

Momentum and Collisions

$$\vec{p} = m\vec{v} \quad \text{kgm/s} \quad \begin{array}{c} \vec{p} \\ \vec{v} \end{array} \rightarrow +$$

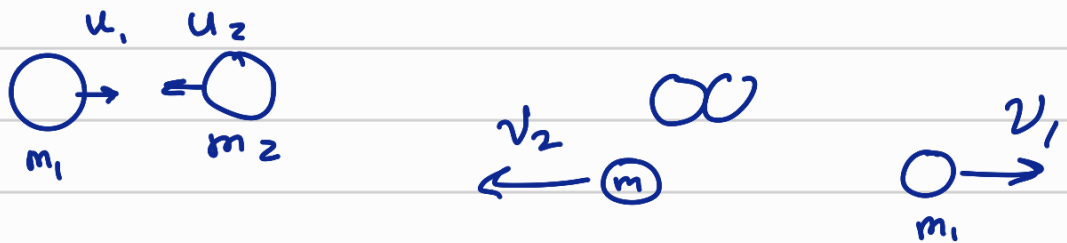
$$\Delta\vec{p} = \text{Impulse (I)} = m \Delta\vec{v}$$



$$\text{Impulse} = F \Delta t \quad \text{Nm} = \text{kgm/s}$$

Collisions

$$\sum p_{\text{before}} = \sum p_{\text{after}}$$



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

momentum is conserved

elastic collision

momentum conserved

kinetic energy conserved

Inelastic

p conserved

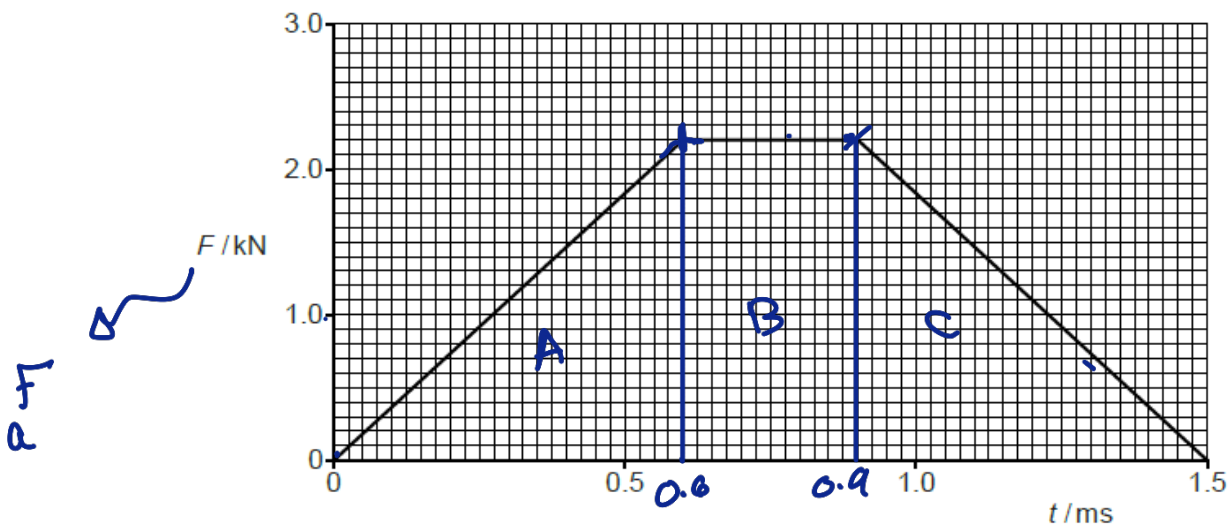
loss of energy

Momentum and Collisions Homework Sheet

- Q1. (a) Collisions between two objects can be either **elastic** or **inelastic**. In the table below there are 4 statements. Place a cross (X) in the relevant column(s) for each statement that is TRUE for that type of collision. [2]

Statement	Elastic collision	Inelastic collision
Total momentum for the objects is conserved.	X	X
Total kinetic energy of the objects is conserved.	X	—
Total energy is conserved.	X	X
The magnitude of the impulse on each object is the same.	X	X

- (b) A steel ball is at rest on a smooth horizontal table. It is hit by a wooden stick. The figure below shows a simplified graph of the force, F , acting on the ball against the time, t .



