جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Introduction to General Physics for Engineers

Physics Department

College of Science

Outline of Course contents

1. Ch 21 Coulomb's Law
2. Ch 22 Electric Fields
3. Ch 25 Capacitance
4. Ch 26 Current and Resistance
5. Ch 27 Circuits
6. Ch 28 The Magnetic Field B
7. Ch 29 Magnetic Field due to currents

Timetable

	Subject	Contact hours	No. of week
1	Ch 21	3	1
2	Ch 22	4.5	1.5
3	Ch 23	6	2
4	Ch 24	3	1
5	Ch 25	4.5	1.5
6	Ch 26	3	1
7	Ch 27	3	1
8	Ch 28	3	1
9	Ch 29	3	1

Course Description

This course will provide a conceptual background in physics sufficient to enable students to take courses that are more advanced in related fields. It covers the following: Electric charge, electric fields, superposition, simple circuits, Ohm's Law, and capacitors, magnetic fields and magnetic field due to currents.

Learning Outcomes

At the end of this course, the student should acquire:

1. Provide and define the fundamental properties of the electric charge, solve technical problems associated with the electrostatic force (Coulomb force).
2. Identify that at every point in the space surrounding a charged particle, the particle sets up an electric field, which is a vector quantity and thus has both magnitude and direction.
3. Identify how an electric field can be used to explain how a charged particle can exert an electrostatic force on a second charged particle even though there is no contact between the particles.
4. Explain how a small positive test charge is used (in principle) to measure the electric field at any given point.
5. Calculate the electric field due to distribution of charges.
6. Define electric capacitance and solve technical problems associated with capacitors of various symmetries, capacitors in series and parallel combination, the microscopic effect of dielectric materials on capacitance and stored energy.
7. Define electric current, current density, and solve technical problems involving DC networks of resistors, batteries, and capacitors, Ohm's Law, Kirchhoff's laws, and RC charging and decay circuits.
8. Calculate the potential difference between any two points in a circuit.
9. Calculate the magnetic field due to passing of an electric current.

Marks Distribution

	Test	Percentage	Date
1	Homework	10%	Continued all over the course
2	Med term Exam.	20%	6^{th} week (for male students)
3	Lab.	20 %	10% for Lab reports. 10% final Lab exam.
4	Final exam.	50%	At the end of the term
5	Total	100 %	

