magnitude is

المقدار

$$|a| = \sqrt{a_t^2 + a_r^2}$$

$$\theta = \tan^{-1}\left(\frac{a_r}{a_t}\right)$$

- The acceleration in terms of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ is:

الاتجاه

$$\boxed{\vec{a} = \left|\frac{dv}{dt}\right|\hat{\boldsymbol{\theta}} - \frac{v^2}{r}\hat{\mathbf{r}}}$$

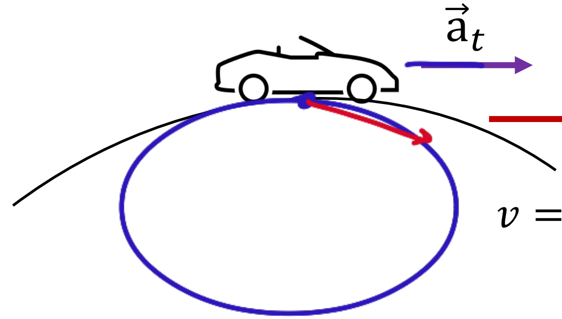
- The origin is taken to be the center of the circle.
- $\hat{\mathbf{r}}$ points radially outword.
- $\hat{\boldsymbol{\theta}}$ points in the direction of increasing θ perpendicular to $\hat{\mathbf{r}}$.

4.5 - Tangential and Radial Acceleration

Example 4.7: Over The Rise

موازي للطريق = شارع مصاحب

A car exhibits a constant acceleration of 0.300 m/s² parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction of the total acceleration vector for the car at this instant?



$$\boxed{a_t = 0.3 \ m/s^2}$$

$$r = 500 \ m$$

$$V = 6 \ m/s$$

$$a_r = -\frac{v^2}{r} = -\frac{(6)^2}{500} = -0.072 \ m/s^2$$

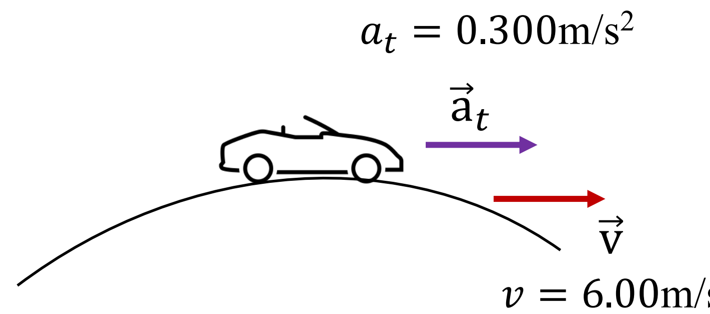
$$|a| = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072)^2 + (0.3)^2} = 0.309 \ m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{-0.072}{0.3}\right) = -13.5^\circ$$

4.5 - Tangential and Radial Acceleration

Example 4.7: Over The Rise

A car exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction of the total acceleration vector for the car at this instant?



Given

$a_t = 0.300 \text{m/s}^2$

$v = 6.00 \text{m/s}$

$r = 500 \text{m}$

Solution

The question asks for the direction of the total acceleration vector

$$\theta = \tan^{-1}\left(\frac{a_r}{a_t}\right)$$

$$a_r = -a_c = -\frac{v^2}{r} = -\frac{6^2}{500} = \underline{-0.072 \text{m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{-0.072}{0.3}\right)$$

$$\Rightarrow \theta = -13.5°$$ (To the horizontal)

Chapter 1- Part 2

Circular Motion and Other Applications of Newton's Laws

Chapter Outline

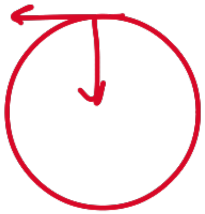
- 6.1 Extending the Particle in Uniform Circular Motion Model
- 6.2 Non-Uniform Circular Motion

Introduction

- Two analysis models using Newton's Laws of Motion have been developed.
- The models have been applied to linear motion.
- Newton's Laws can be applied to other situations: such as objects traveling in circular paths

6.1 - Particle in Uniform Circular Motion Model

Acceleration



$$a_c = \frac{v^2}{r}$$

- A particle moves with a constant speed in a circular path of radius r with an acceleration.

- The magnitude of the acceleration is given by

$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration, $\vec{\mathbf{a}}_C$, is directed ***toward the center*** of the circle.

- The centripetal acceleration is always perpendicular to the velocity.