- (i) Describe how the **velocity of the ball** varies between $t = 0.6$ ms and $t = 0.9$ ms. *velocity increased with constant acceleration* [1]
- (ii) Use the graph on the previous page to calculate the impulse acting on the ball. $I = F\Delta t$ *area under F vs t Curve* [3]
- (iii) The mass of the steel ball is 140 g. Calculate the final speed of the steel ball as it leaves the end of the stick. [2]

ii) $Area = A + B + C$
 $= (\frac{1}{2} \times 0.6 \times 2.2) + 0.3 \times 2.2 + (\frac{1}{2} \times 0.6 \times 2.2) = 1.98 \text{ N s}$

iii) $I = m\Delta v$ $I = m(v - 0)$ $v = \frac{I}{m} = \frac{1.98}{140} = 14.14 \text{ m/s}$

(P)

Q2. A student does an experiment to determine whether momentum is conserved when a pair of trolleys on a track collide head-on. The figure below shows the trolleys on a level track approaching each other.

before

$$m_1u_1 + m_2u_2$$

$$1.6(0.56) + 2.4(-0.41)$$

$$0.896 - 0.984$$

$$= -0.088 \text{ kg m/s}$$



KE before

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$\frac{1}{2}(1.6)(0.56)^2 + \frac{1}{2}(2.4)(0.41)^2$$

$$0.4526 \text{ J}$$

after

$$m_1v_1 + m_2v_2$$

$$(0.55 \times 1.6) + (2.4 \times 0.33)$$

$$0.88 + 0.792$$

$$= 0.088 \text{ kg m/s}$$

The figure below shows the trolleys after the collision.



KE after

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\frac{1}{2}(1.6)(0.55)^2 + \frac{1}{2}(2.4)(0.33)^2$$

$$0.3726 \text{ J}$$

$$\Sigma P_{\text{before}} = \Sigma P_{\text{after}}$$

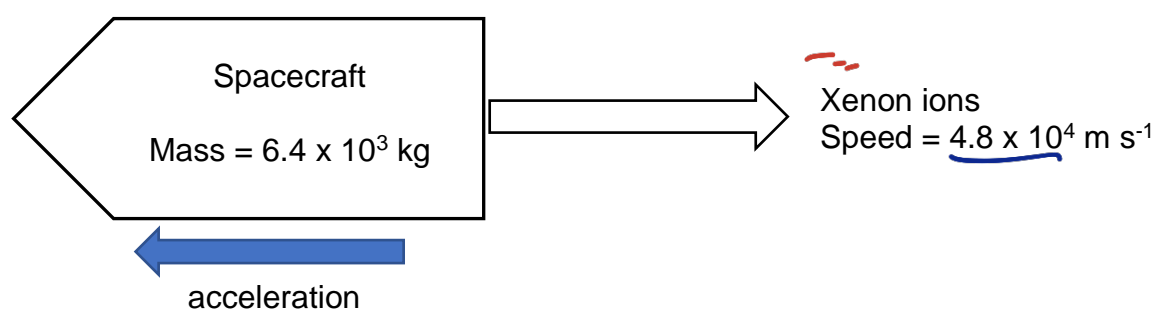
momentum is conserved

- (a) Show that momentum is conserved. [2]
 (b) Determine whether the collision is elastic or inelastic. [3]

$$\Sigma KE_{\text{before}} \neq \Sigma KE_{\text{after}}$$

this collision is inelastic

- Q3. (a) State the conservation of momentum. [2]
 (b) The figure below shows a spacecraft being accelerated to the left by ejecting xenon ions from its rear to the right.



