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Chapter 10

One-Sample Tests of Hypothesis

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Learning Objectives

LO10-1 Explain the process of testing a hypothesis.

- LO10-2 Apply the six-step procedure for testing a hypothesis.
- LO10-3 Distinguish between a one-tailed and a two-tailed test of hypothesis.
- LO10-4 Conduct a test of a hypothesis about a population mean.
- LO10-5 Compute and interpret a p-value.
- LO10-6 Use a t-statistic to test a hypothesis.

Introduction

- We continue to apply statistics and statistical inference to the research process.
- In the research process, we often start with a hypothetical statement.
- Then we define a population to sample, collect data on the variables of interest, and then conduct statistical analysis.
- This analysis performs statistical tests of the hypothesis using the sample data.
- The results of the analysis provide the evidence used to make inferences about the population.

What is Hypothesis Testing?

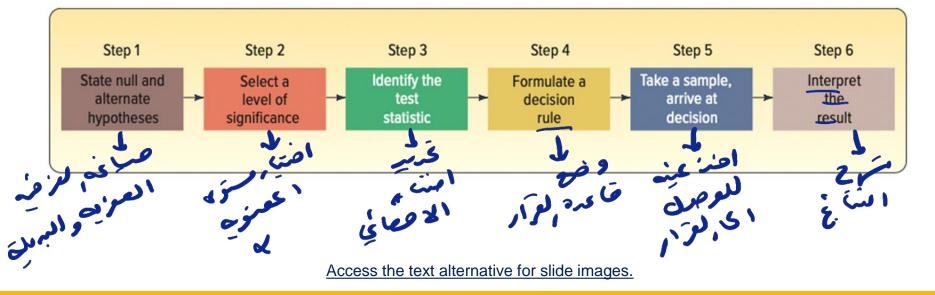
محمدت اعمد العمدي العمد العمدي الع subject to verification. المريد الشريك العمدي ال

- Select sample data.
- Use statistics to conduct the hypothesis test.
- Based on sample evidence, decide to reject or fail to reject the statement.

Hypothesis Testing A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.

ختار عنيه وغرى منصق (اعتسار الارضه) معن لاده الماتي عذالعيد مقرر مع منعن فرخه عنا العيد مقرر مله مفش من معن المرضه

- Procedure that systematizes hypothesis testing.
- At step 6, interpret the results based on the decision to reject or no reject the hypothesis.
- No proof something is true. ا صنب ر الرزمب ت التبات وجر در منبو But "proof beyond a reasonable doubt" like a court.



Six-Step Procedure for Testing a Hypothesis ² Ho H

• Step 1: State the Null and Alternative hypothesis.

 Null Hypothesis
 A statement about the value of a

 population parameter developed for the purpose of testing

 numerical evidence.

 الماده هود معاملات الحجيم في الحي العضية

 • Depoted

- Denoted H_0 .
- The subscript of **O** denotes no change.
- If sample data provide convincing evidence it is false, then the null hypothesis is rejected.
- Otherwise reject the null hypothesis.

ادا العبِّ اعضًا أدله على كامنيه ليتم رغب هذه الزخم

• Step 1: State the Null and Alternative hypothesis.

Alternate Hypothesis A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false. آفاده ميت حبولها ادر حتاطت

- Denoted H_1 .
- Research hypothesis.
- If the null hypothesis is rejected, we conclude the alternative is supported by the sample data.
- The sign will not appear in the alternative.

• Step 2: Select a Level of Significance.