6.1 - Particle in Uniform Circular Motion Model

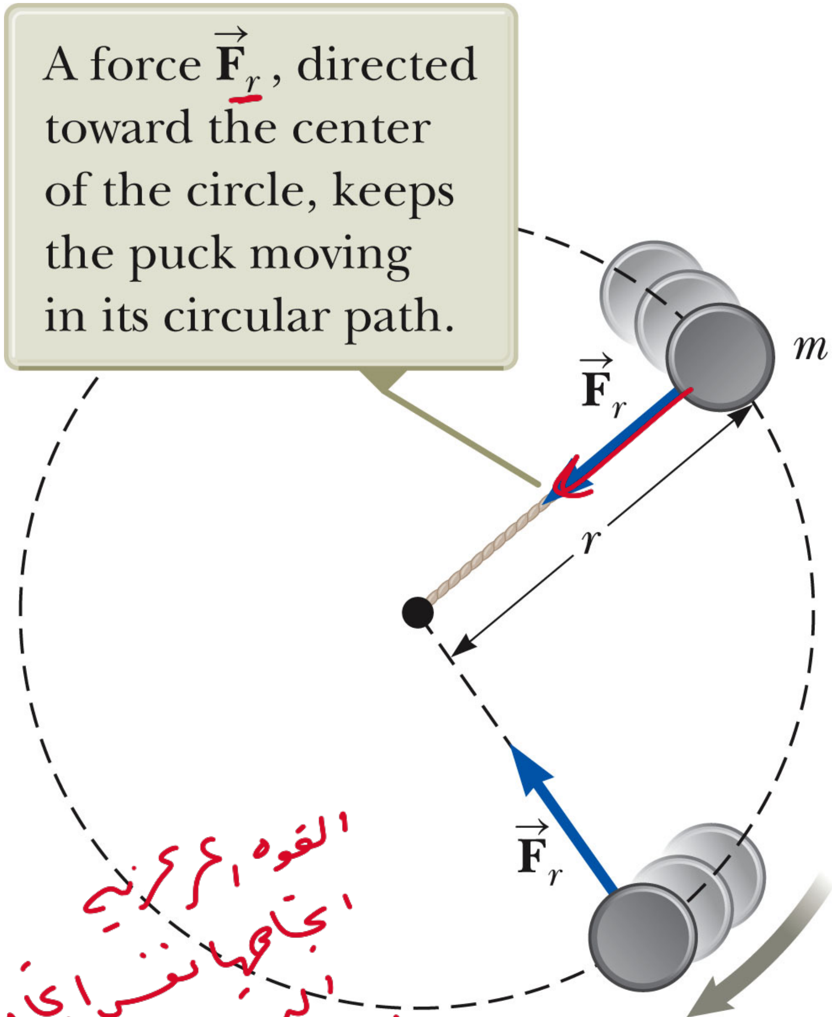
Force

$F_r = ma_c$

- A force, $\vec{\mathbf{F}}_r$, is associated with the centripetal acceleration.
- The force is also directed toward the center of the circle. نحو المركز.
- Applying Newton's Second Law along the radial direction gives

$$\sum F = ma_c = m\frac{v^2}{r}$$

$$F_r = ma_c = \frac{mv^2}{r}$$



القوه المركزيه
اتجاهها نفس اتجاه
المركز

بندول مخروطي

Example 6.1: The Conical Pendulum

A small ball of mass m is suspended from a string of length L. The ball revolves with a constant speed v in a horizontal circle of radius r as shown in the Figure. (This system is called the *Conical Pendulum*). Show that: $v = \sqrt{Lg sin\theta \tan\theta}$

اشتقاق القانون

$$\sum F_y = T\cos\theta - mg = 0$$

(1) $T\cos\theta = mg$

(2) $\sum F_x = T\sin\theta = ma_c = \dfrac{mv^2}{r}$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

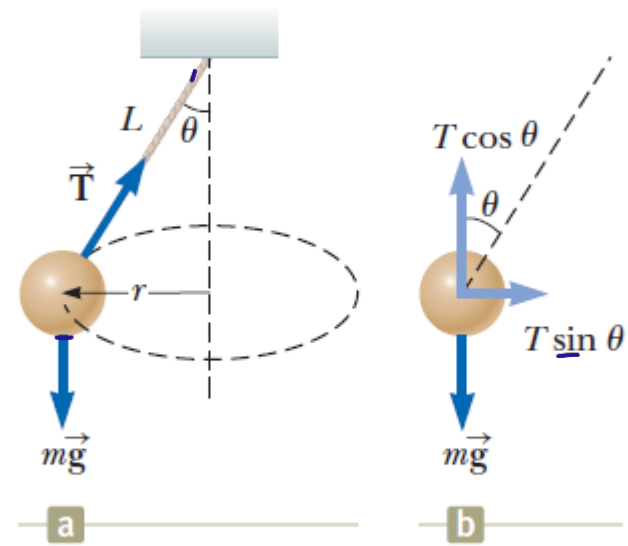
$T\cos\theta = mg$

$T\sin\theta = \dfrac{mv^2}{r}$

$\tan\theta = \dfrac{v^2}{gr}$

$v = \sqrt{gr\tan\theta}$

$v = \sqrt{l\sin\theta\, g\tan\theta}$



$$\boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

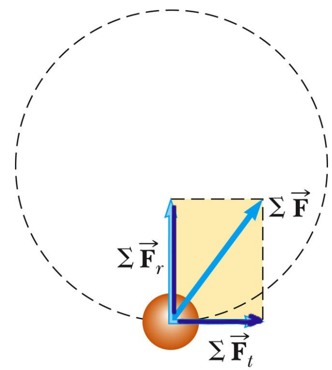
القوه الكليه = القوه المماسيه F_t
القوه الدورانيه F_r

