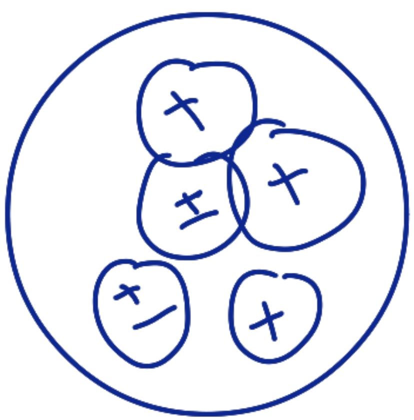
قوى الربط النووية

NUCLEAR BINDING ENERGY



The Strong Nuclear Force

- The force that binds the protons and neutrons together in the nucleus, despite the electrical repulsion of the protons, is an example of the strong nuclear force.
- Characteristics:
  - ❑ Does not depend on charge; neutrons and protons are bound and the binding is the same for both.
  - ❑ Short range; about 10^{-15} m. Within this range, the nuclear strong force is greater than the electrical force of repulsion.
  - ❑ Physicists still do not fully understand the dependence of the strong nuclear force on the separation r.

The Strong Nuclear Force

- The nearly constant density of nuclear matter and the nearly constant binding energy per nucleon of larger nuclides show that a particular nucleon cannot interact simultaneously with all the other nucleons in a nucleus, but only with those directly around it.

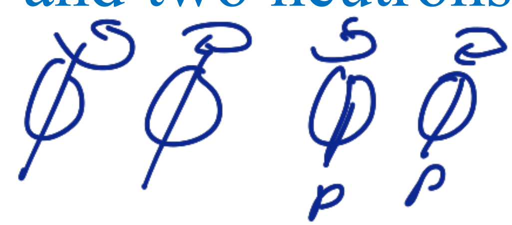
هي القوى النووية التي تكون فقط في ما حوله و الجسيمات البعيدة تتأثر بطريقة مختلفة

- This is different from the electrical force; every proton in the nucleus repels every other proton in the nucleus.

وهذا يختلف عن القوى الكهربائية حيث ينفر البروتون كل الباقين

- The nuclear strong force favors (تفضل) binding of pairs of protons or neutrons with opposite spins and of pairs of pairs (a pair of protons and a pair of neutrons, each having opposite spins). This is why the alpha particle (two protons and two neutrons) is a stable nucleus.

(2p 2n)



Yukawa Strong Forces

- In 1935, the Japanese physicist Hideki Yukawa proposed that a force between protons that is stronger than the electrostatic repulsion can exist between protons.

- Later research showed a similar attraction between two neutrons and between a proton and a neutron.

- This force is called the force and is exerted by nucleons only when they are very close to each other.

Yukawa Strong Forces

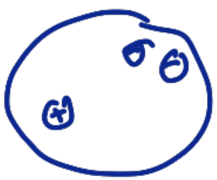
- All the protons and neutrons of a stable nucleus are held together by this strong force.

- Although the strong force is much stronger than electrostatic repulsion, the strong force acts only over very short distances.

- The nucleons are close enough for each nucleon to attract all the others by the strong force.

Yukawa Strong Forces

- In larger nuclei, some nucleons are too far apart to attract each other by the strong force.



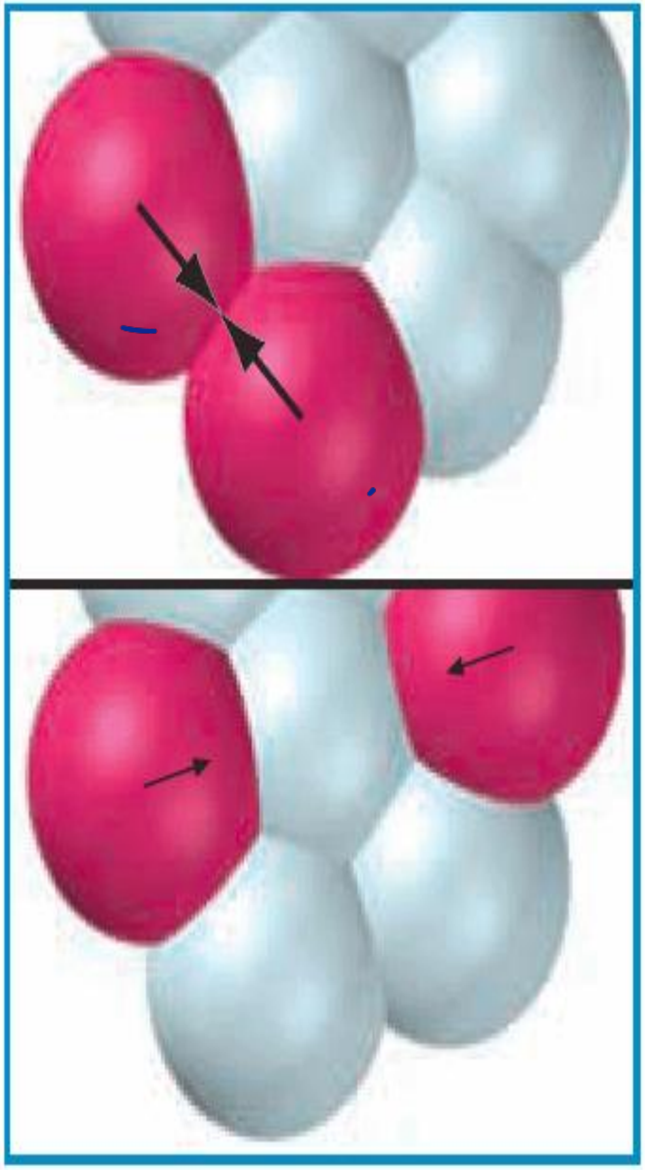
- Although forces due to charges are weaker, they can act over greater distances.

- If the repulsion due to charges is not balanced by the strong force in a nucleus, the nucleus will break apart.

حتى تكون النواة مستقرة يجب ان تتوازن قوة التنافر الكهربائي
مع القوى النووية وإذا كبرت النواة تصبح غير تقل
القوى النووية وتزداد قوى التنافر الكهربائي الى تفكك النواة

Yukawa Strong Forces

- In the nucleus, the nuclear force acts only over a distance of a few nucleon diameters.
- Arrows describe magnitudes of the strong force acting on the protons.