Level of Significance The probability of rejecting the null hypothesis when it is true. احتراف وحمل جور حمل محد العزامية العزام

V = 0.01

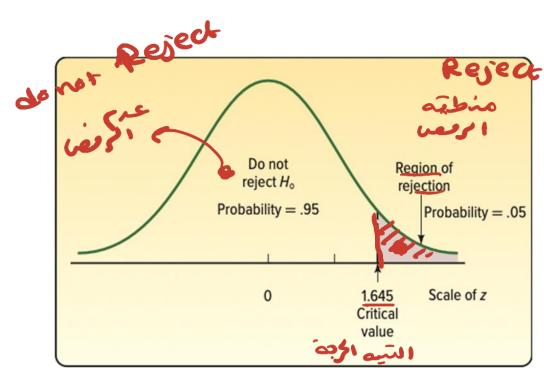
- Due to sampling, there is a risk the sample indicates the null is false when it is true.
- α denotes the probability of this error.
- Determine this error before proceeding.
- Traditionally, use 0.01, 0.05 and 0.10.

Step 3: Select the Test Statistic.
Test Statistic A value, determined from sample information, used to determine whether to reject the null hypothesis.

• When testing a mean when σ is known: $z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$.

- Used because of the central limit theorem.
- The number of standard errors that separate the sample and population values.
- Determine probabilities the sample mean is within a specified number of standard errors.

Step 4: Formulate the Decision Rule.
 Critical Value The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.



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Step 5: Make a Decision.

Compute the value of the test statistic.

Compare it to the critical value.

لحربة

Make a decision.

- Reject: Improbable the test statistic is large due to sampling error. مين في المنه معلي من ح
- Fail to Reject H_0 : A small test statistic attributed to sampling error.

It is possible to make one of two errors.

• Step 5: Make a Decision.

الغابي الخية

Type one Error Rejecting the null hypothesis, H_0 ,

when it is true.

Type two Error Not rejecting the null hypothesis when it is false.

احتمايه رمف، الوض العونة ومس جلس

- α Probability of making a Type one error.
- Probability of making a Type two error.

Six-Step Procedure for Testing a Hypothesis ³

• Step 5: Make a Decision.

Null Hypothesis	Researcher DoesNot Reject H ₀	Researcher Rejects H ₀	ا فد
H_0 is true	Correct decision 🥢	Type I error	×
H_0 is false	Type II error 🗙	Correct decisio	n

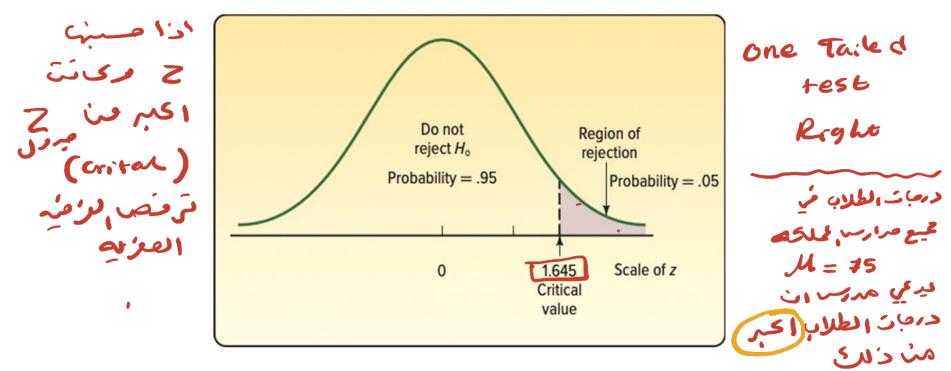
عرم رحن

• Step 6: Interpret the Result.

ترج . لمنابخ

• What can we say or report based on the results of the statistical test?

One-Tailed and Two-Tailed Hypothesis Tests



- This is an example of a one-tailed test.
- The rejection region is only in one tail (right).
- Corresponds to > in the alternative.

