

Discrete Structures

Course Code: CS-1004

Al-Baha University
Faculty of Computing and Information
Department of Computer Science
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Dr. Ahmed Youssef

Chapter 1

Mathematical Logic

Mathematical Logic

Definition: Mathematical Logic is a Method of reasoning; it provides rules and techniques to determine whether an (argument == statement

$$X = Q$$

$$x - 1 = 5$$

Example: If x is an even integer, then x + 1 is an odd integer

- The statement x + 1 is an odd integer is true under the condition that x is an even integer is true.
 - التعريف: المنطق الرياضي هو طريقة للتفكير المنطقي؛ حيث يوفر قواعد وتقنيات لتحديد ما إذا كان (الحجة == العبارة == الاقتراح) صحيحًا أم لا.
 - مثال: إذا كان x عددًا صحيحًا زوجيًا، فإن x+1 عدد صحيح فردي.
 - العبارة: x عددًا صحيحًا زوجيًا.

Mathematical Logic

عله عربة عله

- A **statement**, or a **proposition**, is a <u>declarative</u> sentence that is either true or false, but not both
- Uppercase letters denote propositions
 - Examples:
 - P: 2 is an even number (true)
 - Q: 7 is an even number (false) F
 - R: A is a vowel (true)
 - The following are not propositions:
- P: My cat is beautiful not Peropostion
 - Q: My house is big

 How beautiful is the rose?
- What time is it? المراكبة المنازلة الم

$$x + y = 3$$

حبه خرب تحمل العراد الخطأ و سب علاج)

$$X = 2 + 1$$

Not proposition

Logical Connectives and Truth Tables

- Each proposition evaluates to either true T or false F.
- T and F are called proposition truth values

روابط المنطفه

In this section we look at how simple propositions can be combined to form more complicated propositions called **compound propositions**.

- Logical connectives {negation, and, inclusive or, exclusive or, implication, biconditional} are used to link pairs of propositions.
- The truth value of any compound proposition is completely determined by (a) the truth values of its component simple propositions, and (b) the particular connective, or connectives, used to link them.

$$\frac{\text{Jegation}}{\bar{p} \text{ (or } \sim p \text{ or } -p \text{ or } \neg p)} \qquad \qquad \bar{p} \stackrel{\text{Torsel}}{\sim} \bar{p} \stackrel{\text{Torsel}}{\sim} \bar{p}$$

The negation of P, written $\neg P$, is the statement obtained by negating statement P

Example:

P: A is a consonant letter. $\neg P$: A is not a consonant letter.

 $\neg \mathbb{Z}$: The circle does not have a volume Q: The circle have a volume

The negation of the proposition reverse the truth value of the proposition which is

shown in the following truth table

Р	$\neg P$
T	F
F	T
l	-

There are several alternative ways of stating the negation of a proposition. If we consider the proposition

- ראולאני איה איי 'All dogs are fierce', בּלוּולאני אייה some examples of its negation are:
- P It is not the case that all dogs are fierce. العبيم ان كاراكلاب المحلوم المحلوم
- ۹ Not all dogs are fierce. خبن کار محلا ج
- p Some dogs are not fierce. من بالمحلاب بالمحلاب بالمحلاب المحلاب بالمحلاب بالمحل بالمحل بالمحلاب بالمحلاب بالمحلاب

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Conjunction ^

 Λ (9)

البوم حميمه والسماء ماطيره P م

• Let P and Q be statements. The conjunction of P and Q, written $P \ ^Q$, is the statement formed by joining statements P and Q using the word "and"

T: 1 F: 6

- P ^ Q is true if both P and Q are true; otherwise P ^ Q is false
- نکو ۸ مجلہ صحبہ اذا کائن المجلب صحبہ منا ۱۳ و The Truth Table for the conjunction is:

Example:

• P: The sun is shining.

• Q: Pigs eat turnips.

• $P \wedge Q$: The sun is shining and pigs eat turnips.

الته مرحه وانخارم تاعل اللغت

P	Q	$\mathbf{P} \wedge \mathbf{Q}$
1	1	1
1	0	0
0	1	0
0	0	0

and V

The Disjunction: Let *P* and *Q* be statements.