Properties of the Strong Force

خصائص القوى النووية القوية

1. Range: less than nuclear radius, <1.4fm

مداها صغير

2. Attractive: on the distance scale of 1fm,

جذب

تتغلب على قوى كولوم لتحافظ على تجاذب البروتونات

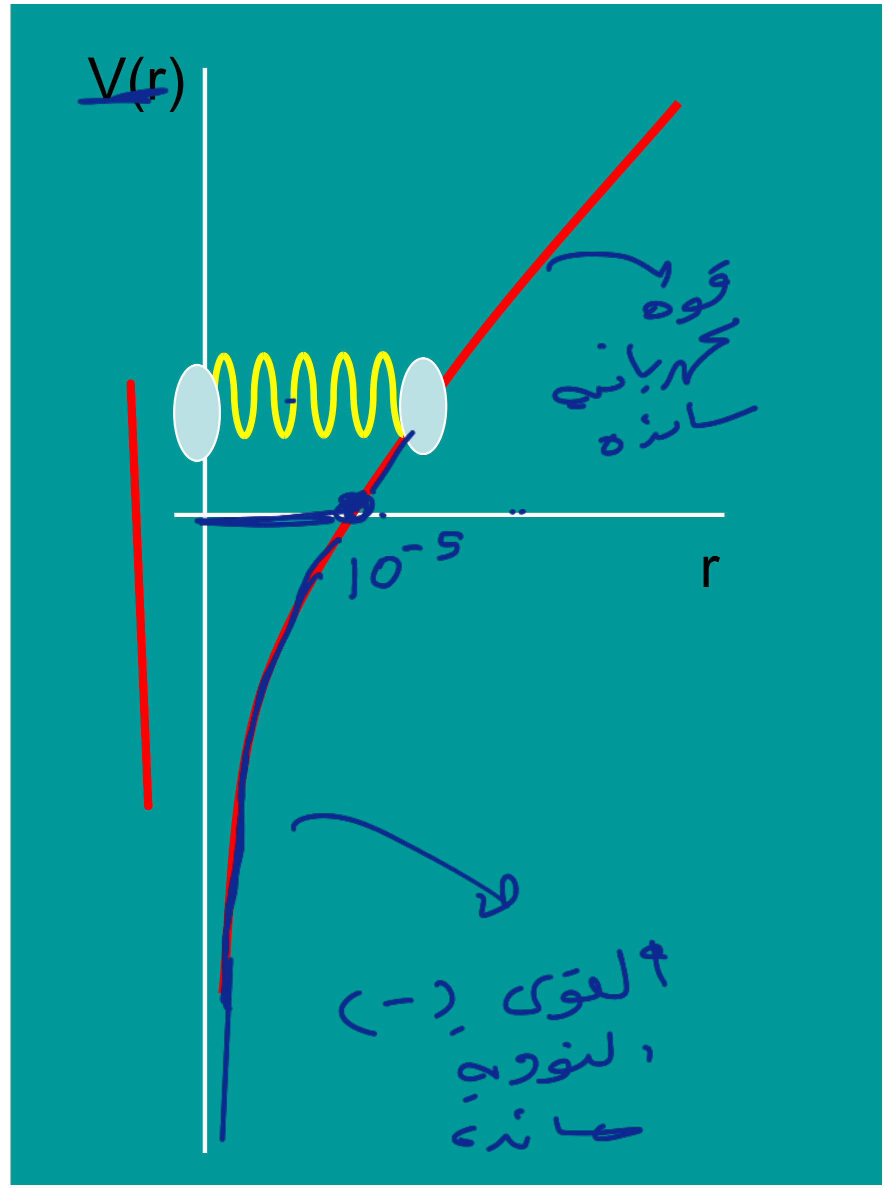
3. overcomes coulombic force to hold charged protons together

4. Repulsive: on the distance scale of <0.5fm

تصبح قوى تنافر عندما تزداد الكثافة

5. Charge Independent: interaction is independent of nucleon electrical charge, ie p-n = p-p= n-n force

لا تعتمد على نوع الشحنة



Charge Independence

الحاصل

A	Nucleus نواه	Total B.E. قوة الجذب النووي (MeV) –	Coulomb Energy قوه كهربائيه (MeV) +	Net Nuclear B.E. (MeV)
3	^{3}H	-8.486	0	-8.486
	^{3}He	-7.723	0.829	-8.552
13	^{13}C	-97.10	7.631	-104.734
	^{13}N	-94.10	10.683	-104.770
23	^{23}Na	-186.54	23.13	-209.67
	^{23}Mg	-181.67	27.75	-209.42
41	^{41}Ca	-350.53	65.91	-416.44
	^{41}Sc	-343.79	72.84	-416.63

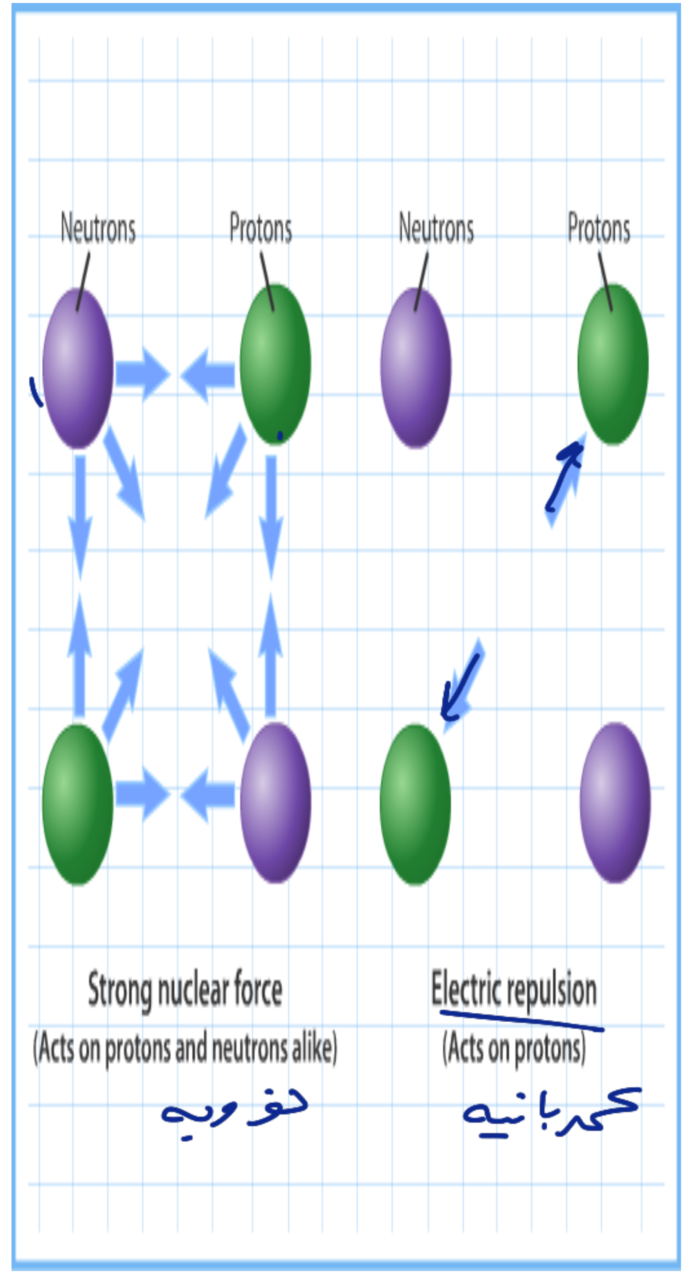
Yukawa Strong Forces

- In the 1960s, scientists discovered that protons and neutrons are made of even smaller particles called quarks.

- Quarks were first identified by observing the products formed in high-energy nuclear collisions.
- When protons and neutrons that are far apart come together and form a nucleus, energy is released.

النواه تحمل طاقة اقل من طاقة مكوناتها منفصله

- As a result, a nucleus is at a lower energy state than the separate nucleons were.



Binding Energy and Nuclear Stability

- A system is always more stable when it reaches a lower energy state.

