Discrete Structures

Course Code: CS-1004

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Chapter 1

Mathematical Logic

Mathematical Logic

- Definition: Mathematical Logic is a Method of reasoning; it provides rules and techniques to determine whether an (argument == statement == proposition) is valid or not

$x = ٤$

2,٤,٦ فردي

$x - 1 = 5$

Example: If x is an even integer, then $x + 1$ is an odd integer

- The statement x + 1 is an odd integer is true under the condition that x is an even integer is true.

- **التعريف**: المنطق الرياضي هو طريقة للتفكير المنطقي؛ حيث يوفر قواعد وتقنيات لتحديد ما إذا كان (الحجة == العبارة == الاقتراح) صحيحًا أم لا.
- **مثال**: إذا كان x عددًا صحيحًا زوجيًا، فإن $x + 1$ عدد صحيح فردي.
- **العبارة**: $x + 1$ عدد صحيح فردي صحيحة بشرط أن يكون x عددًا صحيحًا زوجيًا.

Mathematical Logic

جملة　　　　جملة خبرية

- A **statement**, or a **proposition**, is a declarative sentence that is either true or false, but not both

جملة خبرية تحتمل الصواب أو الخطأ وليس كلاهما

- Uppercase letters denote propositions
  - Examples:

امثلة عن جمل خبرية

    - P: 2 is an even number (true) T
    - Q: 7 is an even number (false) F
    - R: A is a vowel (true) T
  - The following are not propositions:
    - P: My cat is beautiful　not Poroposition
    - Q: My house is big　رأي

      How beautiful is the rose?　سؤال

      What time is it?　سؤال

      Take a cup of tea.　امر

1- سؤال
2- امر
3- رأي
4- معادلة فيها متغيرات

$$x + y = 3$$

$$Q = \underbrace{\text{Today}}_{\text{True}} \text{ is } \underbrace{\text{Friday}}_{\text{False}}$$

$$\underbrace{x}_{} = \underbrace{2+1}_{}$$

not proposition

Logical Connectives and Truth Tables

- Each proposition evaluates to either true T or false F.

قيم الصدق

- T and F are called proposition truth values

جمله جزئيه بسيطه

In this section we look at how simple propositions can be combined to form more complicated propositions called **compound propositions**. جمله جزئيه مركبة

روابط المنطقيه

نفي

- **Logical connectives** {negation, and, inclusive or, exclusive or, implication, biconditional} are used to link pairs of propositions.

$\sim$ $\vee$ $\veebar$ $\rightarrow$ $\leftrightarrow$

هذه الادوات بنستخدم لدمج الجمل

- The truth value of any compound proposition is completely determined by (a) the truth values of its component simple propositions, and (b) the particular connective, or connectives, used to link them.

النفي

Negation

$\bar{p}$ (or $\sim p$ or $-p$ or $\neg P$)

$P \to -P,\ \sim P,\ \neg P \qquad \bar{P}$

اليوم الجمعة

اليوم ليس الجمعة

The negation of P, written $\neg P$, is the statement obtained by negating statement P

Example:

P: A is a consonant letter. $\neg P$:A is not a consonant letter.

Q: The circle have a volume $\neg Q$: The circle does not have a volume

The negation of the proposition reverse the truth value of the proposition which is shown in the following truth table

جدول الصدق

P	$\neg P$
T	F
F	T

جدول النفي

There are several alternative ways of stating the negation of a proposition.

If we consider the proposition

p ‘**All dogs are fierce**’, كل الكلاب شرسة

حرف آخر للنفي

some examples of its negation are:

-p - **It is not the case that all dogs are fierce.** ليس من الصحيح أن كل الكلاب شرسة

-p - **Not all dogs are fierce.** ليست كل الكلاب شرسة

-p - **Some dogs are not fierce.** بعض الكلاب ليست شرسة

Note that the proposition ‘No dogs are fierce’ is not the negation of ‘All dogs are fierce’.