The inclusive disjunction of P and Q, written $P \vee Q$ is true if at least one of the statements P and Q is true; otherwise, P VQ is false. The Truth Table for the inclusive disjunction is shown left below.

The exclusive disjunction of P and Q, written P VQ is true if either P or Q is true but not both. The Truth Table for the inclusive disjunction is shown right نحرم کیلہ صحیح فقط اذا کانن اص سرح الجملس الجملس الجملس الجملس الجملس الجملس الجملس

P	Q	P∨ Q
1	1	1
1	0	1
0	1	1
0	0	0

Exclusive or

F

Examples:

Assume the propositions:

P: 'Tomorrow I will go swimming.

Q: 'I will play golf'.

Then the proposition:



'Tomorrow I will go swimming or 'I will play golf'. and not both seems to suggest that I will not do both and therefore points to an exclusive interpretation P VQ

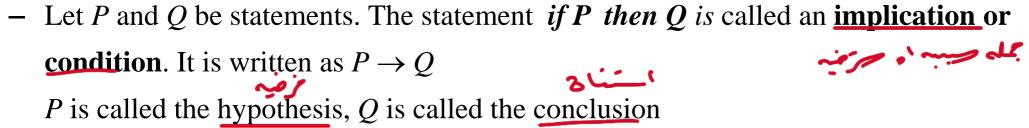
But the propositions



'Applicants for this post must be over 25 or have at least 3 years relevant.

experience' suggests that applicants who satisfy both criteria will also be considered, therefore 'or' should be interpreted inclusively $P \nabla Q$.





Truth Table for Implication:

Tradit rable for implication.	P		$P \to O$
If I pass then I will go for to	P	•	- · · · ·
	• 1		
p ~ Q	1	0	0
	^	4	4

	ن ، مبب	
مَلُوب	خاخة	ر سنیم
	خاف	ملحرا

- 'if P then Q' is false only when P is true and Q is false, i.e. a true Notice that statement cannot imply a false one.
- If P is false, the compound proposition is true no matter what the truth value of Q.

Examples:

- P: I eat breakfast.
- Q: I don't eat lunch. F: T Par lunch
- $P \rightarrow Q$: If I eat breakfast then I don't eat lunch

P	Q	$\mathbf{P} o \mathbf{Q}$
1	1	1
1	0	0
0	1	1
0	0	1

- Let *P* : Today is Sunday.
 - Q: I will wash the car.
- $P \rightarrow Q$: If today is Sunday, then I will wash the car

- The **contrapositive** of this implication is $\neg Q \rightarrow \neg P^{\mathsf{TF}} \neg Q \longrightarrow \mathsf{th} \neg P$ If I do not wash the car, then today is not Sunday
 - If I don't wash the cour then today is

Biimplication (Biconditional:)

if and only if

Let P and Q be statements. The statement "P if and only if Q" is called the biimplication or biconditional of P and Q.

It is written as $P \leftrightarrow Q$

Example:

P: I eat breakfast.



- $P \leftrightarrow Q$: I eat breakfast if and only if I don't eat lunch (or <u>alternatively</u>, 'If and only if I eat breakfast, then I don't eat lunch').
- The truth table for $P \leftrightarrow Q$ is given by:

کنن اکمیس	13	له طهم	نگو~,ک
۔ صو' ب	ىن	اد الجما	آئے

p	\boldsymbol{q}	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

P→9 ~ 9→P

Examples 1.1

Consider the following propositions:

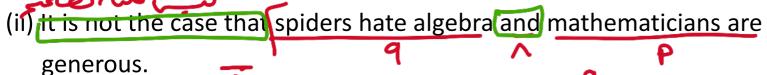
- p: Mathematicians are generous.
- q : Spiders hate algebra.

Write the compound propositions symbolized by:

- (i) $p \vee \bar{q}$ (ii) $(q \wedge p)$
- (iii) $\bar{p} \rightarrow q$
- (iv) $\bar{p} \bigoplus \bar{q}$

Solution

(i) Mathematicians are generous or spiders don't hate algebra (or both).



- (iii) If mathematicians are not generous then spiders hate algebra.
- (iv) Mathematicians are not generous if and only if spiders don't hate algebra.

if and only if

Example 1.2

Let *p* be the proposition 'Today is Monday' and *q* be 'I'll go to London'.