The ions have a fixed mean speed of $4.8 \times 10^4 \text{ m s}^{-1}$ relative to the spacecraft. The initial mass of the spacecraft is $6.4 \times 10^3 \text{ kg}$.

- (i) Calculate the mass of one xenon ion, the molar mass of xenon is $0.131 \text{ kg mol}^{-1}$. [2]
 (ii) The spacecraft ejects 9.5×10^{18} xenon ions per second. Determine the initial acceleration of the spacecraft. [4]

Q3 (a) In closed system the total momentum remains constant

(b) (i)

$$m = \frac{\text{Molar mass}}{N_A}$$



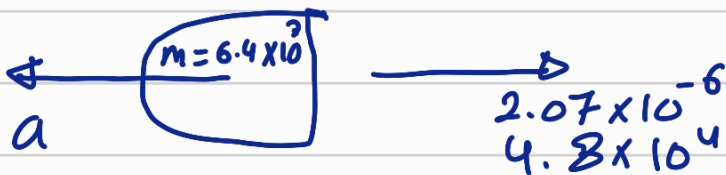
$$m_{\text{(one Ion)}} = \frac{0.131}{6.022 \times 10^{23}} = 2.17 \times 10^{-25} \text{ kg}$$

(c) total mass of xenon

= number of Ions \times mass of one Ion

$$= 9.5 \times 10^{18} \times 2.17 \times 10^{-25} \text{ kg}$$

$$= 2.07 \times 10^{-6} \text{ kg}$$



$$F_{\text{xenon}} = m(v - 0)$$

$$= 2.07 \times 10^{-6} \times 4.8 \times 10^4$$

$$0.099 \text{ N}$$

$$F_{\text{xenon}} = F_{\text{spacecraft}} = ma$$

$$a = \frac{F}{m} = \frac{0.099}{6.4 \times 10^3} = 1.55 \times 10^{-5} \text{ m/s}^2$$

(iii) State and explain how and why the acceleration of the spacecraft changes while the engine is running.

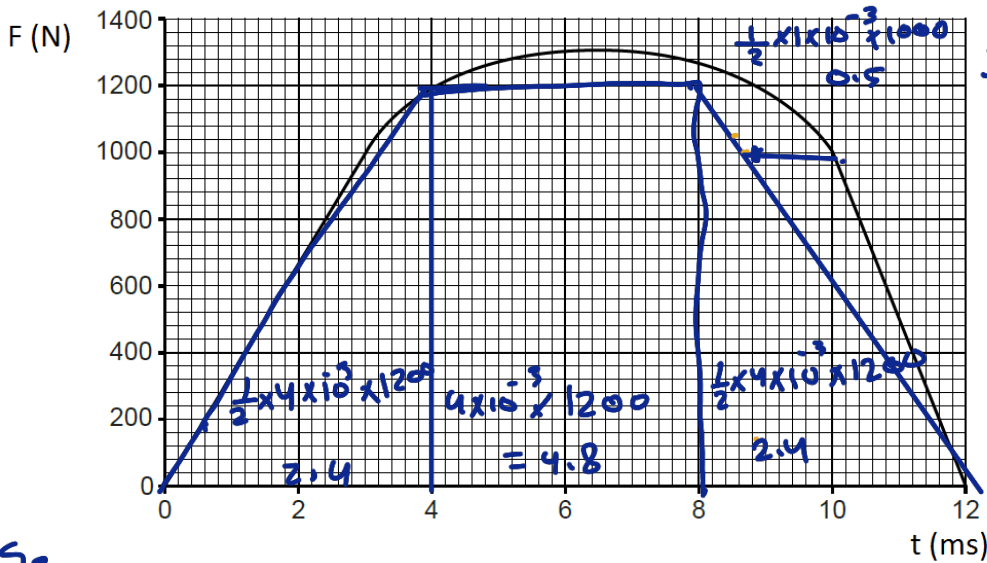
(iii) the mass of the spacecraft decreases as the xenon is ejected [3]

Q4. (a) (i) Define the term **impulse of a force**.

the product of the force and over period of time [1]

(ii) A satellite of mass 180kg uses a small rocket to disconnect (detach) from a space craft with a mass of 6.4×10^3 kg. The figure below shows a graph of how the force, F , on the satellite changes with time, t .