الكلاب ليست شرسة

ليس نفي صحيح

اليوم جمعه $\boxed{و}$ السماء ماطره

$P \wedge Q$

Conjunction ^

∧ (و)

- Let *P* and *Q* be statements. The conjunction of *P* and *Q*, written *P* ^*Q* , is the statement formed by joining statements *P* and *Q* using the word “and”
- P ^ Q is true if both P and Q are true; otherwise P ^ Q is false

تكون نتيجه and صحيحه اذا كانت الجملتين صحيحتين فقط

- The Truth Table for the conjunction is :

T: 1   F: 0

<u>Example:</u>

- P : The sun is shining.
- Q : Pigs eat turnips.
- P ∧ Q : The sun is shining and pigs eat turnips.

الشمس مشرقة و الخنازير تاكل اللفت

P	Q	P ∧ Q
1	1	1
1	0	0
0	1	0
0	0	0

جدول ∧
and

$\vee$ او or

The Disjunction: Let P and Q be statements.

- The **inclusive disjunction** of P and Q, written $P \vee Q$ is true if at least one of the statements P and Q is true; otherwise, $P \vee Q$ is false. The Truth Table for the inclusive disjunction is shown left below.

على الاقل

- The **exclusive disjunction** of P and Q, written P $\underline{\vee}$Q is true if either P or Q is true but not both. The Truth Table for the inclusive disjunction is shown right below.

ليس كلاهما

تكون الجمله صحيحه فقط اذا كانت احدى الجملتين صحيحه

P	Q	P∨ Q
1	1	1
1	0	1
0	1	1
0	0	0

Inclusive or

تكون الجمله خاطئة فقط عندما تكون الجملتين خطأ

p	q	$p \underline{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

Exclusive or

Examples:

Assume the propositions:

P: ‘Tomorrow I will go swimming.

Q: ‘I will play golf’.

Then the proposition: $\underline{\vee}$

‘Tomorrow I will go swimming or ‘I will play golf’. and not both

seems to suggest that I will not do both and therefore points to an exclusive interpretation $P \underline{\vee} Q$

But the propositions $\vee$

‘Applicants for this post must be over 25 or have at least 3 years relevant.

experience’ suggests that applicants who satisfy both criteria will also be considered, therefore ‘or’ should be interpreted inclusively $P \boxed{\vee} Q$.

Implication or condition

If P ⟶ then Q

جمله ل اداه، شرط

– Let P and Q be statements. The statement ***if P then Q is*** called an **implication or condition**. It is written as $P \rightarrow Q$

جمله سببيه او فرضيه

فرضيه   استنتاج

P is called the hypothesis, Q is called the conclusion

- Truth Table for Implication:

If I pass then I will go for trip

P ⟶ Q

P	Q	P → Q
1	1	1
1	0	0
0	1	1
0	0	1

اذا كان السبب صحيح و النتيجه خاطئه تكون الجمله خاطئه

- Notice that ‘if P then Q’ is false only when P is true and Q is false, i.e. a true statement cannot imply a false one.
- If P is false, the compound proposition is true no matter what the truth value of Q.

Examples:

P	Q	P → Q
1	1	1
1	0	0
0	1	1
0	0	1

- P : I eat breakfast.
- Q : I don't eat lunch. F : I eat lunch
- P → Q : If I eat breakfast then I don't eat lunch

- Let P : Today is Sunday.
  Q : I will wash the car.
- $P \to Q$: If today is Sunday, ~~then~~ I will wash the car

if $P \to$ then Q

  – The **contrapositive** of this implication is $\neg Q \to \neg P$ If $\neg Q \to$ th $\neg P$

    If I do not wash the car, then today is not Sunday

If I don't wash the car then today is not sunday

Biimplication (Biconditional:)

$\leftrightarrow$ if and only if

Let P and Q be statements. The statement "P if and only if Q" is called the biimplication or biconditional of P and Q.

It is written as P $\leftrightarrow$ Q

Example:

$p \rightarrow q$ ✓

$q \rightarrow p$

- P : I eat breakfast.
- Q : I don't eat lunch. $\longleftrightarrow$
- P $\leftrightarrow$ Q : I eat breakfast if and only if I don't eat lunch (or alternatively, 'If and only if I eat breakfast, then I don't eat lunch').
- The truth table for P $\leftrightarrow$ Q is given by:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

✓

تكون الجملة صحيحة إذا كانا الجملتين
خطأ او الجملتين صواب

Examples 1.1

Consider the following propositions:

p : Mathematicians are generous.

q : Spiders hate algebra.