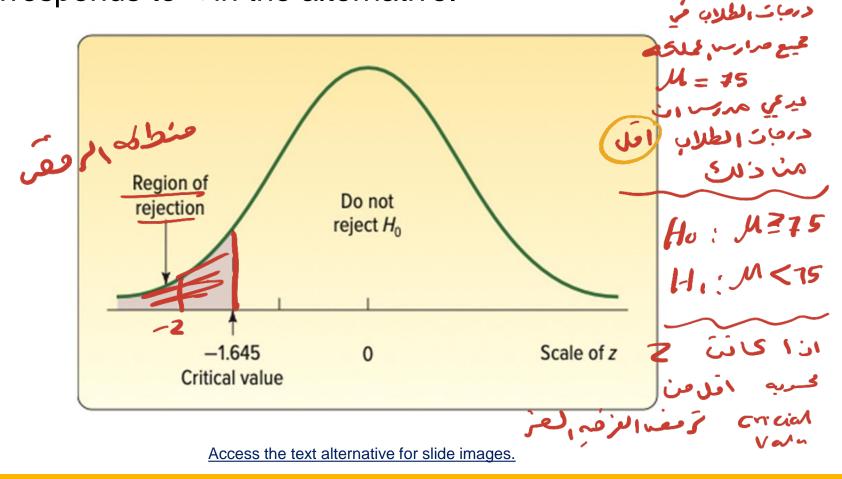
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Ho : M = 75

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One-Tailed and Two-Tailed Hypothesis Tests 2

- The rejection region is only in the left tail.
- Corresponds to < in the alternative.

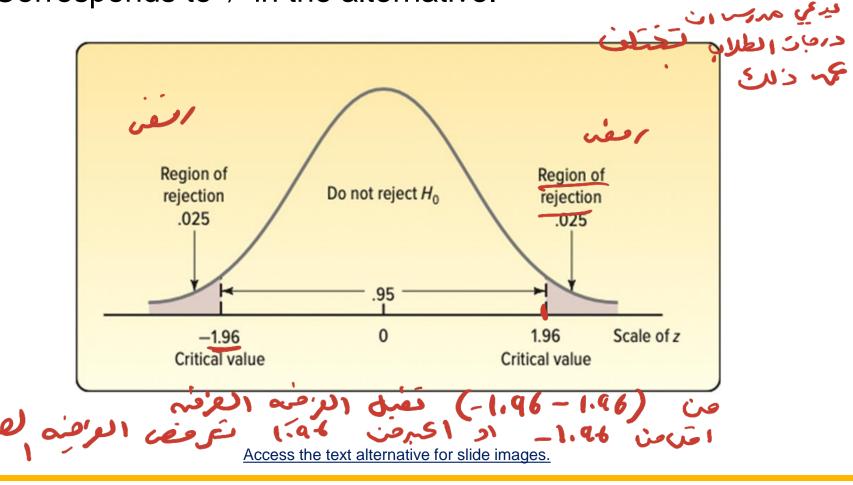


tailed test

1ef+

One-Tailed and <u>Two-Tailed Hypothesis</u> <u>Tests</u> ،

- Two-tailed test: The rejection region is both tails.
- Corresponds to \neq in the alternative.



M = 75

- Example 1: Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in New York State.
- At the Fredonia plant, the weekly production of the Model A325 desk follows a normal distribution with a mean of 200 and a standard deviation of 16.
- New production methods have been introduced and the vice president of manufacturing would like to investigate whether there has been a change in weekly production of the Model A325.
- Is the mean number of desks produced different from 200 at the .01 significance level?

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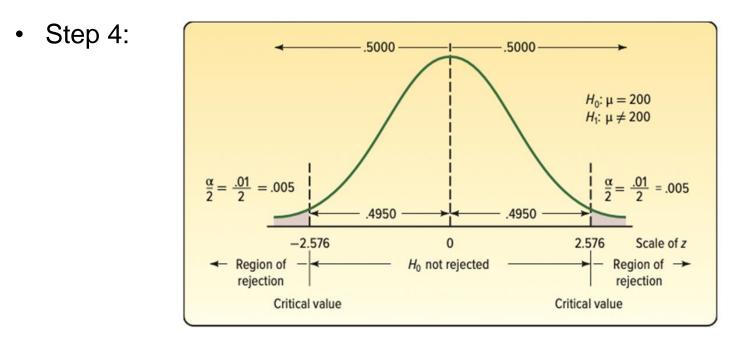
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α level	One-tailed (left)	One-tailed (right)		→
α = 0.05	z = -1.64 5	z=1.645 \$1.65	z = ± 1.96	
α = 0.01	z = -2.33	z = 2.33	$z = \pm 2.57$	
α = 0.001	z = -3.08	z = 3.08	z = ± 3.32	
\$ = 0,1	-1.28	1.28	1.645	