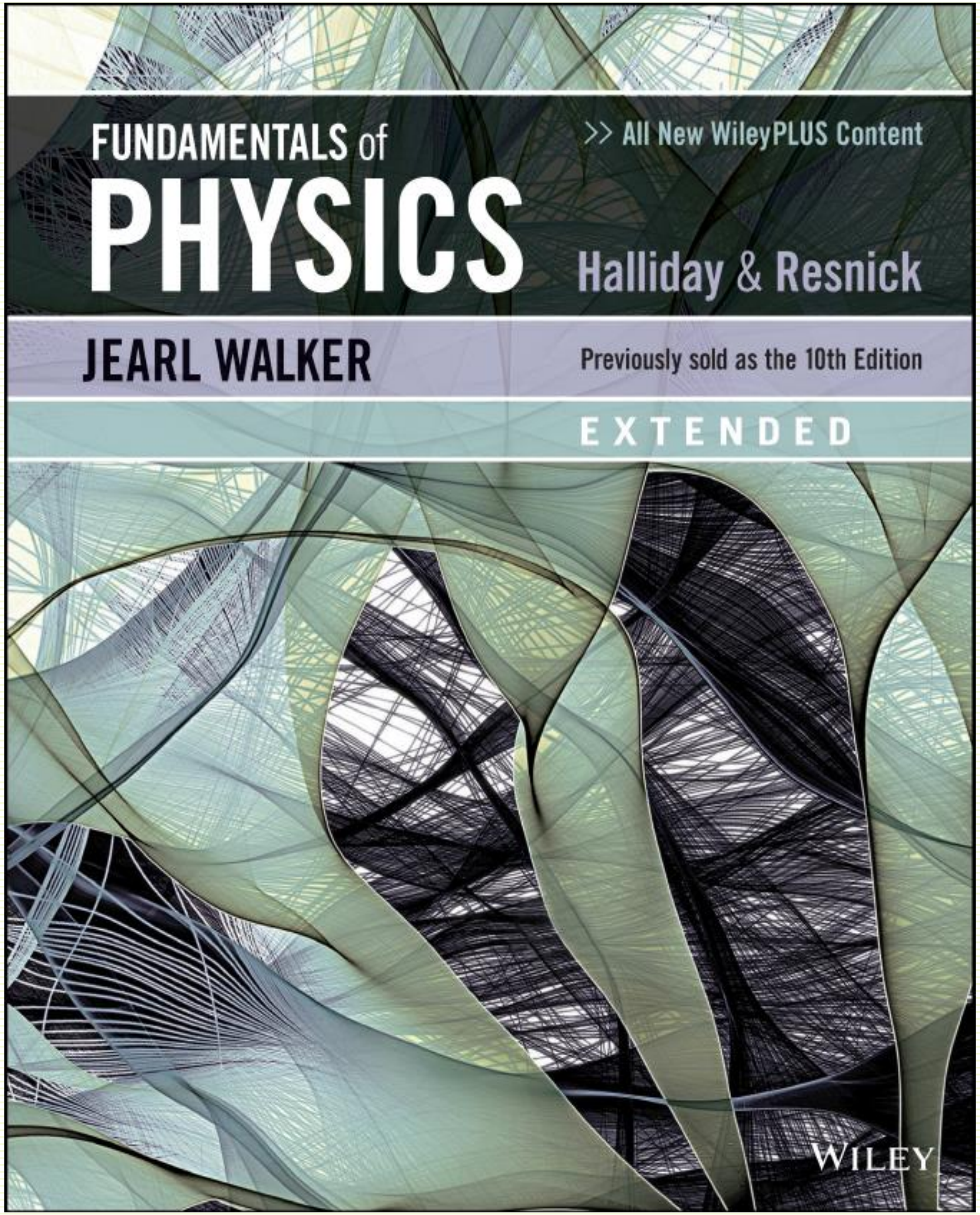
General Physics for Engineering
Code: 4031112

Textbook:

Fundamentals of Physics,

By

Halliday & Resnick, and Jearl Walker



جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch 21

Coulomb's Law

قانون كولوم

Physics Department

College of Science

Contents

Introduction

- The physics of electricity and magnetism was first studied by the early Greek philosophers, who discovered that if a pieces of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an electric force. القوه الكهربائيه

- The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between the magnet and iron is due to a magnetic force.

القوه المغناطيسيه

تمت دراسة فيزياء الكهرباء والمغناطيسية لأول مرة من قبل الفلاسفة اليونانيين القدماء، الذين اكتشفوا أنه إذا تم فرك قطعة من الكهرمان ثم تقريبها من قطع القش، فإن القش سوف يقفز نحو الكهرمان. نحن نعلم الآن أن الجذب بين الكهرمان والقش يعود إلى قوة كهربائية.

□كما اكتشف الفلاسفة اليونانيون أن هناك نوعًا معينًا من الحجر (مغناطيس طبيعي) إذا تم تقريبُه من قطع الحديد، فإن الحديد سوف يقفز نحو الحجر. نحن نعلم الآن أن الجذب بين المغناطيس والحديد يعود إلى قوة مغناطيسية.

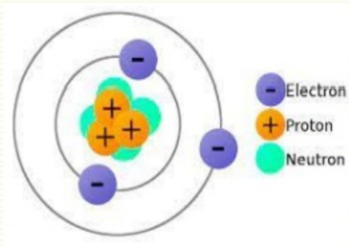
Electric Charge

الشحنة الكهربائية

- Electric charge: (symbolized q) is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. That is, the strength of a particle's electrical interaction with objects around it depends on its electric charge.

الشحنة مكماة

- Charge is quantized, that is it comes in integer multiples of individual small units called the elementary charge (electronic charge), (symbolized e, about 1.602×10^{-19} C). In fact, the charge of an electron is $-e$, while the charge of a proton is $+e$.