Write the compound propositions symbolized by:

(i) $p \vee \bar{q}$

(ii) $\overline{(q \wedge p)}$

(iii) $\bar{p} \rightarrow q$

(iv) $\bar{p} \leftrightarrow \bar{q}$.

if and only if

Solution

(i) Mathematicians are generous or spiders don't hate algebra (or both).

(ii) It is not the case that spiders hate algebra and mathematicians are generous.

q ∧ p

$\bar{p}$ q

(iii) If mathematicians are not generous then spiders hate algebra.

(iv) Mathematicians are not generous if and only if spiders don't hate algebra.

$\bar{p}$ $\bar{q}$

Example 1.2

Let p be the proposition 'Today is Monday' and

q be 'I'll go to London'.

Write the following propositions symbolically.

(i) If today is Monday then I won't go to London. $p \rightarrow \bar{q}$

(ii) Today is Monday or I'll go to London, but not both. $p \veebar q$

(iii) I'll go to London and today is not Monday. $q \wedge \neg p$

(iv) If and only if today is not Monday then I'll go to London. $\bar{p} \leftrightarrow q$

(i) $p \to \bar{q}$

(ii) $p \veebar q$

(iii) $q \wedge \bar{p}$

(iv) $\bar{p} \leftrightarrow q$.

$\vee$ or $\veebar$ or but not both

$\wedge$ and $\rightarrow$ if then $\leftrightarrow$ if and only if

Example 1.3:

Construct the Truth Table for the following compound propositions

(i) $\bar{p} \vee q$

(ii) $\bar{p} \wedge \bar{q}$

(iii) $\bar{q} \rightarrow p$

(iv) $\bar{p} \leftrightarrow \bar{q}$.

Solution

(i)

p	q	$\bar{p}$	$\bar{p} \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

p	q	$\bar{p}$	$\bar{p} \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

(ii)

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \wedge \bar{q}$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(iii)

p	q	$\bar{q}$	$\bar{q} \to p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

(iv)

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \leftrightarrow \bar{q}$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ملاحظة اذا احتوت الجملة المركبة على جملتين فقط يكون
جدول الصدق مكون من ٤ صفوف

اما اذا كانت الجملة فيها 3 جمل سيكون جدول الصدق من 8 صفوف

Example 1.4. Construct truth tables for:

(i) $p \rightarrow (q \wedge r)$

(ii) $(\bar{p} \vee q) \leftrightarrow \bar{r}$.

Solution

(i)

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

(ii) Homework

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

$p \rightarrow (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

ii $(\bar{p} \vee q) \leftrightarrow \bar{r}$

p	q	r	$\bar{p}$	$\bar{r}$	$(\bar{p} \vee q)$	$(\bar{p} \vee q) \leftrightarrow \bar{r}$
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	F	F

	p	q		$p \vee q$		$p \wedge q$		$(p \vee q) \rightarrow (p \rightarrow q)$	
1	T	T	1	T	1	T	1	T	1
1	T	F	0	T	1	F	0	F	0
0	F	T	1	T	1	F	0	F	0
0	F	F	0	F	0	F	0	T	1

Example 1.5 Construct the truth table for $(p \vee q) \to (p \wedge q)$

Solution

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
1	1	1	1	1
1	0	1	0	0
0	1	1	0	0
0	0	0	0	1

<u>Example 1.6</u> Construct the truth table for $(p \to q) \to (q \to p)$

p	q	p→q	q→p	(p→q) → (q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Solution

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
1	1	1	1	1
1	0	0	1	1
0	1	1	0	0
0	0	1	1	1

Tautology

هي جملة دائماً T
العمود الاخير في جدول الصدق = T

- A **tautology** is a compound proposition which is true no matter what the truth values of its simple components.

In other words A compound proposition $p \equiv p\ (p_1, p_2, \ldots, p_n)$ where $p_1, p_2, \ldots, p_n$ are variables is called a tautology if it is true for every truth assignments for $p_1, p_2, \ldots, p_n$.