- One way to describe this reaction is as follows:
- separate nucleons $\rightarrow$ nucleus + energy

- The energy released in this reaction is enormous compared with the energy changes that take place during chemical reactions.

- The energy released when nucleons come together is called nuclear binding energy.

- The source of the energy is found by comparing the total mass of the nucleons with the nucleus they form.

Binding Energy and Nuclear Stability

- The mass of any atom is less than the combined masses of its separated parts.

- This difference in mass is known as the also called mass loss.

- Electrons have masses so small that they can be left out of mass defect calculations.

Binding Energy and Nuclear Stability

معادله $E=mc^2$ تقول كدا ان الكتله ممكن ان تتحول الى طاقه و الطاقه تتحول الى كتله

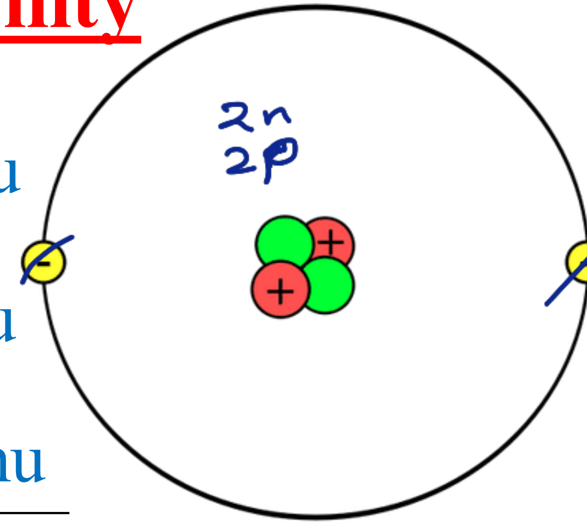
- The equation $E = mc^2$ shows energy can be converted into mass, and mass can be converted into energy.
- A small quantity of mass is converted into an enormous quantity of energy when a nucleus forms.

الكمية الصغيره من الكتله تتحول لتعطي طاقه هائله

$$E=mc^2$$

طاقه	E:	energy (J)
وزن الكتله	m:	mass defect (kg)
سرعه الضوء	c:	speed of light (3.00×10^8 m/s)

Example : Mass Defect and Nuclear Stability



2 protons: (2 x 1.007276 amu) = 2.014552 amu

2 neutrons: (2 x 1.008665 amu) = 2.017330 amu

2 electrons: (2 x 0.0005486 amu) = 0.001097 amu

Total combined mass: 4.032979 amu / = 4.002602 amu

The atomic mass of He atom is 4.002602 amu.
This is 0.030366 amu *less* than the combined mass.

This difference between the mass of an atom and the sum of the masses of its protons, neurons, and electrons is called the mass defect and its denoted by symbol Δ = (m-A).

احسب طاقة الربط النووي لنواة He

« nuclear Binding energy »

2p
2n

① مجموع كتل المكونات

= كتلة البروتون x عدد البروتونات + كتلة النيوترون x عدد النيوترونات

كتلة المكونات $= N m_n + Z m_p$

$= 2 \times 1.008665 + 2 \times 1.007276$

كتلة المكونات 4.031882

② كتلة النواة

$m(He) = 4.002602$

③ فرق الكتلة

$\Delta m = Z m_p + N m_n - m(He)$

$= 4.031882 - 4.002602$

$\Delta m = 0.02928 \text{ amu}$

④ نطبق معادلة اينشتين $E = mc^2$

للتحويل من amu لـ kg نضرب بـ 1.6605×10^{-27}

$B = (m \times 1.6605 \times 10^{-27}) \times c^2$

$B = 0.02928 \times 1.6605 \times 10^{-27} \times (3 \times 10^8)^2$

$B = 4.375 \times 10^{-12} \text{ J}$

What causes the loss in mass?

According to Einstein's equation $E = mc^2$

Convert mass defect to energy units

للتحويل الى kg

$$0.030377 \text{ amu} \left(\frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ amu}} \right) = 5.0441 \times 10^{-29} \text{ kg}$$

The energy equivalent can now be calculated

$$E = m\,c^2$$

$$E = (5.0441 \times 10^{-29} \text{ kg})\,(3.00 \times 10^{8} \text{ m/s})^2$$

$$E = (4.54 \times 10^{-12} \text{ kg m}^2/\text{s}^2) = 4.54 \times 10^{-12} \text{ J}$$

طاقة الربط النووي

Ex(2): Calculate the Binding energy for $^{7}_{3}Li$ per nucleon

1) Calculate mass defect

protons: 1.007276 amu

neutrons: 1.008665 amu

electrons: 0.0005486 amu

mass number (# of protons + neutrons) → 7

atomic number (# of protons) → 3

$^{7}_{3}$Li

Li - 7

2) Convert amu ⟶ kg

$$\_\_\_\_\_\_\_\_ \text{ amu} \left(\frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ amu}} \right) = \_\_\_\_\_\_\_\_ \text{ kg}$$

3) $E = mc^2$ speed of light (c) 3.00×10^{8} m/s

4) Divide binding energy by number of nucleons

$^{7}_{3}Li$

$Z = 3$ عدد البروتونات

$Z = 3$ عدد الالكترونات

$N = 4$ عدد النيوترونات

m_{Li} 7.01818

① فرق الكتلة mass defect

$$\Delta m = Zm_p + Nm_n + Zm_e - m(Li)$$

$$= 3(1.007267) + 4(1.008665) + 3(0.0005486) - 7.01818$$

في بعض الاحيان تهمل لان كتلته قليل

$$\Delta m = 0.03992 \text{ amu}$$

③ نحول الكتلة الى kg ثم نطبق معادلة اينشتين

② او نضرب مباشرة بـ 931.5 لتكون النتيجة بوحدة MeV

$$B = mc^2$$

$$= 0.03992 \times 1.6605 \times 10^{-27} \times (3 \times 10^8)^2$$

$$\boxed{5.965 \times 10^{-12} \text{ J}}$$

Binding energy per nucleon

④ نقسم طاقة الربط على العدد الكتلي

$$= \frac{5.965 \times 10^{-12}}{7} = 8.5 \times 10^{-13} \text{ J}$$

Binding Energy and Nuclear Stability

- A system's stability depends on the amount of energy released as the system is established.

- When 16 g of oxygen nuclei is formed, 1.23×10^{13} J of binding energy is released. This amount of energy is about equal to the energy needed to heat 4.6×10^{6} L of liquid water from $0°C$ to $100°C$ and to boil the water away completely.

- The binding energy per nucleon rises rapidly among the lighter nuclei.

تزداد طاقة الربط بشكل كبير للأنوية الصغيرة

- The greater the binding energy per nucleon is, the more stable the nucleus is.

كلما كانت طاقة الربط النووية أكبر كان الاستقرار أكبر

Binding Energy and Nuclear Stability

- The most stable nuclei are:

1. These isotopes are relatively abundant in the universe in comparison to other heavy metals, and they are the major components of Earth's core.
2. Atoms that have larger mass numbers than $^{56}_{26}Fe$ and $^{58}_{28}Ni$ have nuclei too large to have larger binding energies per nucleon than these iron and nickel isotopes.
3. In these cases, the net attractive force on a proton is reduced because of the increased distance of the neighboring protons.
4. The repulsion between protons results in a decrease in the binding energy per nucleon.
5. Nuclei that have mass numbers greater than 208 and atomic numbers greater than 82 are never stable.