- Depending on the number and type of the elementary charge, there are two types of electric charge: positive charge ($+q$) and negative charge ($-q$).

متعادل كهربائي

- An object with equal amounts of the two kinds of charge is electrically neutral.

$4e^+$
$4p^+$

تجاذب

- Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other. يتنافر
- The SI unit of electric charge is Coulomb (C).

الشحنة الكهربائية: يرمز لها بالرمز (q) هي خاصية فيزيائية للمادة تتسبب في تجربتها لقوة عند وضعها في مجال كهرومغناطيسي. أي أن قوة التفاعل الكهربائي للجسيم مع الأجسام المحيطة به تعتمد على شحنته الكهربائية.

□الشحنة مكممة، أي أنها تأتي في مضاعفات صحيحة من وحدات صغيرة فردية تسمى الشحنة الأولية (الشحنة الإلكترونية)، يرمز لها بالرمز e ، وتقدر بحوالي 1.602×10^{-19} كولوم.

في الواقع، شحنة الإلكترون هي$-e$ ، بينما شحنة البروتون هي$+e$.

□اعتمادًا على عدد ونوع الشحنة الأولية، هناك نوعان من الشحنات الكهربائية: شحنة موجبة ($+q$) وشحنة سالبة.($-q$)

□الجسم الذي يحتوي على كميات متساوية من النوعين من الشحنات يكون متعادل كهربائيًا.

□الجسيمات ذات نفس نوع الشحنة الكهربائية تتنافر مع بعضها البعض، بينما الجسيمات ذات الشحنات المعاكسة تتجاذب.

□وحدة الشحنة الكهربائية في النظام الدولي للوحدات هي الكولوم.(C)

e $2e$ $100e^-$

$e^- = 1.6\times10^{-19}$ C

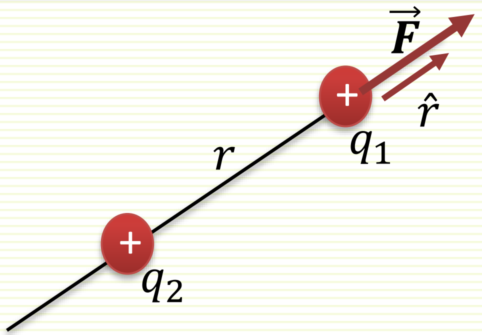
$p^+ = +1.6\times10^{-19}$ C

قانون كولوم

Coulomb’s Law

القوه الكهروسكونيه

- Coulomb’s law describes the electrostatic force (or electric force) between two charged particles. If the particles have charged q_1 and q_2, are separated by distance r, and are at rest (or moving only slowly) relative to each other, then the force acting on particle 1 in terms of a unit vector $\hat{r}$ that points along a radial axis extending through the two particles, radially away from particle 2 is given by



K

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\,\hat{r}$$

نفاذية الفراغ

- In which ε_o is the permittivity of free space $= 8.85 \times 10^{-12}\ C^2/N.m^2$.

ثابت الكهروسكونيه

- And k is electrostatic constant (or Coulomb constant)
- $k = 1/4\pi\varepsilon_o = 8.99 \times 10^9\ N.m^2/C^2$

Charles-Augustin de Coulomb
(14 June 1736 – 23 August 1806)
a French officer, engineer, and physicist.

قانون كولوم :- يدرس القوه الكهربائيه (F) المتبادلة بين شحنتين نقطيتين

كلما كانت مقدار الشحنتين $q_1 q_2$ اكبر كانت القوه اكبر (طردي)

كلما كانت r^2 المسافه اقل كانت القوه اكبر (عكسي)

تعتمد القوه على الوسط الموجود فيه الشحنات

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$

كولوم² ، نيوتن ، m^2

$$K = \frac{1}{4\pi \varepsilon_0}$$

K ثابت كولوم (ثابت الكهرباء السكونيه)

ε_0 النفاذيه الكهربائيه للوسط

في حالة الهواء $\Rightarrow \varepsilon_0 = 8.85 \times 10^{-12} \; C^2/Nm^2$

$$K = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 8.99 \times 10^{9} \; N m^2/C^2$$