هي جملة مركبة صحيحة دائماً بغض النظر عن الجمل البسيطة صح أو خطأ

Example 1.7 Show that $p \vee \bar{p}$ is a tautology.

tautology اثبت ان الجملة تاتيه

Solution

نعمل جدول صدق

Constructing the truth table for $p \vee \bar{p}$, we have:

اذا كان العمود الاخير كله T تكون Tautology

p	$\bar{p}$	$p \vee \bar{p}$
T	F	T
F	T	T

T als

p	$\bar{p}$	$p \vee \bar{p}$
T	F	T
F	T	T

Note that $p \vee \bar{p}$ is always true (no matter what proposition is substituted for p) and is therefore a tautology.

p	q	$p \wedge q$	$\overline{p \wedge q}$	$(p \wedge q) \vee (\overline{p \wedge q})$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Example 1.8 Show that $(p \wedge q) \vee (\overline{p \wedge q})$ is a tautology.

Solution

The truth table for $(p \wedge q) \vee (\overline{p \wedge q})$ is given below.

p	q	$p \wedge q$	$\overline{p \wedge q}$	$(p \wedge q) \vee (\overline{p \wedge q})$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

العمود الاخير كله T

اذا Tautology

The last column of the truth table contains only the truth value T and hence we can deduce that $(p \wedge q) \vee (\overline{p \wedge q})$ is a tautology.

Note that The proposition $(p \wedge q) \vee (\overline{p \wedge q})$ is said to be a **substitution instance** of the proposition $p \vee \bar{p}$

الجمله أو نفيها دائماً صواب

دائماً تكون خاطئة
F = العمود الاخير بالجدول

مغالطة

Contradiction

- A **contradiction** is a compound proposition which is false no matter what the truth values of its simple components.

يعني دائماً خطأ بغض النظر عن الاجزاء بحمل بسيطة

- **The proposition** $p \wedge \sim p$ is a contradiction which is shown in next truth table

p	q	$\bar{p}$	$\bar{q}$	$(p \wedge \bar{q})$	$(\bar{p} \vee q)$	$(p \wedge \bar{q}) \wedge (\bar{p} \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

العمود الاخير كله F

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

الجملة و نفيها دائماً غلط

Example 1.9 Show that $(p \wedge \bar{q}) \wedge (\bar{p} \vee q)$ is a contradiction.

Solution

p	q	$\bar{q}$	$p \wedge \bar{q}$	$\bar{p}$	$\bar{p} \vee q$	$(p \wedge \bar{q}) \wedge (\bar{p} \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

contradiction

Assignment 1- Question 1:

Attempt any two of the following questions

كله T — كله F — يوجد T,F

Determine whether each of the following is a tautology, a contradiction or neither:

1. $p \to (p \vee q)$

2. $(p \to q) \wedge (\bar{p} \vee q)$

3. $(p \vee q) \leftrightarrow (q \vee p)$

4. $(p \wedge q) \to p$

5. $(p \wedge q) \wedge (\overline{p \vee q})$

$(p \wedge q) \wedge -(p \vee q)$

(1)

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology

p	q	$p \to q$	$-p$	$\neg p \vee q$	$(p \to q) \wedge (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

neither Tautology nor contradiction

(3)

p	q	$p \vee q$	$q \vee p$	$(p \vee q) \leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Tautology

(4)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

(5)

p	q	$p \vee q$	$\neg(p \vee q)$	$p \wedge q$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	F	T	F
T	F	T	F	F	F
F	T	T	F	F	F
F	F	F	T	F	F

Contradiction

مسبب منطقي

Logically Implies

P ⊢ q

– A proposition P is said to **logically imply** a proposition Q if, whenever P is true, then Q is also true. If P logically implies Q, then symbolically we write P → Q .

نسميه بجملة P مسبب منطقي للجملة Q
اذا كانت P صحيحة تكون Q صحيحة

– In Other words: A proposition P is said to logically imply proposition Q if the proposition P → Q is a tautology.

P→Q
ممنوع Q تكون خطأ اذا كانت P صحيحة

Note that the converse does not apply, i.e. Q may also be true when P is false. For logical implication all we insist on is that Q is never false when P is true. We shall symbolize logical implication by ⊢ so that 'P logically implies Q' is written $P \vdash Q$.