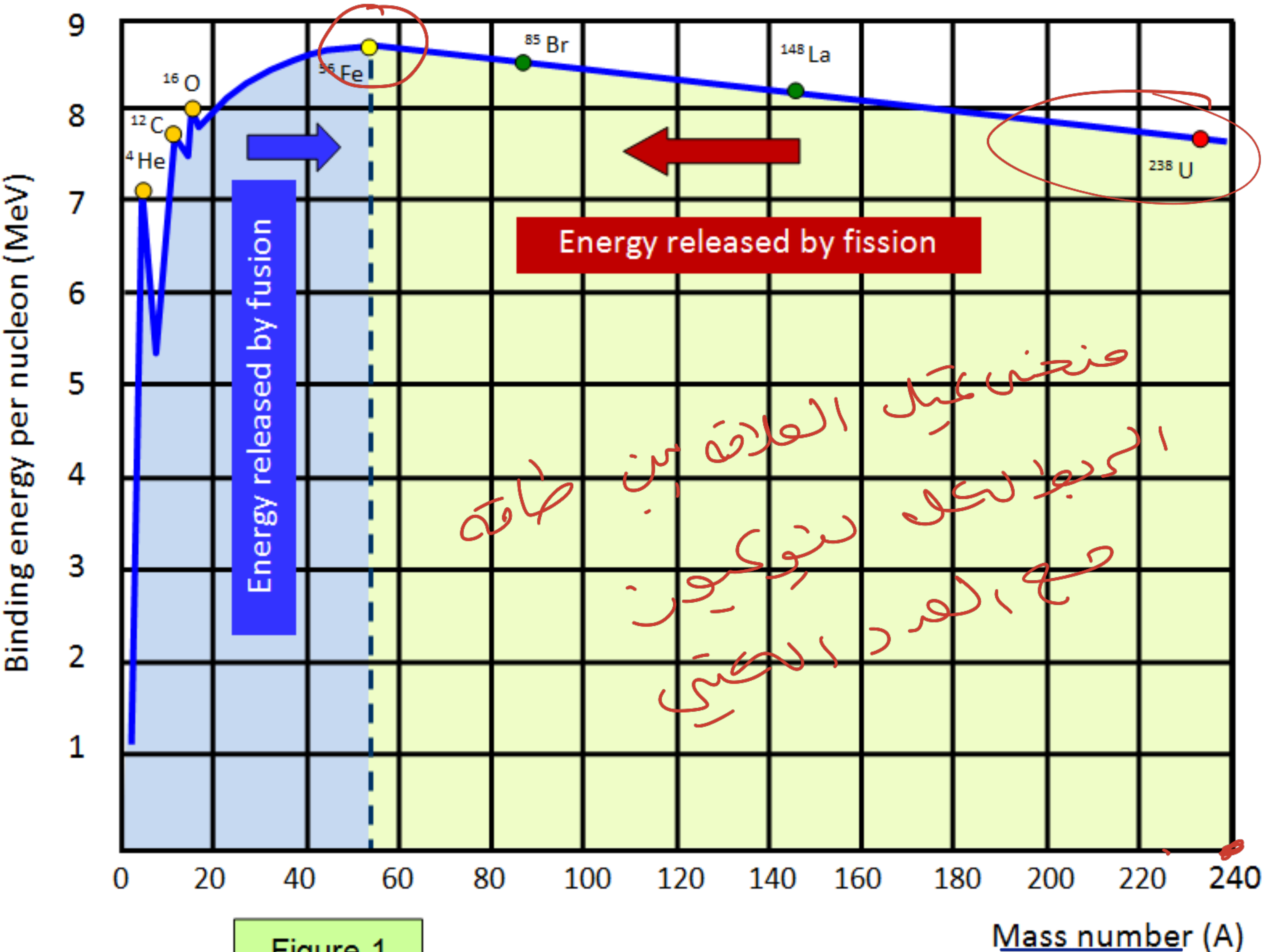

Binding energy per nucleon (MeV)
Mass number (A)
^{4}He
^{12}C
^{16}O
^{56}Fe
^{85}Br
^{148}La
^{238}U
Energy released by fusion
Energy released by fission
منحنى يمثل العلاقة بين طاقة الربط لكل نيوكليون مع العدد الكتلي

Figure 1

- **The binding energy (B) of a nucleus is**

❑ The energy required to separate all of the constituent nucleons from the nucleus so that they are all unbound and free particles

or, طاقة الربط النووية هي الطاقة اللازمة لفض جميع مكونات النواة لتصبح حرة

❑ The difference in mass energy between a nucleus ^{A}X and its constituent Z protons and N neutrons.

$$B = \{\underbrace{Zm_p + Nm_n - m(^{A}_{Z}X)}_{\Delta m}\}C^2$$

عدد كتلة عدد كتلة n كتلة النواة

Where

Z is the atomic number.
N is the number of neutrons.
m is the mass.
C is the speed of light.

B/A is denoted to the binding energy per nucleon

$B/A \Rightarrow$ بعد حساب B نقسمها على العدد الكتلي

Δm   B

EX: calculate the mass defect and the binding energy of Boron nuclei ($^{10}_{5}B$)?

$m_p = 1.007825$

$m_n = 1.008665$

$m(B) = 10.012936$

$\Delta = (m - A)$.

$\Delta = (10.012936 - 10) = 0.012936$ amu.

$\Delta = 0.012936 \times C^2 = 0012936 \times 931.5 = 12.04$ MeV

$$B = \{Zm_p + Nm_n - m(^{A}_{Z}X)\}\, C^2$$

$$B = \{(5 \times 1.007825) + (5 \times 1.008665) - 10.012936\}\, 931.5$$

$$B = 64.75\ MeV$$

B/A = 64.75/10 = 6.5 MeV

$^{10}_{5}B$

mass defecect =

كتلة النواة − العدد الكتلي

$$\Delta = m(B) - 10 =$$

$$10.012936 - 10 = 0.012936 \text{ amu}$$

$$\Delta = 0.012936 \times 1.6605 \times 10^{-27} \times c^2$$

لا تحول

$$= 0.012936 \times \boxed{1.6605 \times 10^{-27}} (3 \times 10^{8})^2$$

$$1.933 \times 10^{-12} \text{ J}$$

لو تحب مباشرة بوحدة MeV لا تحول

الى kg وانما نضرب الكتلة

مباشرة ب 931.5

بوحدة eV

$$\Delta = 0.012936 \times \boxed{931.5}$$

$$12.04 \text{ MeV}$$

Binding energy

تستعمل مرة للفجوة 2 و تستعمل لتحويل الطاقة الى eV

$$B = [Zm_p + Nm_n - m(B)] \boxed{931.5}$$

$$= [5(1.007825) + (5 \times 1.008665) - 10.012936] \boxed{931.5}$$

$$= 64.75 \text{ MeV}$$

$$B/A = \frac{B}{10} = 6.475 \text{ MeV}$$

Separation Energies

طاقة الفصل

- *Neutron separation energy:*

نيوترون

The energy needed to remove a neutron

الطاقة اللازمة لفصل نيوترون

$$S_n = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z}X_{N-1})$$

$$S_n = \{m({}^{A-1}_{Z}X_{N-1}) - m({}^{A}_{Z}X_N) + m_n\}C^2$$

- *Proton separation energy:*

بروتون

The energy needed to remove a proton

الطاقة اللازمة لفصل بروتون

$$S_p = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z-1}X_N)$$

$$S_p = \{m({}^{A-1}_{Z-1}X_N) - m({}^{A}_{Z}X_N) + m_p\}C^2$$

931.5

EX: Calculate the separation energy for last proton and neutron?