لا نعوض الاشارة السالبه في قانون كولوم

$$\boxed{\vec{F} = 8.99 \times 10^{9} \frac{q_1 q_2}{r^2} \hat{r}}$$

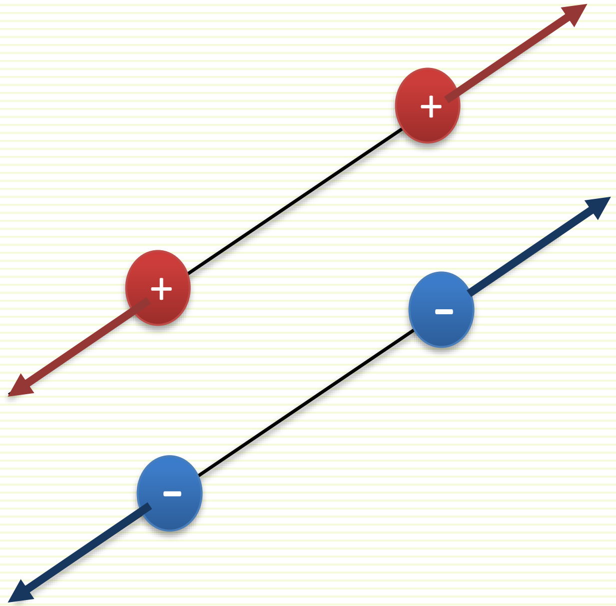
Coulomb's Law

- The direction of the force vectors depends on the signs of the charges. اتجاه القوة تعتمد على نوع الشحنات

نفس الاشارة قوة تنافر

- Repulsion force: If the particles have the same sign of charge, they repel each other (this means that the force vector on each is directly away from the other particle).

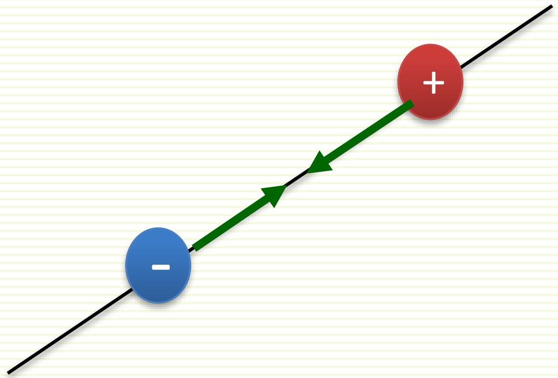
الاتجاهات خارج الخط الوصل بين q_1 و q_2



الاشارات متعاكسة قوة التجاذب

- Attractive force: If the particles have opposite signs of charge, they attract each other (this means that the force vector on each is directly toward the other particle)

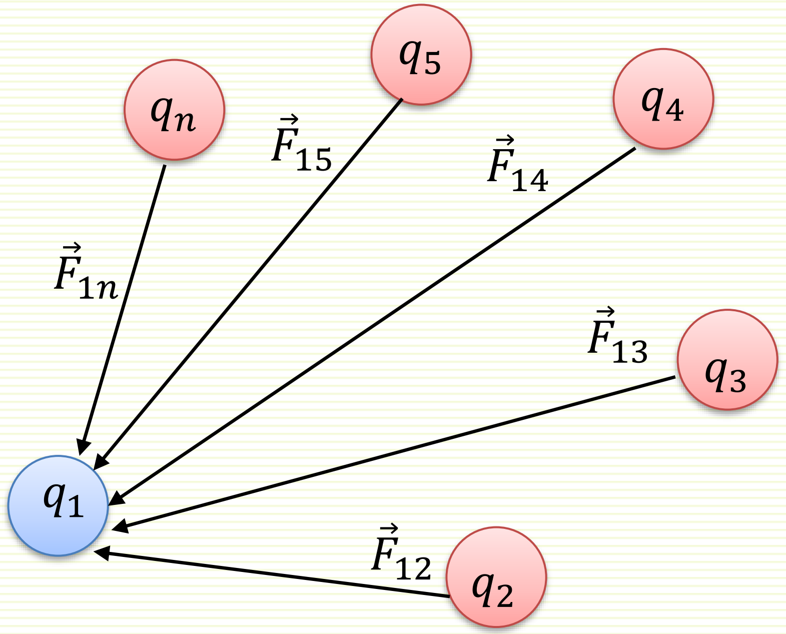
الاتجاهات نحو بعضها



Coulomb’s Law

- Multiple forces: The electrostatic force obeys the principle of superposition. مبدأ التراكب