اثبت ان الجملة (مسبب منطقي)
نفحص جدول صدق p→q ونبحث عن صفين P الصحيحة
ونتميز ما يقابلها من q اذا كانت صحيحة نعتبر مسبب منطقي

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

الجملة، الصحيحة
عمرها ما بتجيبها
جملة صحيحة
دايماً

Example 1.10

Show that $q \vdash (p \vee q)$.

Solution

We must show that, whenever q is true, then $p \vee q$ is true. Constructing the truth table gives:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$q \vdash p \vee q$

q is logically implies $(p \vee q)$

From a comparison of the second and third columns we note that, whenever q is true (first and third rows), $p \vee q$ is also true. Note that $p \vee q$ is also true when q is false (second row) but this has no relevance in establishing that q logically implies $p \vee q$.

Example 1.11

Show that $(p \leftrightarrow q) \wedge q$ logically implies p.

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$	p
T	T	T	T	T
T	F	F	F	T
F	T	F	F	F
F	F	T	F	F

Solution

As with example 1.5 we can show that $[(p \leftrightarrow q) \wedge q] \vdash p$ in one of two ways. We can either show that p is always true when $(p \leftrightarrow q) \wedge q$ is true or we can show that $[(p \leftrightarrow q) \wedge q] \rightarrow p$ is a tautology.

yes it is logically implie

The truth table for $(p \leftrightarrow q) \wedge q$ is given by:

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	F

$((p \leftrightarrow q) \wedge q) \vdash p$

Comparing the fourth column with the first, we see that p is true whenever $(p \leftrightarrow q) \wedge q$ is true (first row only). Therefore $[(p \leftrightarrow q) \wedge q] \vdash p$.

Alternatively, we could complete a further column of of the truth table for $[(p \leftrightarrow q) \wedge q] \rightarrow p$ and show this to be a tautology.

جمل متكافئة منطقياً

Logically Equivalent

لهم نفس صدق متطابقة

Two propositions are said to be **logically equivalent** if they have identical truth values for every set of truth values of their components. we write $P \equiv Q$ if P and Q are logically equivalent. In other words a proposition P is said to be **logically equivalent** to a propositions Q if the statement formula $P \leftrightarrow Q$ is a tautology

لإثبات تكافؤ جملتين ننشئ جدول صدق للجملتين ونلاحظ إذا تطابق

لإثبات تكافؤ جملتين ننشئ جدول صدق $P \leftrightarrow Q$ إذا كان صحيح دائماً

نعين صدق تطابق

Example 1.12

Show that $\bar{p} \vee \bar{q}$ and $\overline{p \wedge q}$ are logically equivalent, i.e. that $(\bar{p} \vee \bar{q}) \equiv (\overline{p \wedge q})$.

Solution

نقارن بين الجملة

We draw up the truth table for $\bar{p} \vee \bar{q}$ and also for $\overline{p \wedge q}$.

$$\bar{p} \vee \bar{q} \equiv \overline{p \wedge q}$$

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee \bar{q}$	$p \wedge q$	$\overline{p \wedge q}$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee \bar{q}$	$p \wedge q$	$\overline{p \wedge q}$
T	T	F	F	F	T	F
T	F	F	F	T	F	T
T	T	T	F	T	F	T
T	F	F	T	T	F	T

Example 1.13 Show that the following two propositions are logically equivalant

(i) If it rains tomorrow then, if I get paid, I'll go to Paris. $p \to (q \to r)$

(ii) If it rains tomorrow and I get paid then I'll go to Paris. $(p \wedge q) \to r$

Solution

Define the following simple propositions:

p : It rains tomorrow.

q : I get paid.

r : I'll go to Paris.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

We are required to show the logical equivalence of $p \to (q \to r)$ and $(p \wedge q) \to r$. We can do this in one of two ways:

(a) establish that $p \to (q \to r)$ and $(p \wedge q) \to r$ have the same truth values,

or

(b) establish that $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$ is a tautology.

Using the first method we complete the truth table for $p \to (q \to r)$ and $(p \wedge q) \to r$.