❖ $S_n = B(^{A}_{Z}X_N) - B(^{A-1}_{Z}X_{N-1})$

❑ For ^{16}O ,

$$B(^{16}_{8}O_8) = \{Zm_p + Nm_n - m(^{A}_{Z}X)\}\, C^2$$

$$B = \{(8 \times 1.007825) + (8 \times 1.008665) - 15.994915\}\, 931.5$$

$$B = \{(8.0626) + (8.06932) - 15.994915\}\, 931.5$$

$$B = 127.62$$

$$B(^{15}_{8}O_7) = \left\{ \begin{matrix} (8 \times 1.007825) + (7 \times 1.008665) \\ -15.003065 \end{matrix} \right\} 931.5$$

$$B(^{15}_{8}O_7) = \{0.12019 \times 931.5\} = 111.957 \text{ MeV}$$

$$S_n = 127.62 - 111.957 = 15.66\ MeV$$

$^{16}_{8}O_{8}$

احسب طاقة فصل البروتون و النيوترون ا

$m(^{16}_{8}O_{8}) = 15.994915$ $\quad$ $m(^{15}_{7}O_{8}) = 15.0001808$

$$^{16}_{8}O_{8} \rightsquigarrow {}^{15}_{7}O_{8}$$

البروتونة

7P
8N

طريقة ١

طاقة الفصل = طاقة الربط قبل - طاقة الربط بعد

$$S = B(^{16}_{8}O_{8}) - B(^{15}_{7}O_{8})$$

$$B(^{16}_{8}O_{8}) = [m_p Z + N m_n - m(^{16}_{8}O_{8})] \times 931.5$$

$$= [8 \times 1.007825 + 8 \times 1.008665 - 15.994915]\, 931.5$$

$$= 127.62 \text{ Mev}$$

$$B(^{15}_{7}O_{8}) = [m_p Z + N m_n - m(^{15}_{7}O_{8})]\, 931.5$$

$$= [7 \times 1.007825 + 8 \times 1.008665 - 15.000108]\, 931.5$$

$$= 115.49 \text{ Mev}$$

$$S = B(^{16}_{8}O_{8}) - B(^{15}_{7}O_{8})$$

$$= 127.62 - 115.94 = 12 \text{ Mev}$$

طريقة ٢

$$S = [m(^{16}_{7}O_{8}) - m(^{15}_{7}O_{8}) + m_p]\, 931.5$$

$$= [15.000108 - 15.994915 + 1.007825]\, 931.5$$

$$= 12 \text{ Mev}$$

$$S_p = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z-1}X_N)$$

طريقة (1)

❑ For ^{16}O ,

$$B({}^{16}_{8}O_8) = \{Zm_p + Nm_n - m({}^{A}_{Z}X)\}\, C^2$$

$$B = \{(8 \times 1.007825) + (8 \times 1.008665) - 15.994915\}\, 931.5$$

$$B = \{(8.0626) + (8.06932) - 15.994915\}\, 931.5$$

$$B = 127.62$$

$$B({}^{15}_{7}N_8) = \left\{ \begin{matrix} (7 \times 1.007825) + (8 \times 1.008665) \\ -15.000108 \end{matrix} \right\} 931.5$$

$B({}^{15}_{7}O_{8}) = 115.49$ MeV

$$S_p = 127.62 - 115.49 = 12\ MeV$$

Using the mass of the elements

$$S_n = \{m({}^{A-1}_{Z}X_{N-1}) - m({}^{A}_{Z}X_N) + m_n\}C^2$$
$$S_n = \{m({}^{15}_{8}O_7) - m({}^{16}_{8}O_8) + m_n\}C^2$$
$$S_n = \{15.003065 - 15.994915 + 1.008665\}931.5$$
$$S_n = 15.66$$

$$S_p = \{m({}^{A-1}_{Z-1}X_N) - m({}^{A}_{Z}X_N) + m_p\}C^2$$
$$S_p = \{m({}^{15}_{7}N_8) - m({}^{16}_{8}O_8) + m_p\}C^2$$
$$S_p = \{15.000108 - 15.994915 + 1.007825\}931.5$$
$$S_p = 12\ MeV$$

$^{16}_{8}O_8$ احسب طاقة فصل نيوترون من نواة

$[^{15}_{8}O_7] = 15.003065$

$$^{16}_{8}O_8 \longrightarrow {}^{15}_{8}O_7$$

طريقة 1

$$S = B[^{16}_{8}O_8] - B[^{15}_{8}O_7]$$

تم حسابها

$$B[^{16}_{8}O_8] = 127.62$$

$$B[^{15}_{8}O_7] = [8 \times 1.00782 + 7 \times 1.008665 - 15.003065] \times 931.5$$

$$= 111.957$$

$$S = B[O_{8\,8}] - B[O_{8\,7}]$$

$$= 127.62 - 111.957 = 15.66 \text{ Mev}$$

طريقة 2

$$S = [m[^{15}_{8}O_7] - m[^{16}_{8}O_8] - m_n]\, 931.5$$

$$= [15.003065 - 15.994915 - 1.008665]\, 931.5$$

15.66 Mev

نموذج قطرة السائل — حرابية تجريبية لحساب طاقة الربط

The Liquid Drop Model and The Semi-Empirical Binding Energy Formula

$a_v = 14\ MeV$

surface Term (السطح)

$a_c = 0.146\ MeV$

pairing Term (تزاوج)

$$B.E = a_v A - 4a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_s A^{\frac{2}{3}} - a_\tau \frac{(A-2Z)^2}{A} \mp E_\delta$$

Volume Term (الحجم)

Coulomb Term (قوة كولوم)

Asymmetry Term (التناظر)