Write the following propositions symbolically.

- (i) If today is Monday then I won't go to London.
- (ii) Today is Monday or I'll go to London, but not both.
- (iii) I'll go to London and today is not Monday. 9, 1 P
- (iv) If and only if today is not Monday then I'll go to London.
 - (i) $p \rightarrow \bar{q}$
 - (ii) $p \vee q$
 - (iii) $q \wedge \bar{p}$
 - (iv) $\bar{p} \leftrightarrow q$.

or V or but not both

- if and offyil

Example 1.3:

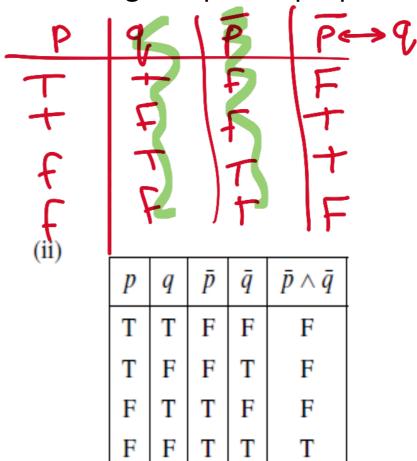
Construct the Truth Table for the following compound propositions

- (i) $\bar{p} \vee q$
- (ii) $\bar{p} \wedge \bar{q}$
- (iii) $\bar{q} \rightarrow p$
- (iv) $\bar{p} \rightarrow \bar{q}$.

Solution

(i)

p	q	\bar{p}	$\bar{p} \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T



(iii)		Γ			(iv	·)					
	p	q	\bar{q}	$\bar{q} \rightarrow p$	(1,	,	p	q	<u></u>	ą	$\bar{p} \leftrightarrow \bar{q}$
	T	T	F	T			Т	T	F	F	T
	T	F	T	T			Т	F	F	Т	F
	F	T	F	T			F	Т	Т	F	F
	F	F	T	F			F	F	Т	T	T
		l					1.	1.	1	1	1

p	q	<u></u>	$ar{q}$	$\bar{p}\leftrightarrow \bar{q}$
T	T	F	F	T
T	F	F	T	F
F	Т	T	F	F
F	F	T	T	T

ملاحده اذا اصور الحبله اع کبه ی تمبین مفط مکوبه عبدوالصرف حکومه ۲ جمنون المادا كانت الحبله فيها 3 على سكُّوله جبر, ل العرن له 8 عمور المادا كانت الحبله فيها 3 على سكُّوله جبر, ل العرن له 8 عمور

Example 1.4. Construct truth tables for:

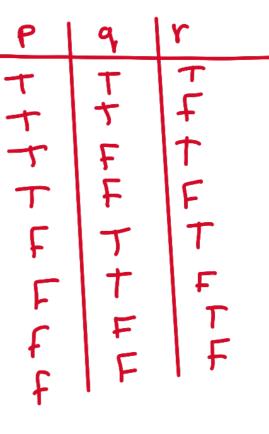
(i)
$$p \to (q \land r)$$

Solution

(i)

p	q	r	$q \wedge r$	$p \to (q \wedge r)$
Т	Т	Т	T	T
T	Т	F	F	F
T	F	Т	F	F
T	F	F	F	F
F	Т	T	T	T
F	Т	F	F	T
F	F	T	F	T
F	F	F	F	T

(ii) Homework



P	9	۲	(GNY)	P -> (any)
+	+	T	+	+
T	十	F	F	Ė
T	F	1	F	Ė.
T	F	F	F	Ė
F	T	1	十	+
	+	Ł	F	T
ţ,	F	T	F	T
Γ	I	F	F	T
1	,			

P	q	Υ	P	Y	(PV9)	(pva) cor
+	十	T	1	F,	+	F
T	十	F	£	+	T	T
T	F	T	F	F	F	T
十	F	F	F	一丁	F	F
	T	1	+	F	1	F
	+	F	+	1 +	+	+
	F	一	+	1	十	F
		F	\	1 +	1 F	F
+	1		1	'	1	
	,					_