Determine the change in the velocity of the satellite as a result of the force being applied for a time of 12ms. [4]



ms

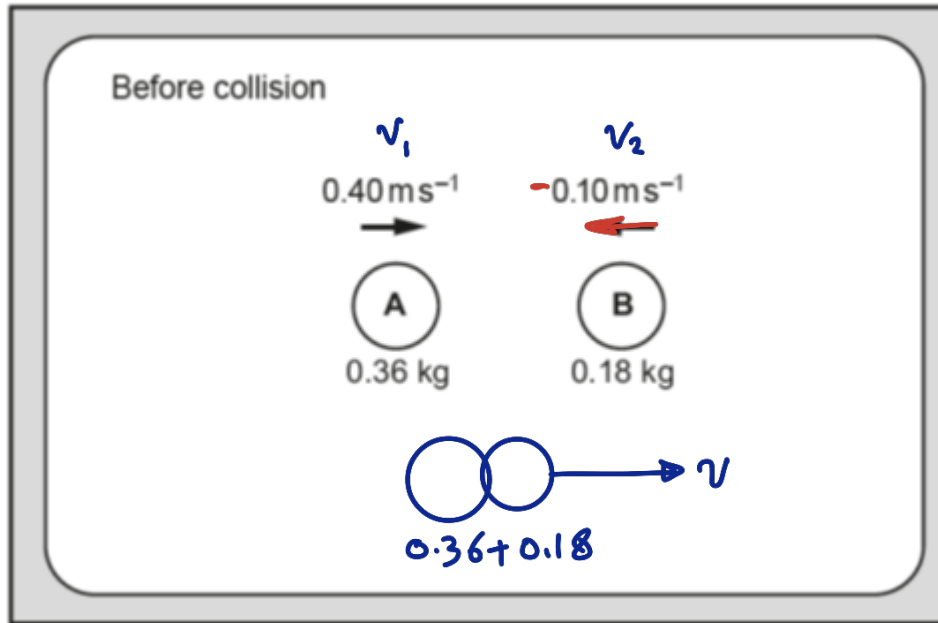
(b) (i) State the principle of conservation of momentum. [2]

(ii) Two balls, **A** and **B**, collide head-on. Ball **A** has a mass of 0.36kg and ball **B** has a mass of 0.18kg. Before the collision ball **A** has a velocity of 0.40 m s⁻¹ and ball **B** has a velocity of 0.10 m s⁻¹ in the opposite direction. This is shown in the figure below.

* $\Sigma P_{\text{before}} = \Sigma P_{\text{after}}$
 $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$
 $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
 $v = \frac{0.36 \times 0.4 + (-0.1)(0.18)}{0.36 + 0.18}$
 $v = 0.23 \text{ m/s}$
 →

$K E_{\text{before}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$
 $= \frac{1}{2} (0.36)(0.4)^2 + \frac{1}{2} (0.18)(0.1)^2$
 $= 0.0297 \text{ J}$
 $K E_{\text{after}} = \frac{1}{2} (m_1 + m_2) (v)^2$
 $= \frac{1}{2} (0.36 + 0.18)(0.23)^2$
 $= 0.0147 \text{ J}$
 $\Delta K E = 0.0297 - 0.0147$
 $= 0.015 \text{ J}$