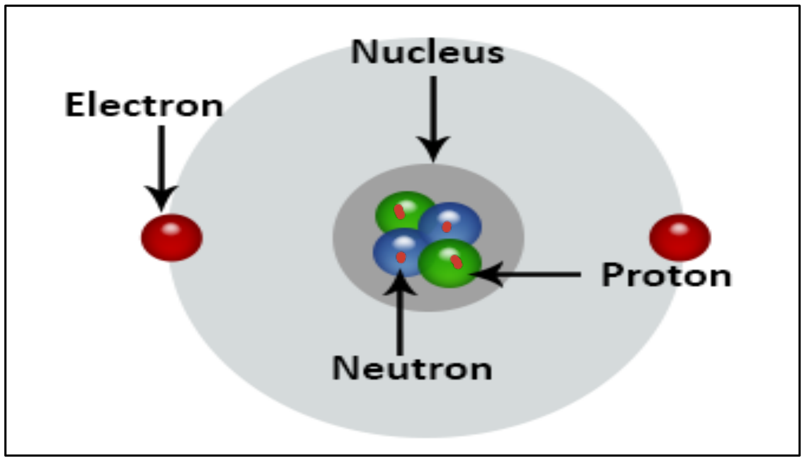
$a_s = 13.1\ MeV$

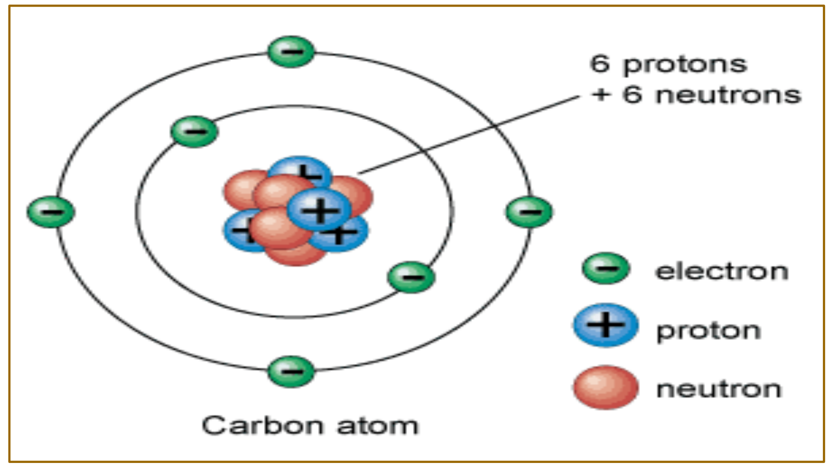
$a_\tau = 19.4\ MeV$

نحصل عليها من ١ قانون تجريبي لحساب طاقة الربط

volume term:

➢ The main contribution to the binding energy of the nucleus comes from a term proportional to the mass number A; since the volume of the nucleus is also proportional to A, this term may be regarded as a volume energy.





$$E_v \propto A$$

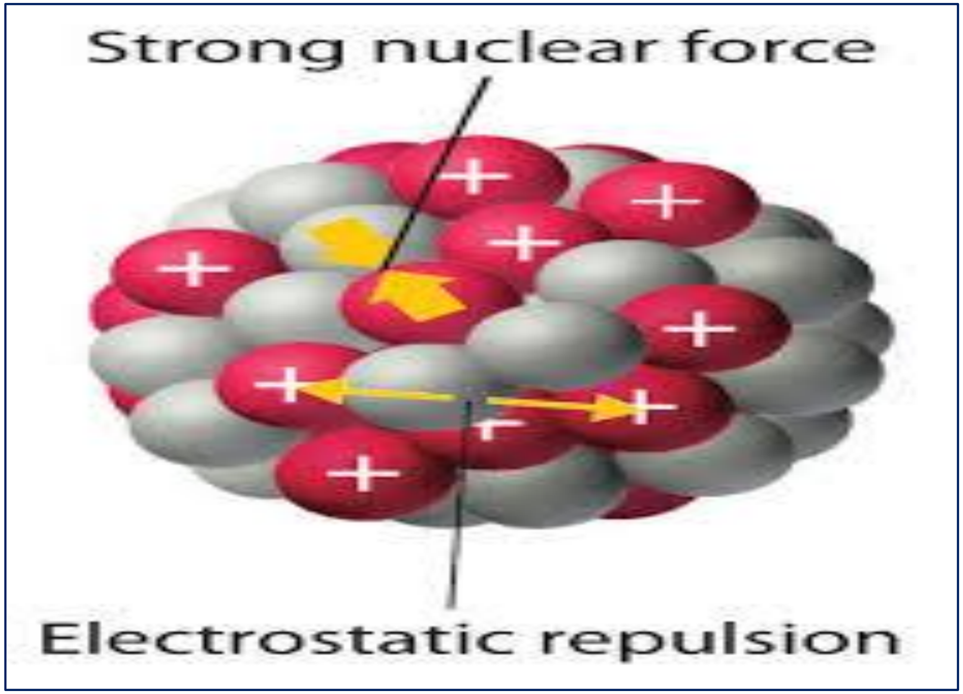
$$E_v = a_v A \quad (1)$$

Where;

E_v is a volume energy, A is the mass number and a_v is a volume constant (14 MeV).

Coulomb term:

- The coulomb energy between the protons tends to lower binding energy and its effect appears as a term with minus sign. The total coulomb energy of a nucleus of charge Z , and its effect on the binding energy is represented by the term



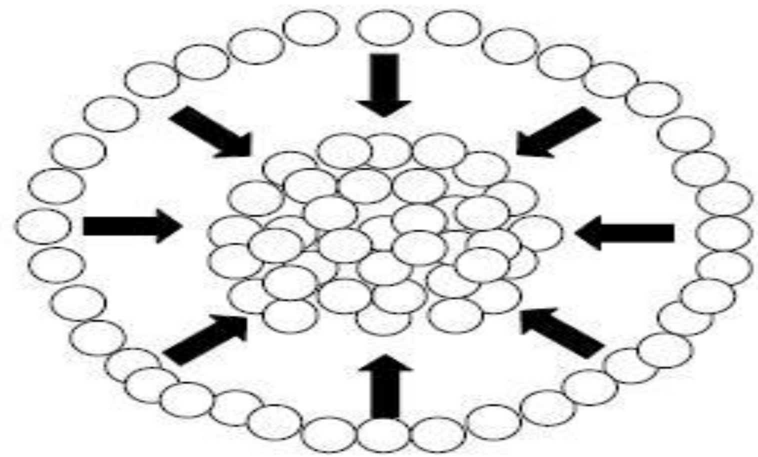
$$E_c = -4\, a_c \frac{Z(Z-1)}{A^{1/3}}$$

Where;
E_c is a coulomb energy, A is the mass number, Z is the charge and a_c is a coulomb energy constant (0.146 MeV).

حد المساحة:

Surface term:

➢ The binding energy is also reduced because the nucleus has a surface, particles at the surface interact, on the average, only with half as many other particles as do particles in the interior of the nucleus. In the attractive term (1), it was assumed that every nucleon has the same access to other nucleons and it necessary, therefore, to subtract a term proportional to the surface area of the nucleus. This surface energy is represented by