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PI	9,	bror	PVa	(Pvq) -> (P-)	9
1 +	+	7 1	+-	+	
1 +	FO	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	f o	Fo	
o F	1 = '	1	· ·	Fo	
0 F	1 + 0	1 F 0	F 0	1 + 1	

Example 1.5 Construct the truth table for $(p \lor q) \to (p \land q)$

Solution

p	q	p∨q	p ∧ q	$(\mathbf{p} \lor \mathbf{q}) \to (\mathbf{p} \land \mathbf{q})$
1	1	1	1	1
1	0	1	0	0
0	1	1	0	0
0	0	0	0	1

Example 1.6 Construct the truth table for $(p \rightarrow q) \rightarrow (q \rightarrow p)$ **Solution** $(\mathbf{p} \to \mathbf{q}) \to (\mathbf{q} \to \mathbf{p})$ $\mathbf{q} \rightarrow \mathbf{p}$ $\overline{\mathbf{p}} \rightarrow \mathbf{q}$ q

Tautology العرد الافير أن عرد العون العرب العون العرب العون العرب العرب

A **tautology** is a compound proposition which is true no matter what the truth values of its simple components.

In other words A compound proposition $p \equiv p(p_1, p_2, ..., p_n)$ where $p_1, p_2, ..., p_n$ are variables is called a tautology if it is true for every truth assignments for $p_1, p_2, ..., p_n$.

دنی منص النی عمر انجل البین صلح او صلا Example 1.7 Show that $p \vee \bar{p}$ is a tautology.

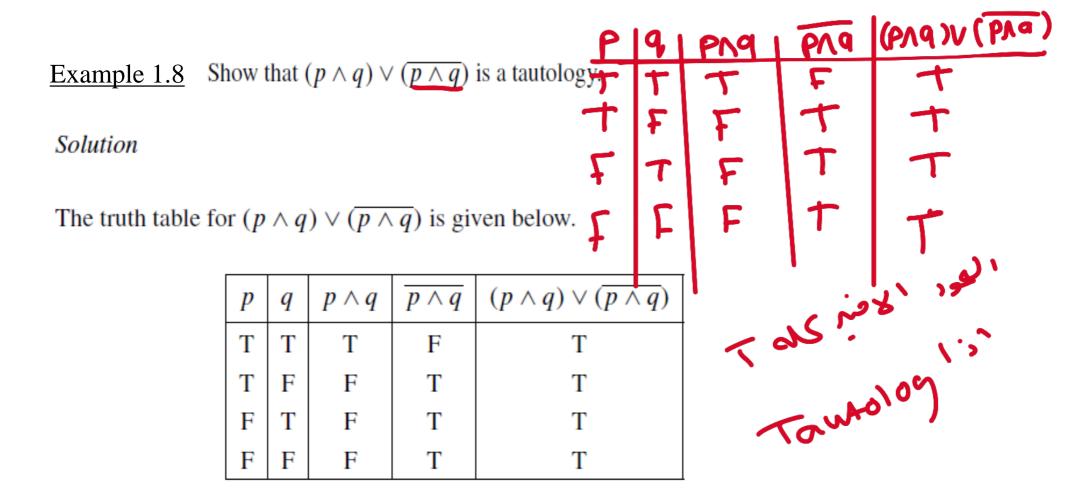
Solution

Constructing the truth table for $p \vee \bar{p}$, we have:

p	\bar{p}	$p \vee \bar{p}$
Т	F	T
F	T	Т

tauto 1699 سالم المجادة المادي Taut 6 1077

Note that $p \vee \bar{p}$ is always true (no matter what proposition is substituted for p) and is therefore a tautology.



The last column of the truth table contains only the truth value T and hence we can deduce that $(p \land q) \lor (\overline{p \land q})$ is a tautology.

Note that The proposition $(p \land q) \lor (\overline{p} \land \overline{q})$ is said to be a <u>substitution instance</u> of the proposition $p \lor \overline{p}$.