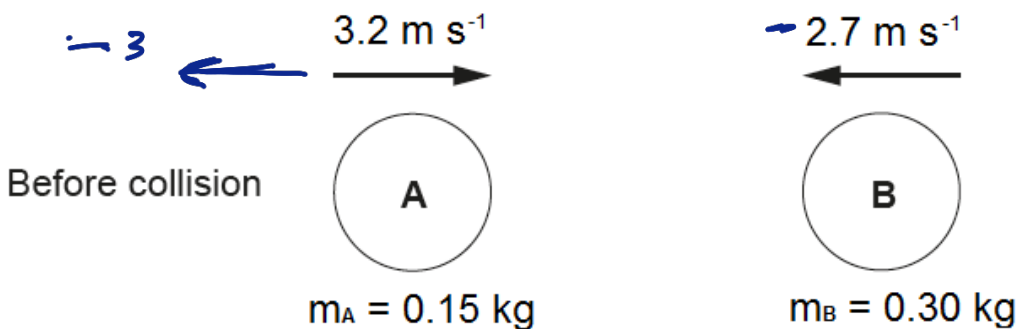
$$\text{Percentage} = \frac{\Delta KE}{KE_{\text{before}}} = \frac{0.615}{0.6297} \times 100 = 56\%$$



After the collision the balls stick together. Calculate the velocity of the balls after the collision **and** the kinetic energy lost during the collision. Express the lost kinetic energy as a percentage of the total kinetic energy of the balls before the collision. [6]

Q5. (a) State the principle of conservation of momentum. [2]

(b) Two discs **A** and **B** of masses $m_A = 0.15 \text{ kg}$ and $m_B = 0.30 \text{ kg}$ slide towards each other on a frictionless horizontal surface and collide head-on. Before the collision the speed of disc **A** is 3.2 m s^{-1} and the speed of disc **B** is 2.7 m s^{-1} . This is shown in the figure below.



(i) After the collision the direction of disc **A** is reversed and its speed is 3.0 m s^{-1} . Determine the speed of disc **B** after the collision. [3]

(ii) Calculate the total kinetic energy **lost** during the collision. [3]

(iii) The contact time between the discs during the collision is 0.28 s . Calculate the average force on disc **A** **and** state its direction. [3]

Q6. (a) State the principle of conservation of momentum. [2]

(b) A rubber ball is suspended by a string to form a pendulum. A student fires an air

$$(5) \text{ a) } \sum p_{\text{before}} = \sum p_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0.15 \times 3.2 + 0.3 \times (-2.7) = 0.15(-3) + 0.3(v_2)$$

$$v_2 = \frac{(0.15 \times 3.2) - (0.3 \times 2.7) + 0.5(3)}{0.3} = 0.4 \text{ m/s}$$

$$\text{b) } \Delta KE_{\text{lost}} = \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \right) - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right)$$

$$= \frac{1}{2} \times 0.15 \times 3.2^2 + \frac{1}{2} \times 0.3 \times (2.7)^2 - \left(\frac{1}{2} \times 0.15 \times 3^2 + \frac{1}{2} \times 0.3 \times 0.4^2 \right)$$