$$E_s = SK = 4\pi R^2 K$$

$$E_s = 4\pi (r_o A^{\frac{1}{3}})^2 K = 4\pi r_o K A^{\frac{2}{3}}$$

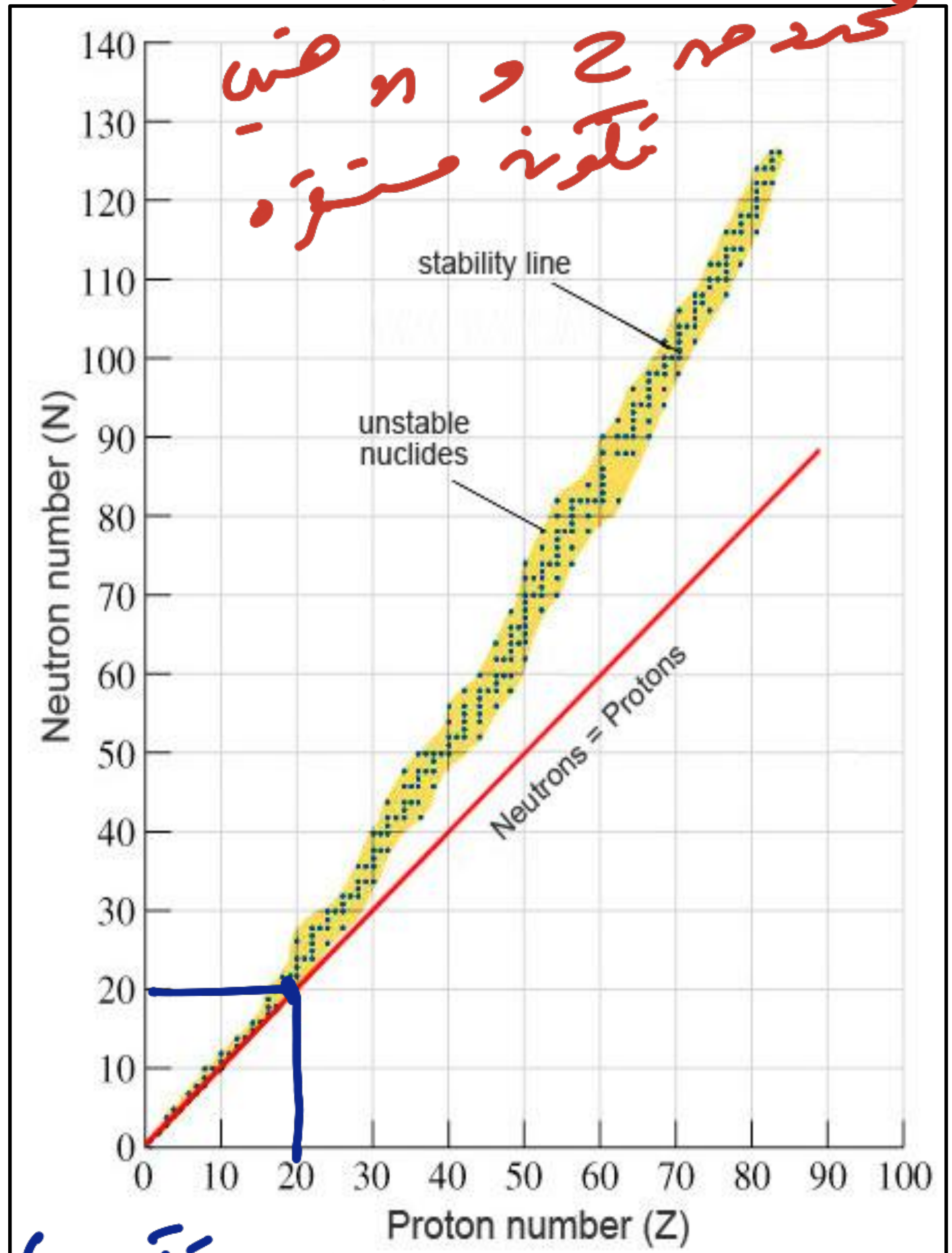
$$E_s = -\, a_s A^{\frac{2}{3}}$$

Where;
E_c is a surface energy, A is the mass number, K is surface tension factor and a_s is a surface energy constant (13.1 MeV).

Symmetry term:

حد التناظر

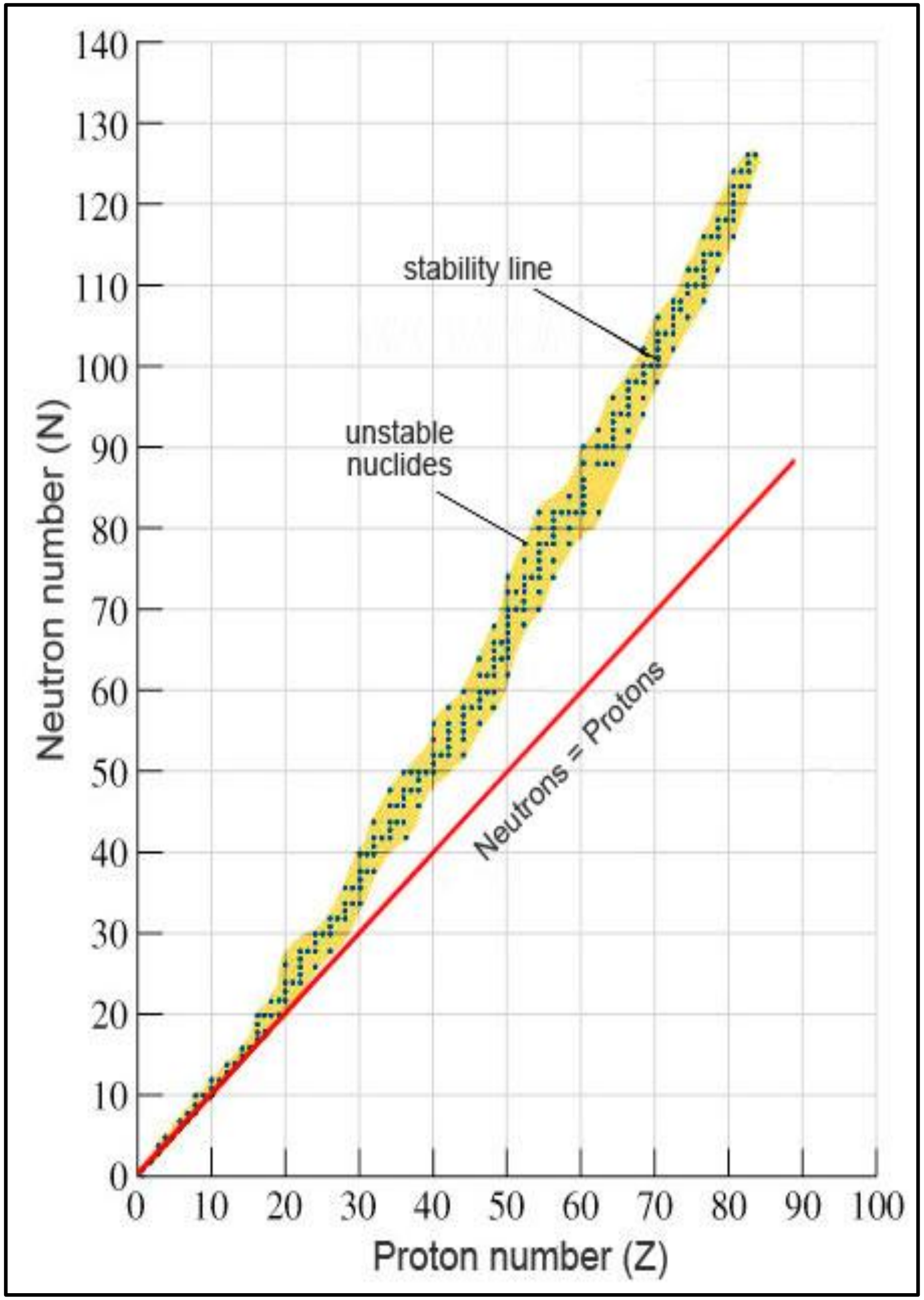
- The binding energy formula needs another term to represent the so called symmetry effect. For a given value of A, there is a particular value of Z which corresponds to the most stable nuclide. For light nuclides, where the coulomb effect is small, this value A/2, as is seen from the fact that the numbers of protons and neutrons are equal in the most abundant light nuclides.