مفاللہ منافقہ کے کا کا کہ مخالفہ کا کہ مخالفہ کے Contradiction

- A contradiction is a compound proposition which is false no matter what the truth values of its simple components.
- The proposition $p \land \sim p$ is a contradiction which is shown in next truth table

P	o _v	P	ā	(P/Q)	(Pva)	(P/19)/ (PV4)
ナナーナー	11 11 11	+1 +1	トナトナ	カイカ	+44+	F right reals

Example 1.9 Show that $(p \wedge \bar{q}) \wedge (\bar{p} \vee q)$ is a contradiction.

p	~p	$p \wedge \sim p$
T	F	F
F	T	F
uż i	156	ه و نن

23

Solution

	$(\bar{p}\vee q)$	$(p \wedge \bar{q}) \wedge ($	$\bar{p} \vee q$	\bar{p}	$p \wedge \bar{q}$	$ar{q}$	q	p
17624		F	Т	F	F	F	T	T
adiction	Conn	F	F	F	T	T	F	T
	. •	F	Т	Т	F	F	T	F
23		F	T	T	F	T	F	F

Assignment 1- Question 1:

Attempt any two of the following questions

1.
$$p \rightarrow (p \lor q)$$

2.
$$(p \rightarrow q) \land (\bar{p} \lor q)$$

3.
$$(p \lor q) \leftrightarrow (q \lor p)$$

4.
$$(p \land q) \rightarrow p$$

5.
$$(p \wedge q) \wedge (\overline{p \vee q})$$

 $(p \wedge q) \wedge - (p \vee q)$

p	q	$p \lor q$	p o (p ee q)	
T	Т	T	Т	
T	F	T	Т	Tantolog
F	Т	Т	Т	
F	F	F	Т	

	$(\neg p \lor q)$	$(p o q) \wedge$	$ eg p \lor q$	-P	p o q	q	p
		T	T	£	Т	Т	Т
Lalous	neither Tank	F	F	F	F	F	Т
adiction		T	T	T	Т	T	F
X II. CHION	7101 (811174	Т	T	+	Т	F	F

(3)

p	q	$p \lor q$	$q \lor p$	$(p \vee q) \leftrightarrow (q \vee p)$
Т	Т	Т	T	T
Т	F	Т	T	Tanology
F	T	Т	Т	Т
F	F	F	F	T

(Y)

p	q	$p \wedge q$	$(p \wedge q) o p$	
T	Т	T	T	
T	F	F	Tau+016	97
F	Т	F	TAUT	
F	F	F	Т	

(5)

p	q	$p \lor g$	$\lnot(p\lor q)$	$p \wedge q$	$(p \wedge q) \wedge \neg (p \vee q)$
Т	Т	Т	F	Т	F 1.000
Т	F	Т	F	F	F Contrad. Cho
F	Т	Т	F	F	F
F	F	F	Т	F	F

مسبب صنعن

Logically Implies



- A proposition P is said to **logically imply** a proposition Q if, whenever P is true, then Q is also true. If P logically implies Q, then symbolically we write $P \rightarrow Q$.
- In Other words: A proposition P is said to logically imply proposition Q if the proposition $P \rightarrow Q$ is a tautology.

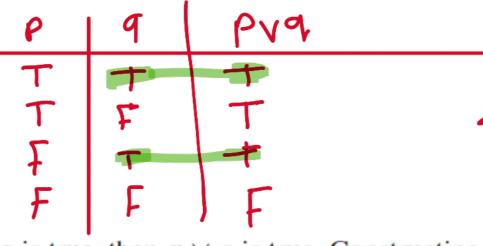
Note that the converse does not apply, i.e. Q may also be true when P is false. For logical implication all we insist on is that Q is never false when P is true. We shall symbolize logical implication by \vdash so that 'P logically implies Q' is written $P \vdash Q$.

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Example 1.10

Show that $q \vdash (p \lor q)$.

Solution



We must show that, whenever q is true, then $p \lor q$ is true. Constructing the truth table gives:

p	q	$p \vee q$						
T	T	T						
T	F	T						
F	T	T						
F	F	F						

q is logically imples (PVa)

From a comparison of the second and third columns we note that, whenever q is true (first and third rows), $p \lor q$ is also true. Note that $p \lor q$ is also true when q is false (second row) but this has no relevance in establishing that q logically implies $p \lor q$.

Example 1.11

Show that $(p \leftrightarrow q) \land q$ logically implies p.