$$= 1.86 - 0.699 = 1.16 \text{ J}$$

$$\text{c) } I = m \Delta v = F \Delta t$$

$$= (0.15) (-3 - 3.2) = F (0.28)$$

$$F = \frac{0.15(-3 - 3.2)}{0.28} = -3.3 \text{ N}$$

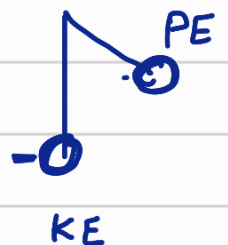
←
left

طاقة الوضع عند أعلى نقطة = طاقة الحركية عند أدنى

$$\text{6) } GPE = KE$$

$$mgh = KE = 91 \times 10^{-3} \times 9.8 \times 0.25$$

$$= 0.223 \text{ J}$$



نستخدم الطاقة الحركية كإجابة. v

$$KE = \frac{1}{2} m v^2$$

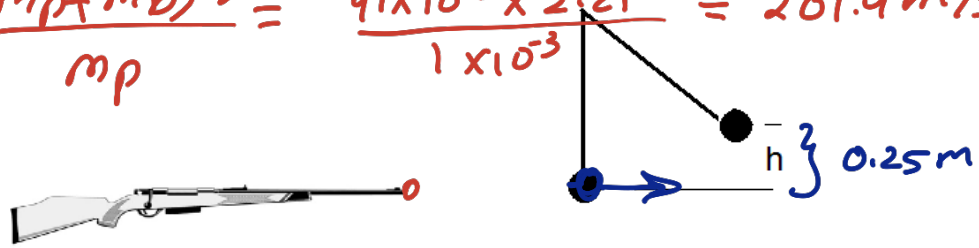
$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \times 0.223}{91 \times 10^{-3}}} = 2.21 \text{ m/s}$$

$$\sum P_{\text{before}} = \sum P_{\text{after}}$$

$$m_p u_p + 0 = (m_p + m_b) v$$

rifle pellet horizontally at the rubber ball. This is shown in the figure below.

$$u_p = \frac{(m_p + m_b) v}{m_p} = \frac{91 \times 10^{-3} \times 2.21}{1 \times 10^{-3}} = 201.4 \text{ m/s}$$



The pellet hits the rubber ball and become stuck inside the ball. The ball swings to the right rising to a height, h , above its initial position.

(i) The height, h , reached by the rubber ball is 0.25m, the mass of the rubber ball is 90g and the mass of the pellet is 1g. Show that the kinetic energy of the rubber ball and pellet is about 0.22J immediately after the pellet hits the ball. [3]

(ii) Use the principle of conservation of momentum and the value of kinetic energy immediately after impact to calculate the speed, u , of the pellet immediately before it hits the rubber ball. [3]

(c) The student claims that the kinetic energy is not conserved in the collision between the pellet and rubber ball in part (b).

$$KE_{\text{after}} = 0.223 \quad KE_{\text{before}} = \frac{1}{2} \times 1 \times 10^{-3} \times (201.4)^2 = 20.3 \text{ J}$$

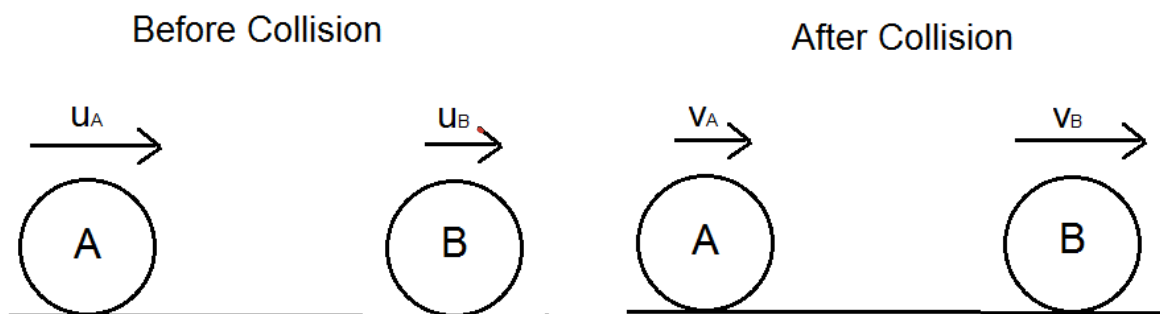
(i) Show that the results of the experiment support the student's claim. [2]

(ii) Describe where the missing energy has gone. [2]

- change of the rubber ball shape
- Heat energy *- Vibration*

Q7. (a) State the principle of conservation of momentum. [2]

(b) Two balls, A and B, are travelling in the same direction along a horizontal track as is shown in the picture below.

