Discrete Structures

Lecture 1

1.1 Propositional Logic

المنطق الظرفي

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- The **rules** of logic give precise meaning to **mathematical statements**.
- The rules are used to distinguish between valid and invalid mathematical statements.

جملة جزمية . جملة خبرية .

- A **proposition** (or **Statement**) is a declarative sentence that is either true or false, but not both. جمله خبريه تحتمل الصواب او الخطأ وليس كلاهما
- Examples of propositions:
  - The Moon is made of green cheese. F
  - Toronto is the capital of Canada. T
  - $1 + 0 = 1$ T
  - $0 + 0 = 2$ F
- Examples of not propositions.
  - Sit down! ✗
  - What time is it? ✗
  - $x + 1 = 2$ ✗
  - $x + y = z$ ✗

لا نعتبر الجمله خبريه عندما

1- امر

2- سؤال

3- متغيرات

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Constructing Propositions

متغير حرفي

- **Propositional Variables or (statement variables):** p, q, r, s, ...
  These variables represent propositions. It is just letters!!
- The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic**. منطق حرفي
- قيمة الصدق
  The truth value of a proposition is true, denoted by (T), if it is a true proposition, and the truth value of a proposition is false, denoted by (F), if it is a false proposition.
  جمل خبرية مركبة
  ادوات الوصل المنطقية
- **Compound Propositions**; constructed from logical connectives and other propositions.
  - النفي Negation $\neg$
  - و Conjunction $\wedge$
  - أو Disjunction $\vee$
  - شرطية Implication $\rightarrow$
  - Biconditional $\leftrightarrow$

  اداه شرط ثنائية

p + q

وصل

Compound Proposition: **Negation**

اداة النفي

- The negation of a proposition p is denoted by ¬p and has this truth table:

جدول

السيارة زرقاء

p	¬p
T	F
F	T

$p \Rightarrow \neg p$

- **Example:** Find the negation of the proposition
  "Faisal's PC runs Linux"
  and express this in simple English.
  Answer: Faisal's PC does not run Linux

على الأقل
at least ⇒ more
على الأكثر
at most
less than

- **Example:** Find the negation of the proposition
  "Ahmed's smartphone has at least 32GB of memory"
  and express this in simple English
  Answer: Ahmed's smartphone has less than 32GB of memory.

Compound Proposition: Conjunction

$p \wedge q$ السماء صافية $\wedge$ الجو حار

- The conjunction of propositions p and q is denoted by $p \wedge q$ "p and q", and has this truth table:

and

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

تكون جمله $\wedge$ صحيحه
اذا كانت الجملتين
T فقط

- **Example:** If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."
- Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

and

Compound Proposition: **Disjunction**

أو
or

- The disjunction of propositions p and q is denoted by $p \vee q$ "p or q", and has this truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

جمله او تكون
خاطئة في حاله
واحدة FF

- **Example:** Students who have taken calculus or computer science can take this class.

$p \vee q$

Inclusive Or

Or

Inclusive exclusive

$\vee$ $\oplus$

يجمع بالاثنين TT ممنوع يكون $\boxed{TT}$

$\vee$

"**Inclusive Or**" - In the sentence "Students who have taken CS202 or Math120 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For p $\vee$ q to be true, either one or both of p and q must be true.

يجوز احدهما او كلاهما

T T $\Rightarrow$ T

T F $\Rightarrow$ T

F T $\Rightarrow$ T

Compound Propositions: **Exclusive Or (Xor)**

- Let p and q be propositions. The exclusive or of p and q (Xor), denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

تكون الجملة صحيحة اذا كان أحدهما صحيح وليس كلاهما

- The truth table of Exclusive Or (Xor) is:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- **Example:** Students who have taken calculus or computer science, but not both, can enroll in this class.

ليس كلاهما

$p \oplus q$

 مسببه $P \to q$

مسبب نتيجة

- Let p and q be propositions. The conditional statement $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the hypothesis فرضية (or antecedent or premise) and q is called the conclusion نتيجه (or consequence)
- The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

If P then q

اذا اتيت سوف اكون سعيد

تكون جملة $P \to q$ خاطئة في حالة واحد

$T \to F$

Implication

$p \rightarrow q$

سبب

نتيجه

- **Example:** If you get 100% on the final, then you will get an A.
- In p→q, there is no needs to be connection between the antecedent or the consequent. For example, If the moon is made of green cheese, then I have more money than Bill Gates.