ماعدة الرّار اذا وفعن 2 128 -1.28 ماعدة الرّار اذا وفعن 2 12 معد من منطقة القبول (2.57 - 2.57) تَعْبِل الرَّخِه H6

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	0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359	
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	0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141	
	0.2	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1020	0.1443	0.1480	0.1517	0.5
	0.3	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1400	0.1879	
	0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224	
	$\frac{0.5}{0.6}$	0.1913	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549	0.45
	0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852	
	0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133	
	0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389	1.65
	1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621	<u> </u>
	1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810		
	1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944		0.3980	0.3997	0.3830 0.4015	.64>
	1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177	
	1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319	
	1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441	
		0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545	
	1.6	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633	
	1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706	
	1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767	
	2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817	
	2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857	
	2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890	
	2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916	
	2.3 2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936	
(2.5 2.6	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952	
	2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964	
	2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974	
	2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981	
	2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986	
	3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990	

- Example continued.
- Step 1: $H_0: \mu = 200 \text{ vs } H_1: \mu \neq 200.$
- Step 2: $\alpha = 0.01$; Step 3: $z = \frac{x \mu}{\sigma / \sqrt{n}}$.



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Example continued.

Step 5:
$$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} = \frac{203.5 - 200}{16 / \sqrt{50}} = 1.547.$$

- 1.547 is in between -2.576 and 2.5760.
- Do not reject H_0 .

Step 6: We fail to reject the null hypothesis, so we did not show that the population mean has changed from 200 desks per week.

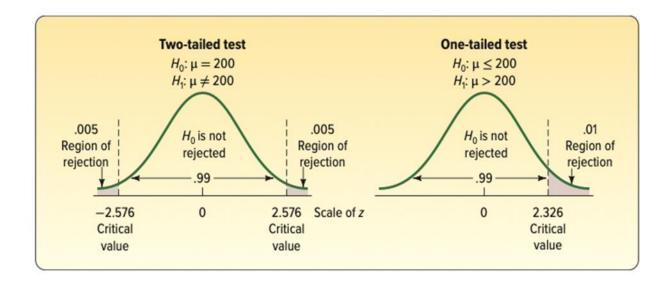
- The difference between the population mean of 200 per week and the sample mean of 203.5 could simply be due to chance.
- The sample information fails to indicate that the new production methods resulted in a change in the 200-desks-per-week production rate.

- In failing to reject the null hypothesis, did we prove that the assembly rate is still 200 per week?
- Absolutely not.
- We did NOT prove it was true.
- The result does not support any conclusion about the null hypothesis.
- The sample data simply do not support the alternate hypothesis.
- Could use a confidence interval approach.
- Example: If the interval captured 200, do not reject H_0 .

The previous example demonstrated a two-tailed test to determine if the mean differed from 200.

We could have tested to see if there was an increase.

- $H_0: \mu \le 200.$
- $H_1: \mu > 200.$



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امتيار الرضت و p-Value in Hypothesis Testing

- p-value is less than α , reject H_0 .
- p-value is greater than $\underline{\alpha}$, do not reject H_0 .

Ho مرمض PLA Ho لازمض PLA

A p-value also gives additional insight about the strength of that decision.
 P-value احتداع احتداع المتعادين المتعادين

p-Value in Hypothesis Testing 2

If the p-value is less than.

a) 0.10, some evidence $H sub_0$ is not true.

b) 0.05, strong evidence H sub ₀ is not true.