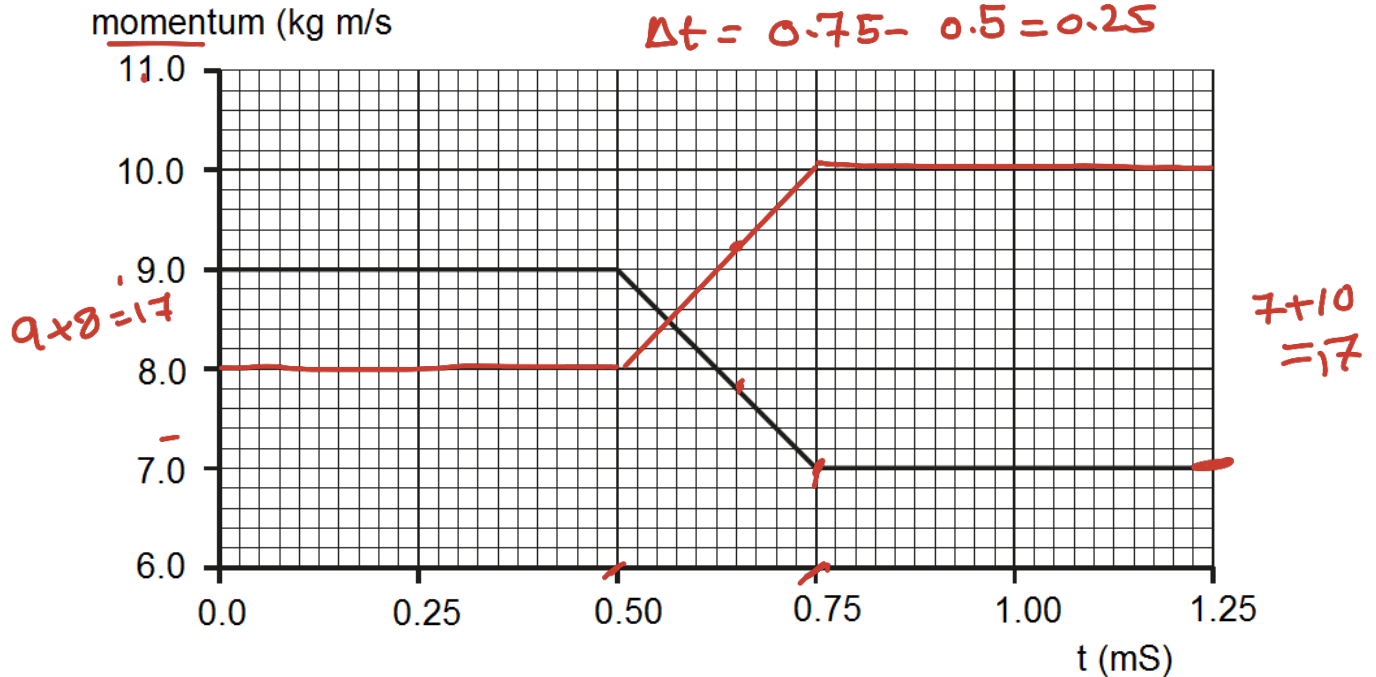


Ball A makes a head-on collision with ball B.

During the collision the momentum of both balls changes. The graph below shows how momentum of ball A against time. The graphs shows the momentum before, during and after the collision.

$$\Delta p = 9 - 7$$

$$\Delta t = 0.75 - 0.5 = 0.25$$



(i) Using the information from the graph calculate the force acting on ball A during the collision. [3]

(ii) The momentum of ball B before the collision is 8.0 kg m/s. On the graph, sketch how the momentum of ball B varies with time during the time interval t = 0ms and t = 1.25ms. [3]

$$i) I = m \Delta v = F \Delta t$$

$$F = \frac{\Delta p}{\Delta t} = \frac{2}{0.25 \times 10^{-3}} = 8000 \text{ N}$$