كل نواة لديها عدد محدد من Z و N حتى تكون مستقرة

في الأنوية الصغيرة تكون عدد البروتونات بساوي عدد نيوترونات

- In the absence of the Coulomb effect, a departure from the condition $Z = A/2$ would tend to instability and a smaller value of the binding energy. A term proportional to some power of the neutron excess $(A-2Z)$ would represent the magnitude of this effect.
- The second power is chosen because the term then vanishes for $Z = A/2$, as does its derivative with respect to Z.



- The latter condition corresponds to the maximum value of the binding energy in the absence of the absence of the Coulomb energy.

- A detailed study of the symmetry effect shows that it is also inversely proportional to A, with the result that the symmetry energy can be written

$$E_{\tau} = -\, a_{\tau} \frac{(A - 2Z)^2}{A}$$

Where;
E_{τ} is a symmetry energy and a_{τ} is a symmetry energy constant (19.4 MeV).

Pairing term:

A	Z p	n N	E_δ
Even	Even	Even	+
Odd	Even	Odd	0
Odd	Odd	Even	0
Even	Odd	Odd	−

زيادة طاقة الربط ، نقصان طاقة الربط   إيجابي ، سالب   زوجي ، فردي   زوجي ، فردي

Where,

$$E_\delta = \begin{cases} \dfrac{135}{A}, \text{even } A, \text{even } Z. \\ 0, \text{odd } A, \text{even } z; \text{odd } A, \text{odd } z. \\ \dfrac{-135}{A}, \text{even } A, \text{odd } Z \end{cases}$$

The Mass Semi-Empirical Formula

كتلة النواة (تجريبية) = كتلة p × عددها + كتلة n × عددها − طاقة الربط / C^2

$$M = \left\{ Z m_p + N m_n - \frac{B.E}{C^2} \right\}$$

$$M = Z m_p + N m_n - a_v \frac{A}{C^2} + 4 a_c \frac{Z(Z-1)}{A^{\frac{1}{3}} C^2} + a_s \frac{A^{\frac{2}{3}}}{C^2} + a_\tau \frac{(A-2Z)^2}{A C^2} \mp \frac{E_\delta}{C^2}$$

Ex: For the $^{21}_{10}Ne$ nuclei, use the semi empirical formula to compute the total binding energy and the mass of neon?

$$\mathrm{B.E} = \left\{ \begin{array}{l} (14 \times 21) - 4 \times 0.146 \dfrac{10(10-1)}{21^{\frac{1}{3}}} - \\ 13.1 \times 21^{\frac{2}{3}} - 19.4 \dfrac{(21 - 2 \times 10)^2}{21} \mp 0 \end{array} \right\}$$

$$\mathrm{B.E} = \{294 - 971 - 19.051 - 0.92\}$$

$$\mathrm{B.E} = 174.315\ MeV$$

$$\frac{\mathrm{B.E}}{\mathrm{A}} = \frac{174.315}{21} = 8.3\ MeV$$

$$^{21}_{10}Ne$$

$A = 21$
$Z = 10$
$N = 11$

$$B \cdot E = a_v A - 4a_c \frac{Z(Z-1)}{A^{1/3}} - a_s A^{2/3} - a_t \frac{(A-2Z)^2}{A} \pm E_s$$

$$B \cdot E = 14(21) - 4(0.146) \frac{10(10-1)}{21^{1/3}} - 13.1(21)^{2/3}$$

$$- \frac{19.4(21-20)}{21} \mp 0$$

$$BE = 294 - 9.71 - 19.051 - 0.92$$

$$= 174.315 \text{ MeV}$$

$$B \cdot E / A = \frac{174.315}{21} = 8.3 \text{ MeV}$$

$$M = Zm_p + Nm_n - \frac{BE}{c^2}$$

$$= 10(1.007276) + 11(1.008665) - \frac{174.315}{931.5}$$

$$20.98694 \text{ amu}$$

$$M = \left\{ Zm_p + Nm_n - \frac{B.E}{C^2} \right\}$$

$$M = \left\{ 10 \times 1.007276 + 11 \times 1.008665 - \frac{174.315}{931.5} \right\}$$

$$M = \{ 10.07276 + 11.095315 - 0.187 \}$$

$$M \approx 20.98094\ amu$$

- **H.W**

1. **Calculate the mass defect and the binding energy of ^{12}C , ^{16}O, ^{24}Mg and ^{238}U?**
2. **Calculate the binding energy per nucleon of the previous elements?**

$^{12}_{6}C$ $Z=6$ $N=6$

In $\boxed{_6C^{12}}$ atom: $p=6$, $n=6$, $m_p = 1.00814\ u$, $M_n = 1.00898\ u$

$M_A = 12\ u$

Now $\Delta m = [6 \times 1.00814 + 6 \times 1.00898 - 12]\ u$

$\Delta m = [12.10272 - 12]\ u$

$\Delta m = 0.10272\ u$

Binding Energy $= (\Delta m \times 931.5)\ MeV = (0.10272 \times 931.5)\ MeV$

$B.E = 95.68\ MeV$

Binding Energy per nucleon $= \frac{B.E}{12} = \frac{95.68\ MeV}{12} = 7.97\ MeV$

In $\boxed{_8O^{16}}$ atom: $p=8$, $n=8$ & $M_A = 15.99492\ u$

Now $\Delta m = [8 \times 1.00814 + 6 \times 1.00898 - 15.99492]\ u$

$\Delta m = [16.13696 - 15.99492]\ u$

$\Delta m = 0.14204\ u$

$B.E. = 0.14204 \times 931.5\ MeV = 132.31\ MeV$

Binding energy per nucleon $= \frac{B.E}{16} = \frac{132.31}{16} = 8.27\ MeV$

In $\boxed{_8O^{16}}$ atom: $p=8$, $n=8$ & $M_A = 16\ u$

Now, $\Delta m = [8 \times 1.00814 + 8 \times 1.00898 - 16]\ u$

$\Delta m = [16.13696 - 16]\ u$

$\Delta m = 0.13696\ u$

$B.E = 0.13696 \times 931.5\ MeV = 127.58\ MeV$

Binding energy per nucleon $= \frac{B.E}{16} = \frac{127.58}{16} = 7.97\ MeV$

In $\boxed{_{12}Mg^{24}}$ atom: $p=12$, $n=12$ & $M_A = 24\ u$

Now, $\Delta m = [12 \times 1.00814 + 12 \times 1.00898 - 24]\ u$

$\Delta m = [24.20544 - 24]\ u$

$\Delta m = 0.20544\ u$

$B.E = 0.20544 \times 931.5\ MeV = 191.37\ MeV$

Binding energy per nucleon $= \frac{B.E}{12} = 15.95\ MeV$