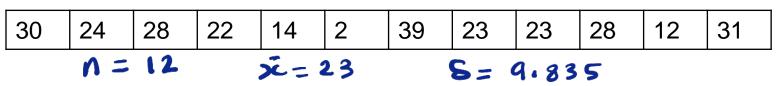
c) 0.01, very strong evidence $H sub_0$ is not true.

d) 0.001, extremely strong evidence *H* sub ₀ is not true.

Example: The p-value of 0.060 is greater than 0.01. Rejection region $\frac{\alpha}{2} = \frac{.01}{2} = .005$ -2.576 -1.55 0 1.55 2.576 Scale of z

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- Example 2: The Myrtle Beach International Airport provides a cell phone parking lot where people can wait for a message to pick up arriving passengers.
- To decide if the cell phone lot has enough parking places, the manager of airport parking needs to know if the mean time in the lot is more than 15 minutes. #>15
- A sample of <u>12 recent customers</u> showed they were in the lot the following lengths of time, in minutes.

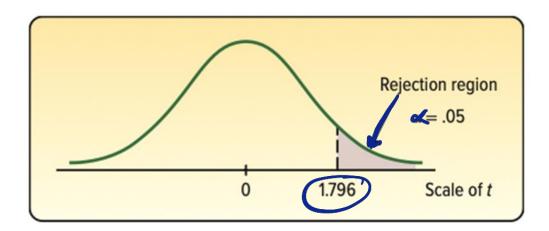


• At the .05 significance level, is it reasonable to conclude that the mean time in the lot is more than 15 minutes?

 صاغه المؤجات Ho: ME 15 H1: M>15 2 خذر مرة المعنوبة d= 0.05 t statistics and 3 اختبار الوص t=x-M 2=50-14 S/Jn t= P-JA t = 23 - 15 = 2.818187'12 (182) 9.835/512 ملیک همود ن P-value من جرل is (4) ŧ — df = 11one tailed X 0.05 Fest (n-i)Critical volare = 1.796 (scale +) = 1.796 1.796 تا ومغدة في منطه الرفق الوضَّ ثَنَّ الرَّاج السرت معف الوطيد العماني ، حبرمن کا دمنیقه

Hypothesis Testing for a Population Mean: Population Standard Deviation Unknown ²

- Example continued.
- Step 1: $H_0: \mu \le 15 \ vs \ H_1: \mu > 15.$
- Step 2: $\alpha = 0.05$.
- Step 3: $t = \frac{\overline{x} \mu}{s / \sqrt{n}}$.
- Step 4: df = 11 and the critical value is t = 1.796.



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Example continued.

Step 5: $\overline{x} = 23$, s = 9.835, n = 12 so that.

•
$$t = \frac{\overline{x} - \mu}{s / \sqrt{n}} = \frac{\underline{23} - 15}{\underline{9.835} / \sqrt{12}} = \underline{2.818}.$$

- 2.818 is larger than 1.796.
- Reject the null hypothesis.

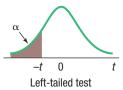
Step 6:

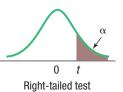
- We conclude that the mean time <u>customers</u> spend in the lot is more than 15 minutes.
- This result indicates that the airport may need to add more parking places.