Implication

- One way to view the logical conditional is to think of an obligation or contract.

  p → q

  - "If I am elected, then I will lower taxes."
  - "If you get 100
- If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

اذا تم انتخابي سوف اخفض الضرائب

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Implication

→ طرق اخرى للتعبير عن

- There are other ways to express the conditional statement $p \to q$ such as:
  - if p, then q
  - p implies q
  - if p, q
  - p only if q
  - q unless $\neg p$
  - q when p
  - q if p
  - q whenever p
  - p is sufficient for q
  - q follows from p
  - q is necessary for p
  - a necessary condition for p is q
  - a sufficient condition for q is p

if then

$p \to q$

سبب

نتيجه

Implication

if p then q
q when p

- **Example:** Let p be the statement "Maria learns discrete mathematics", and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

  Answer:
  - If Maria learns discrete mathematics, then she will find a good job
  - Maria will find a good job when she learns discrete mathematics.
  - For Maria to get a good job, it is sufficient for her to learn discrete mathematics.
  - Maria will find a good job unless she does not learn discrete mathematics

$q \rightarrow \neg p$

سوف تحصل على وظيفه جيده الا اذا لم تدرس جيدا

Converse, Contrapositive, and Inverse

$p \rightarrow q$

$q \rightarrow p$ $\neg q \rightarrow \neg p$ $\neg p \rightarrow \neg q$

جملة (Contrapositive) نفس قيمة الصدق للجملة الاصلية

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

- The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$.
- The proposition $\neg q \rightarrow \neg p$ is called the **contrapositive** of $p \rightarrow q$.
- The proposition $\neg p \rightarrow \neg q$ is called the **inverse** of $p \rightarrow q$.
- The contrapositive has the same truth value as the $p \rightarrow q$.
- The inverse and converse has the same truth value. We called them inverse and converse are equivalent.

و جمله (inverse) و جمله (converse) لها نفس قيمة الصدق

$$q \rightarrow p \equiv \neg p \rightarrow \neg p$$

Continue

If p then q

$\neg q \to \neg p$ $q \to p$ $-p \to -p$

- **Example:** What are the contrapositive, the converse, and the inverse of the conditional statement
  "The home team wins whenever it is raining?"
  الفريق سوف يفوز في حال كانت تمطر
  Answer: Because "q whenever p" is one of the ways to express the conditional statement p → q, the original statement can be rewritten as
  الخطوة الأولى حولناها إلى شكل If, then
  "If it is raining, then the home team wins"
  إذا كانت تمطر سوف يفوز الفريق
  $-q \to -p$ **contrapositive:** "If the home team does not win, then it is not raining."
  $q \to p$ **converse:** "If the home team wins, then it is raining."
  inverse: "If it is not raining, then the home team does not win."
  $-p \to -q$
  loss

Biconditional $\longleftrightarrow$ If and only If

- Let p and q be propositions. The **biconditional** statement $p \leftrightarrow q$ is the proposition "p if and only if q." The **biconditional** statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. **Biconditional** statements are also called **bi-implications**. ثنائية التبعية
- The Truth Table for the Biconditional $p \leftrightarrow q$ is

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\longleftrightarrow$ بدأله

If and only If

- There are other ways to express the bioconditional statments
  - p is necessary and sufficient for q
  - if p then q, and conversely
  - p iff q "iff= if and only if"

$p \leftrightarrow q$

$p \leftrightarrow q =$

- $p \leftrightarrow q$ has exactly the same truth value as $\boxed{(p \to q) \land (q \to p)}$.
- **Example:** Let p be the statement "You can take the flight," and let q be the statement "You buy a ticket."
  Then $p \leftrightarrow q$ is the statement
  "You can take the flight if and only if you buy a ticket."

$\longleftrightarrow$

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Truth Tables of Compound Propositions

جدول الصدق

الجمل المركبة

يستخدم للتعرف على قيمة الصدق للجمل المركبة

- **Truth table** uses for determining the **truth values** of compound propositions.
- Construction of a truth table as follows:

  كل خيارات الجمل المركبة

  صفوف
  - **Rows:** it needs row for every possible combination of values for the atomic propositions.