6.2 Nonuniform Circular Motion

From Newton's 2nd law of motion, $\sum \vec{F} = m\vec{a}$, the existence of nonzero $\vec{a}$ implies that a net force $\sum \vec{F}$ also exists and in the same direction. Therefore, if the total acceleration $\vec{a}$ has radial and tangential components, $\vec{a} = \vec{a}_r + \vec{a}_t$, this implies the net F

exerted on the particle also has net radial and tangential components, as shown in the figure, i.e.:

$$\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t$$



a_r

- The vector $\sum \vec{F}_r$ is directed toward the center of the circle and is responsible for the centripetal acceleration (i.e. responsible for changing the direction of v).

a_r مماس

- The vector $\sum \vec{F}_t$ tangent to the circle is responsible for the tangential acceleration, which represents a change in the particle's speed with time.

Example 6.6: Keep Your Eye on the Ball

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a vertical circle about a fixed point O as illustrated in the figure. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

دايره طوليه

a_t

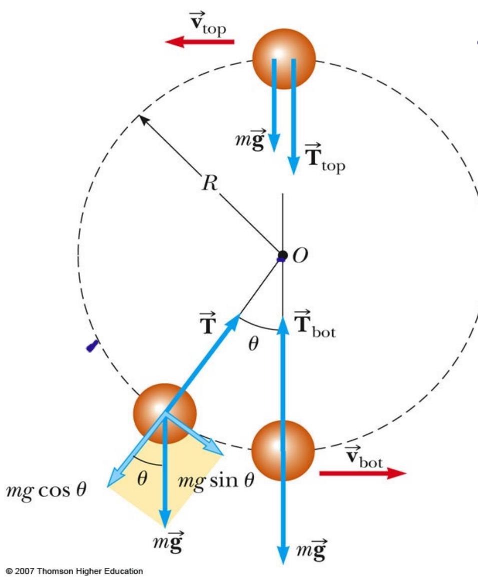
T

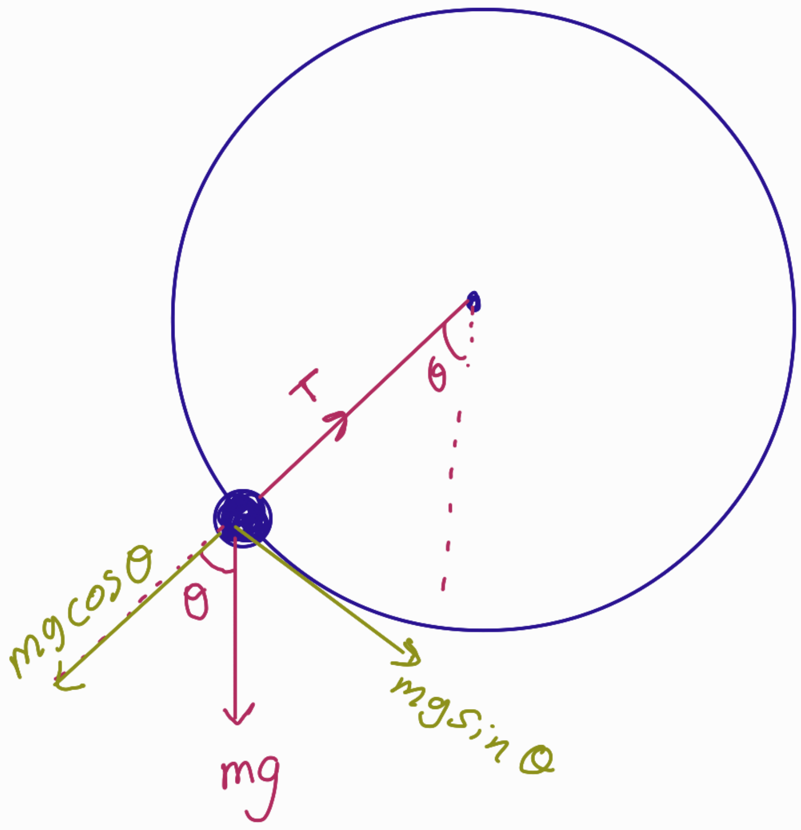
$$\sum F_t = mg\sin\theta = ma_t$$

$$a_t = \boxed{g\sin\theta}$$

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = \boxed{mg\left(\frac{v^2}{Rg} + \cos\theta\right)}$$





محصلة القوى باتجاه مركزي

$$F_r = \frac{mv^2}{R}$$

F_t : القوه المماسيه

$$F_t = mg\sin\theta$$

$$\frac{F_t}{m} = a_t$$

$$\boxed{a_t = g\sin\theta}$$

$$F_r = T - mg\cos\theta$$

$$\frac{mv^2}{R} = T - mg\cos\theta$$

$$T = \frac{mv^2}{R} + mg\cos\theta$$