4. What is the negation of each of these propositions?

a) Jennifer and Teja are friends.
b) There are 13 items in a baker's dozen.
c) Abby sent more than 100 text messages every day.
d) 121 is a perfect square.

a) Jennifer and Teja are not friends.
b) There are not 13 items in a baker's dozen.
c) Abby sent fewer than 101 text messages yesterday.
d) 121 is not a perfect square.

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \rightarrow \neg q$ **e)** $\neg q \rightarrow p$ **f)** $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

a) Sharks have not been spotted near the shore.
b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.
c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.
d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.
e) If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.
f) If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.
g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.
h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed
or sharks have not been spotted near the shore

12. Let p, q, and r be the propositions

p : You have the flu.
q : You miss the final examination.
r : You pass the course.

Express each of these propositions as an English sentence.

a) $p \rightarrow q$
b) $\neg q \leftrightarrow r$
c) $q \rightarrow \neg r$
d) $p \vee q \vee r$
e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
f) $(p \wedge q) \vee (\neg q \wedge r)$

a) If you have the flu, then you miss the final exam.

b) You do not miss the final exam if and only if you pass the course.

c) If you miss the final exam, then you do not pass the course.

d) You have the flu, or miss the final exam, or pass the course.

e) It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course

f) Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

14. Let p, q, and r be the propositions

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

a) $r \wedge \neg q$

b) $p \wedge q \wedge r$

c) $r \rightarrow p$

d) $p \wedge \neg q \wedge r$

e) $(p \wedge q) \rightarrow r$

f) $r \leftrightarrow (q \vee p)$

23. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements.]

a) It snows whenever the wind blows from the northeast.

b) The apple trees will bloom if it stays warm for a week.

c) That the Pistons win the championship implies that they beat the Lakers.

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

e) To get tenure as a professor, it is sufficient to be world-famous.

f) If you drive more than 400 miles, you will need to buy gasoline.

g) Your guarantee is good only if you bought your CD player less than 90 days ago.

h) Jan will go swimming unless the water is too cold.

a) If the wind blows from the northeast, then it snows

b) If it stays warm for a week, then the apple trees will bloom

c) If the Pistons win the championship, then they beat the Lakers.

d) If you get to the top of Long's Peak, then you must have walked eight miles.

e) If you are world famous, then you will get tenure as a professor.

f) If you drive more than 400 miles, then you will need to buy gasoline.

g) If your guarantee is good, then you must have bought your CD player less than 90 days ago.

h) If the water is not too cold, then Jan will go swimming.

27. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

a) converse : "I will ski tomorrow only if it snows today."
contrapositive : ''If I don't ski tomorrow, then it will not have snowed today."
inverse :"If it does not snow today, then I will not ski tomorrow."
b) "If there is going to be a quiz, then I will come to class."
converse : "If I come to class, then there will be a quiz."
contrapositive :"If I don't come to class, then there won't be a quiz."
inverse : "If there is not going to be a quiz, then I don't come to class.''
c) converse : "A positive integer is a prime if it has no divisors other than 1 and itself."
contrapositive : "If a positive integer has a divisor other than 1 and itself, then it is not prime
The inverse : "If a positive integer is not prime, then it has a divisor other than 1 and itself."

28. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

a) Converse: If I stay home, then it will snow tonight.
Contrapositive: If I do not stay at home, then it will not snow tonight.
Inverse: If it does not snow tonight, then I will not stay home.
b) Converse: Whenever I go to the beach, it is a sunny summer day.
Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day.
Inverse: Whenever it is not a sunny day, I do not go to the beach.
c) Converse: If I sleep until noon, then I stayed up late.
Contrapositive: If I do not sleep until noon, then I did not stay up late.
Inverse: If I don’t stay up late, then I don’t sleep until noon.

35. Construct a truth table for each of these compound propositions.

a) $p \to \neg q$ **b)** $\neg p \leftrightarrow q$

c) $(p \to q) \vee (\neg p \to q)$ **d)** $(p \to q) \wedge (\neg p \to q)$

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

a

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

b

p	q	$\neg p$	$\neg p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

c

p	q	$p \to q$	$\neg p$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

e

p	q	$p \to q$	$\neg p$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

d

p	q	$p \to q$	$\neg p$	$\neg p \to q$	$(p \to q) \wedge (\neg p \to q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

f

p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

36. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$ **b)** $(p \vee q) \wedge r$

c) $(p \wedge q) \vee r$ **d)** $(p \wedge q) \wedge r$

e) $(p \vee q) \wedge \neg r$ **f)** $(p \wedge q) \vee \neg r$

a)

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

b)

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

c)

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

d)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

e)

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \wedge \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

f)

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T