    تعابير الجمل الفرعيه المركبة
  - **Columns:** it needs column for each compound proposition and each expression that occurs in the compound proposition as it is built up.

p
T
F

p	q
T	T
T	F
F	T
F	F

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Example of Truth table

اولاً الاقواس تسبق الجمع

: النفي

$\wedge$ $\vee$

$\leftrightarrow$ $\rightarrow$

Construct a truth table for $\mathbf{p \vee q \rightarrow \neg r}$

p	q	r	¬r	p ∨ q	p∨q →¬r
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

Example

Construct a truth table for $\mathbf{(p \vee \neg q) \rightarrow (p \wedge q)}$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

- By using the **parentheses** to specify the order in which logical operators in a compound proposition are to be applied.
- Precedence of Logical Operators

اولويه
العمليات

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Exercise 1

Q1(page 12): Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts. T

b) Miami is the capital of Florida. F

c) $2 + 3 = 5$. T

d) $5 + 7 = 10$. F

e) $x + 2 = 11$. not poroposition

Exercise 2

نفي

Q3(page 12) What is the negation of each of these propositions?
a) Mei has an MP3 player.
b) There is ~~no~~ pollution in New Jersey.
c) $2 + 1 = 3$.
d) The summer in Maine is hot and sunny.

a) Mei does not have an MP3 player.
b) There is pollution in New Jersey.
c) $2 + 1 \neq 3$
d) It is not the case that the summer in Maine is hot and sunny.

Exercise 3

Q8(page 13) Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of these propositions as an English sentence. a) $\neg p$

b) $p \vee q$

c) $p \to q$

d) $p \wedge q$

e) $p \leftrightarrow q$

f) $\neg p \to \neg q$

g) $\neg p \wedge \neg q$

h) $\neg p \vee (p \wedge q)$

a) I did not buy a lottery ticket this week.

b) Either I bought a lottery ticket this week or I won the million-dollar jackpot on Friday.

c) If I bought a lottery ticket this week, then I won the million-dollar jackpot on Friday.

d) I bought a lottery ticket this week and I won the million-dollar jackpot on Friday.

e) I bought a lottery ticket this week if and only if I won the million-dollar jackpot on Friday.

f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday

g) I did not buy a lottery ticket this week, and I did not win the million-dollar jackpot on Friday.

h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million-dollar jackpot on Friday

Exercise 4

Q11(page 13): Let p and q be the propositions

but ⇒ and

p: It is below freezing.

q: It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing. $p \wedge q$

b) It is below freezing but not snowing. $p \wedge \neg q$

c) It is not below freezing and it is not snowing. $\neg p \wedge \neg q$

d) It is either snowing or below freezing (or both). $p \vee q$

e) If it is below freezing, it is also snowing. $p \to q$

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing. $(p \vee q) \wedge (p \to \neg q)$

g) That it is below freezing is necessary and sufficient for it to be snowing.

$p \leftrightarrow q$

Exercise 5

Q16(page 13): Determine whether these biconditionals are true or false.
a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
c) $1 + 1 = 3$ if and only if monkeys can fly.
d) $0 > 1$ if and only if $2 > 1$.

a) This is $\mathbf{T} \leftrightarrow \mathbf{T}$, which is true.
b) This is $\mathbf{T} \leftrightarrow \mathbf{F}$, which is false.
c) This is $\mathbf{F} \leftrightarrow \mathbf{F}$, which is true.
d) This is $\mathbf{F} \leftrightarrow \mathbf{T}$, which is false.

Q31(page 13): Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

a)

p	$-p$	$p \wedge \neg p$
T	F	F
F	T	F

b)

p	$-p$	$p \vee -p$
T	F	T
F	T	T

c)

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

$(p \vee q) \to (p \wedge q)$

d)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

$(p \to q) \Leftrightarrow (\neg q \to \neg p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

$(p \to q) \to (q \to p)$

Questions: 4,9,12,14,23,27,28, 35 and 36 - page number 12 and 13

Reference

Discrete Mathematics and its applications, by Kenneth H Rosen, Seventh Edition