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p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

Since the truth values of $p \to (q \to r)$ and $(p \wedge q) \to r$ are the same for each set of truth values of p, q and r, we can deduce the logical equivalences of these compound propositions. $p \to (q \to r) \equiv (p \wedge q) \to r$

Assignment 1- Question 2:

Attempt any two of the following questions

1. Prove that $(p \to q) \equiv (\bar{p} \vee q)$.

2. Prove that $(p \wedge q)$ and $(\overline{p \to \bar{q}})$ are logically equivalent propositions.

3. Prove that $(\overline{p \veebar q}) \equiv (p \veebar \bar{q})$.

4. Prove that p logically implies $(q \to p)$.

①

p	q	$p \to q$	$\bar{p}$	$\bar{p} \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

logically equivalent

②

p	q	$\neg q$	$p \to \neg q$	$\neg(p \to \neg q)$	$p \wedge q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

logically equiv

③

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg q$	$p \oplus \neg q$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	T	F	F	T
F	F	F	T	T	F

logically equiv

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	F

$P \vdash (P \to q)$

The Algebra of Propositions

The following is a list of some important logical equivalences, all of which can be verified using one of the techniques described previously. These laws hold for any simple propositions *p*, *q* and *r* and also for any substitution instance of them.

The **duality principle** states that, if two propositions are logically equivalent, then so are their duals.

The Duality Principle

Given any compound proposition P involving only the connectives denoted by $\wedge$ and $\vee$, the **dual** of that proposition is obtained by replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$, t by f and f by t. For example, the dual of $(p \wedge q) \vee \bar{p}$ is $(p \vee q) \wedge \bar{p}$. The dual of $(p \vee f) \wedge q$ is $(p \wedge t) \vee q$.

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Laws of algebra of proposition

	Primal form	Dual Form
Idempotent Law	$p \vee p \equiv p$	$p \wedge p \equiv p$
Dominant Law	$p \vee T \equiv T$	$p \wedge F \equiv F$
Complement Law	$p \vee \sim p \equiv T$	$p \wedge \sim p \equiv F$
Commutative Law	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Associative Law	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive Law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption Law	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
DeMorgan's Law	$\sim(p \vee q) \equiv \sim p \wedge \sim q$	$\sim(p \wedge q) \equiv \sim p \vee \sim q$ مهم

Example 1.14

Prove that $(\bar{p} \wedge q) \vee \overline{(p \vee q)} \equiv \bar{p}$. without using truth table

$$(\bar{p} \wedge q) \vee \neg(p \vee q) \rightsquigarrow (\bar{p} \wedge q) \vee (\bar{p} \wedge \bar{q})$$

$$\bar{p} \wedge (q \vee \bar{q})$$

Solution

$$
\begin{aligned}
(\bar{p} \wedge q) \vee \overline{(p \vee q)} &\equiv (\bar{p} \wedge q) \vee (\bar{p} \wedge \bar{q}) && \text{(De Morgan's laws)} \\
&\equiv \bar{p} \wedge (q \vee \bar{q}) && \text{(distributive laws)} \\
&\equiv \bar{p} \wedge T && \text{(complement laws)} \\
&\equiv \bar{p}. && \text{(identity laws)}
\end{aligned}
$$

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قانون التوزيع

قانون المتمم

قانون التطابق

$p \wedge T$

p

قانون التطابق

Assignment 1- Question 3:

Prove the following logical equivalences using the method of the above example

(i) $(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t$.

(ii) $(p \wedge q) \wedge q \equiv p \wedge q$.

(iii) $\bar{p} \wedge \overline{(p \wedge q)} \equiv \bar{p}$.

Assignment 1- Question 3:

Prove the following logical equivalences using the method of the above example

(i) $(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t$.

$(p \wedge p) \vee (\bar{p} \vee \bar{p})$

$p \vee \bar{p}$ Idemponent law

$= T$ complement

(ii) $(p \wedge q) \wedge q \equiv p \wedge q$.

$p \wedge (q \wedge q)$ associatiltive law

$p \wedge q$ Idemponent law

(iii) $\bar{p} \wedge \overline{(p \wedge q)} \equiv \bar{p}$.

$\bar{p} \wedge -(p \wedge q) \equiv \bar{p}$

$\bar{p} \wedge (\bar{p} \vee \bar{q})$ de morgan law

$\bar{p}$ Absorbtion law