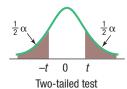
Appendix B

B.2 Student's t Distribution









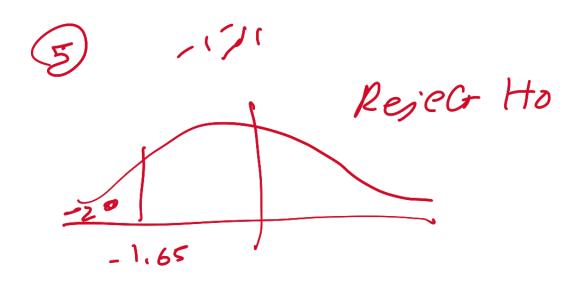
	Confidence Intervals, c								Co	onfidence In	tervals, <i>c</i>			
	80%	90%	95%	98%	99%	99.9%		80%	90%	95%	98%	99%	99.9%	
		Level of Significance for One-Tailed Test, α						Level of Significance for One-Tailed Test, α						
df	0.10	0.05	0.025	0.01	0.005	0.0005	df	0.10	0.05	0.025	0.01	0.005	0.0005	
		Level of Significance for Two-Tailed Test, α						Level of Significance for Two-Tailed Test, α						
	0.20	0.10	0.05	0.02	0.01	0.001		0.20	0.10	0.05	0.02	0.01	0.001	
1	3.078	6.314	12.706	31.821	63.657	636.619	36	1.306	1.688	2.028	2.434	2.719	3.582	
2	1.886	2.920	4.303	6.965	9.925	31.599	37	1.305	1.687	2.026	2.431	2.715	3.574	
3	1.638	2.353	3.182	4.541	5.841	12.924	38	1.304	1.686	2.024	2.429	2.712	3.566	
4	1.533	2.132	2.776	3.747	4.604	8.610	39	1.304	1.685	2.023	2.426	2.708	3.558	
5	1.476	2.015	2.571	3.365	4.032	6.869	40	1.303	1.684	2.021	2.423	2.704	3.551	
6	1.440	1.943	2.447	3.143	3.707	5.959	41	1.303	1.683	2.020	2.421	2.701	3.544	
7	1.415	1.895	2.365	2.998	3.499	5.408	42	1.302	1.682	2.018	2.418	2.698	3.538	
8	1.397	1.860	2.306	2.896	3.355	5.041	43	1.302	1.681	2.017	2.416	2.695	3.532	
9	1.383	1.833	2.262	2.821	3.250	4.781	44	1.301	1.680	2.015	2.414	2.692	3.526	
10	1.372	1.812	2.228	2.764	3.169	4.587	45	1.301	1.679	2.014	2.412	2.690	3.520	
11	1.363	1.796	2.201	2.718	3.106	4.437	46	1.300	1.679	2.013	2.410	2.687	3.515	
12	1.356	1.782	2.179	2.681	3.055	4.318	47	1.300	1.678	2.012	2.408	2.685	3.510	
13	1.350	1.771	2.160	2.650	3.012	4.221	48	1.299	1.677	2.011	2.407	2.682	3.505	
14	1.345	1.761	2.145	2.624	2.977	4.140	49	1.299	1.677	2.010	2.405	2.680	3.500	
15	1.341	1.753	2.131	2.602	2.947	4.073	50	1.299	1.676	2.009	2.403	2.678	3.496	
16	1.337	1.746	2.120	2.583	2.921	4.015	51	1.298	1.675	2.008	2.402	2.676	3.492	
17	1.333	1.740	2.110	2.567	2.898	3.965	52	1.298	1.675	2.007	2.400	2.674	3.488	
18	1.330	1.734	2.101	2.552	2.878	3.922	53	1.298	1.674	2.006	2.399	2.672	3.484	
19	1.328	1.729	2.093	2.539	2.861	3.883	54	1.297	1.674	2.005	2.397	2.670	3.480	
20	1.325	1.725	2.086	2.528	2.845	3.850	55	1.297	1.673	2.004	2.396	2.668	3.476	
21	1.323	1.721	2.080	2.518	2.831	3.819	56	1.297	1.673	2.003	2.395	2.667	3.473	
22	1.321	1.717	2.074	2.508	2.819	3.792	57	1.297	1.672	2.002	2.394	2.665	3.470	
23	1.319	1.714	2.069	2.500	2.807	3.768	58	1.296	1.672	2.002	2.392	2.663	3.466	
24	1.318	1.711	2.064	2.492	2.797	3.745	59	1.296	1.671	2.001	2.391	2.662	3.463	
25	1.316	1.708	2.060	2.485	2.787	3.725	60	1.296	1.671	2.000	2.390	2.660	3.460	
26	1.315	1.706	2.056	2.479	2.779	3.707	61	1.296	1.670	2.000	2.389	2.659	3.457	
27	1.314	1.703	2.052	2.473	2.771	3.690	62	1.295	1.670	1.999	2.388	2.657	3.454	
28	1.313	1.701	2.048	2.467	2.763	3.674	63	1.295	1.669	1.998	2.387	2.656	3.452	
29	1.311	1.699	2.045	2.462	2.756	3.659	64	1.295	1.669	1.998	2.386	2.655	3.449	
30	1.310	1.697	2.042	2.457	2.750	3.646	65	1.295	1.669	1.997	2.385	2.654	3.447	
31	1.309	1.696	2.040	2.453	2.744	3.633	66	1.295	1.668	1.997	2.384	2.652	3.444	
32	1.309	1.694	2.037	2.449	2.738	3.622	67	1.294	1.668	1.996	2.383	2.651	3.442	
33	1.308	1.692	2.035	2.445	2.733	3.611	68	1.294	1.668	1.995	2.382	2.650	3.439	
34	1.307	1.691	2.032	2.441	2.728	3.601	69	1.294	1.667	1.995	2.382	2.649	3.437	
35	1.306	1.690	2.030	2.438	2.724	3.591	70	1.294	1.667	1.994	2.381	2.648	3.435	