Solution

P	9	Pera	(Pe=4)19	
T	+	T	+	1
+	F	F	F	T
ŗ	1+	F	Ė	1F
F	F	T	F	F

As with example 1.5 we can show that $[(p \leftrightarrow q) \land q] \vdash p$ in one of two ways. We can either show that p is always true when $(p \leftrightarrow q) \land q$ is true or we can show that $[(p \leftrightarrow q) \land q] \rightarrow p$ is a tautology.

The truth table for $(p \leftrightarrow q) \land q$ is given by:

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \land q$	109:(0.2
T	T	Т	Т	
T	F	F	F	Such
F	T	F	F	(10 ara) D
F	F	T	F	((P ← 9) A 9) HP

Comparing the fourth column with the first, we see that p is true whenever $(p \leftrightarrow q) \land q$ is true (first row only). Therefore $[(p \leftrightarrow q) \land q] \vdash p$.

Alternatively, we could complete a further column of the truth table for $[(p \leftrightarrow q) \land q] \rightarrow p$ and show this to be a tautology.

جِل منظ به منظمني Logically Equivalent

Two propositions are said to be **logically equivalent** if they have identical truth values for every set of truth values of their components. we write $P \equiv Q$ if P and Q are logically equivalent. In other words a proposition P is said to be **logically** equivalent to a propositions Q if the statement formula $P \leftrightarrow Q$ is a tautology

Example 1.12 (1, q) (1, q)

Solution

We draw up the truth table for $\bar{p} \vee \bar{q}$ and also for $\bar{p} \wedge q$.

Pra = Pra

							1 6	0.1	<u></u>	q	Pra	DA4	200
\boldsymbol{p}	a	\bar{p}	ā	$\bar{p} \vee \bar{q}$	$p \wedge q$	$\overline{p \wedge q}$	P	7	-		11.0		PAT
Г	1	Г	7	r · · · · · · · · · · ·	r	r · · · · · · · · · · · · ·	T	T	F	F	F	+	F
T	T	F	F	F	T	F	🛨	F	t	٦	+	F	T
T	F	F	T	Т	F	Т	_	+	T	F	T	F	+
F	T	T	F	T	F	Т	+	F	lt	T	T	ا عا	†
F	F	T	T	T	F	T			J			'	78

لم منم صف مدم بقه

Example 1.13 Show that the following two propositions are logically

- If it rains tomorrow then, if I get paid, I'll go to Paris. $P \rightarrow (q \rightarrow r)$ (i)
- If it rains tomorrow and I get paid then I'll go to Paris. (PAQ) (ii) P1911 9-1 P-19-1) PA9 (PA9)-r

Solution	P	·	V	

Define the following simple propositions:

T	1	-	4	F
p: It rains tomorrow.	T	<u>t</u> +	+	F
q: I get paid.	+		7	f 1
r: I'll go to Paris.	F	FT	7	F 7

T T T T F

We are required to show the logical equivalence of $p \to (q \to r)$ and $(p \land q) \to q$ r. We can do this in one of two ways:

- establish that $p \to (q \to r)$ and $(p \land q) \to r$ have the same truth values, (a) or
- establish that $[p \to (q \to r)] \leftrightarrow [(p \land q) \to r]$ is a tautology. (b)

Using the first method we complete the truth table for $p \to (q \to r)$ and $(p \land q)$ $q) \rightarrow r$.

حَمِ الصعف للجملين متفاقعه (زا الجل منفاعيد

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \land q) \rightarrow r$
T	T	T	T		Т	T
T	T	F	F	F	Т	E
T	F	T	T		F	7
T	F	F	T	4	F	9
F	T	T	T	Т	F	T
F	T	F	F	Т	F	T
F	F	T	T	Т	F	T
F	F	F	T	Т	F	T

Since the truth values of $p \to (q \to r)$ and $(p \land q) \to r$ are the same for each set of truth values of p, q and r, we can deduce the logical equivalences of these compound propositions. $(p \land q) \to (p \land$

Assignment 1- Question 2:

Attempt any two of the following questions

- 1. Prove that $(p \to q) \equiv (\bar{p} \lor q)$.
- 2. Prove that $(p \land q)$ and $(\overline{p \to \overline{q}})$ are logically equivalent propositions.
- 3. Prove that $(\overline{p \vee q}) \equiv (p \vee \overline{q})$.
- 4. Prove that p logically implies $(q \rightarrow p)$.

		7
/	1	
	١	
1	٠	/

p	q	p o q	\overline{p}		$\overline{p} ee q$
Т	Т	Т	F	T	. 114
T	F	F	F	F	logically whenh
F	T	Т	Т	Т	equi
F	F	Т	T	T	

2

p	q	-9,	p ightarrow eg q		$\lnot(p ightarrow \lnot q)$		$p \wedge q$
Т	T	F	F	Т		Т	C: col
T	F	T	Т	F		F	109 W.V
F	T	F	T	F		F	640
F	F	T	T	F		F	

 $\neg(p\oplus q)$ $p \oplus q$ $p \oplus \neg q$ qT T F F T T F F T F T T F F T T T F T F F

109° CANIU

p	q	q o p	
T	T	/T)	P - (P-09)
T	F	Т	
F	Т	F	
F	F	F	

The Algebra of Propositions

The following is a list of some important logical equivalences, all of which can be verified using one of the techniques described previously. These laws hold for any simple propositions p, q and r and also for any substitution instance of them.

The **duality principle** states that, if two propositions are logically equivalent, then so are their duals.

The Duality Principle

Given any compound proposition P involving only the connectives denoted by \land and \lor , the **dual** of that proposition is obtained by replacing \land by \lor , \lor by \land , t by f and f by t. For example, the dual of $(p \land q) \lor \bar{p}$ is $(p \lor q) \land \bar{p}$. The dual of $(p \lor f) \land q$ is $(p \land t) \lor q$.

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	<u>Primal form</u>	<u>Dual Form</u>
Idempotent Law	$p \lor p \equiv p$	$p \wedge p \equiv p$
Dominant Law	$p \lor T \equiv T$	$p \wedge F \equiv F$
Complement Law	$p \lor \sim p \equiv T$	$p \land \sim p \equiv F$
Commutative Law	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Associative Law	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$
Distributive Law	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
Absorption Law	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
DeMorgan's Law	$\sim (p \lor q) \equiv \sim p \land \sim q$	$\sim (p \land q) \equiv \sim p \lor \sim q$

Example 1.14

Prove that
$$(\bar{p} \wedge q) \vee (\bar{p} \vee q) \equiv \bar{p}$$
. without using that table $(\bar{p} \wedge q) \vee \neg (p \vee q) \equiv \bar{p}$. without using that $(\bar{p} \wedge q) \vee \neg (p \vee q) \implies (\bar{p} \wedge q) \vee (\bar{p} \wedge q)$

Solution

 $(\bar{p} \wedge q) \vee \neg (p \vee q) \implies (\bar{p} \wedge q) \vee (\bar{p} \wedge q)$
 $(\bar{p} \wedge q) \vee \neg (p \vee q) \implies (\bar{p} \wedge q) \vee (\bar{p} \wedge q)$

$$(\bar{p} \wedge q) \vee (\bar{p} \vee q) \equiv (\bar{p} \wedge q) \vee (\bar{p} \wedge \bar{q})$$
 (De Morgan's laws) المرابع $\bar{p} \wedge (q \vee \bar{q})$ (distributive laws) المرابع $\bar{p} \wedge (q \vee \bar{q})$ (complement laws) المرابع $\bar{p} \wedge (q \vee \bar{q})$ (identity laws) المرابع المرابع

Assignment 1- Question 3:

Prove the following logical equivalences using the method of the above example

(i)
$$(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t$$
.

(ii)
$$(p \wedge q) \wedge q \equiv p \wedge q$$
.

(iii)
$$\bar{p} \wedge (\overline{p \wedge q}) \equiv \bar{p}$$
.

Assignment 1- Question 3:

Prove the following logical equivalences using the method of the above example

(i)
$$(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t$$
.

PVP

Idemponent law

complement

(ii)
$$(p \wedge q) \wedge q \equiv p \wedge q$$
.

PA (9A9)

associantive law

PNOV

Idemponent law

(iii)
$$\bar{p} \wedge (\bar{p} \wedge q) \equiv \bar{p}$$
.
 $\bar{p} \wedge -(\bar{p} \wedge q) \equiv \bar{p}$

de morgan law

P

Absorbtion low