(continued)

Chapter 10 Practice Problems

A recent national survey found that high school students watched an average (mean) of 6.8 movies per month with a population standard deviation of 1.8. The distribution of number of movies watched per month follows the normal distribution. A random sample of <u>36</u> college students revealed that the mean number of movies watched last month was 6.2. At the .05 significance level, can we conclude that college students watch fewer movies a month than high school students? P K = 0.05one tailed $H_0: \mathcal{M} \ge 6.8$ $H_1: \mathcal{M} < 6.8$ -1.65 Ho Z <-1.65 Keje (+ 1.65

 $\begin{array}{rcl}
(4) & Z &= & \overline{X} - & M \\ & & S \\ & & & Jn \\ & = & 6.2 - 6.8 \\ & & 1.8 \\ & & J36 \\ & & & J36 \end{array}$



E=1.833 Reject Ho t>1.832

J The mean income per person in the United States is \$60,000, and the distribution of incomes follows a normal distribution. A random sample of (10) residents of Wilmington, Delaware, had a mean of \$70,000 with a standard deviation of \$10,000. At the .05 level of $\sim \frac{1}{2}$ significance, is that enough evidence to conclude that residents of Wilmington, Delaware, have more income than the national average? r = 0r = 10-1() Ho: M≤ 60 000 H. U>60000

 $t = \frac{x - h}{5h} = \frac{70000 - 60000}{10000}$ = 3.16 $t > t_{u,v}$ $p_{v,v}$ Fejeck Ho

LO10-2,3,4,5,6

A Washington, D.C., "think tank" announces the typical teenager sent 67 text messages per day in 2017. To update that estimate, you phone a sample of 12 teenagers and ask them how many text messages they sent the previous day. Their responses were:

51	175	47	44	54	145	203	21	59	42	100	49	
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At the .05 level, can you conclude that the mean number is greater than 67? Compute the *p*-value and describe what it tells you.



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Ho:
$$\mu \leq 67$$

 $\mu > 67$
 π
 $f = \frac{x - M}{\sqrt{n}}$
 $\frac{82.5 - 67}{\sqrt{12}} = 0.963$
 $\frac{5a.49}{\sqrt{12}}$
 $\sqrt{12}$
 $\sqrt{12}$

∑ = 82.5 S = 5q.uq

flo fail to reject

Pualue is ,

0.2 - 0.1

Proce = 0.1931