- Suppose we have n charged particles near a chosen particle called particle 1, then the net force on the particle 1 is given by the vector sum



$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}$$

- Where $\vec{F}_{12}$ is the force on particle 1 due to the presence of particle 2, and so on.

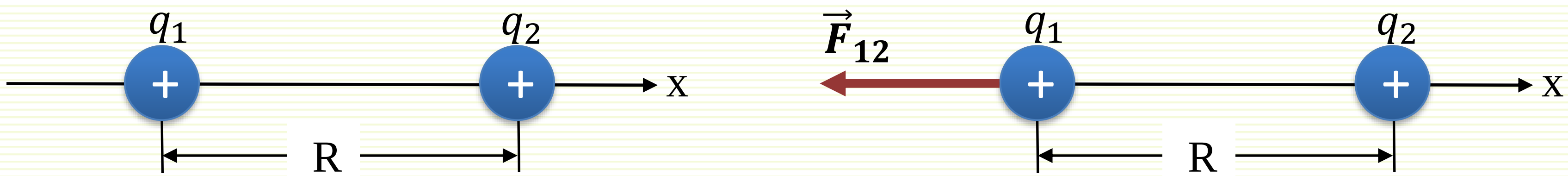
لحساب محصلة القوى المؤثرة في
شحنة ∴ نحسب كل قوه لوحدها باستخدام
قانون كولوم ثم نحسب محصلة القوى من خلال جمع المتجهات

Sample Problem 21.01

Finding the net force due to two other particles

- (a) Two positively charged particles fixed in place on an x −axis. The charges are $q_1 = 1.60 \times 10^{-19} C$ and $q_2 = 3.2 \times 10^{-19} C$, and the particle separation is $R = 0.0200\, m$. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

مقدار الاتجاه



$r = 0.02\ m$

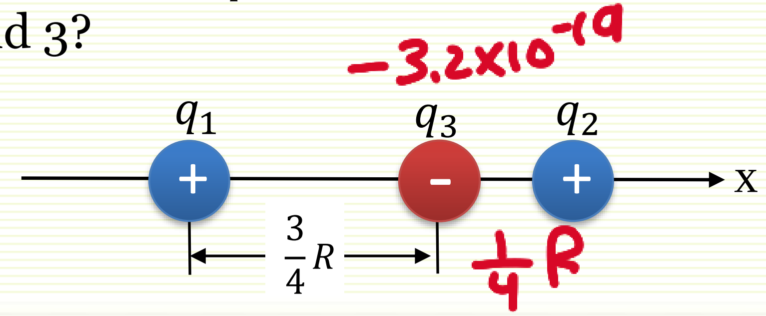
$$F = K\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^{9} \times 1.6 \times 10^{-19} \times 3.2 \times 10^{-19}}{(0.02)^2} = 1.15 \times 10^{-24}\ N$$

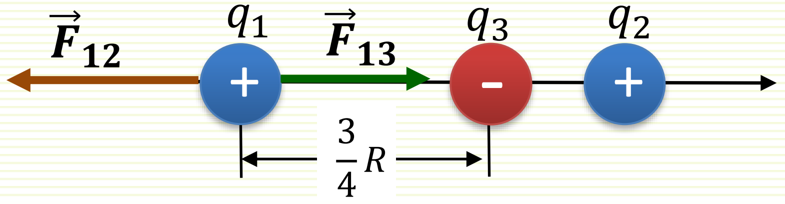
$$F_{12} = -1.15 \times 10^{-24}\, \hat{i}\ \ N$$

Sample Problem 21.01

Finding the net force due to two other particles

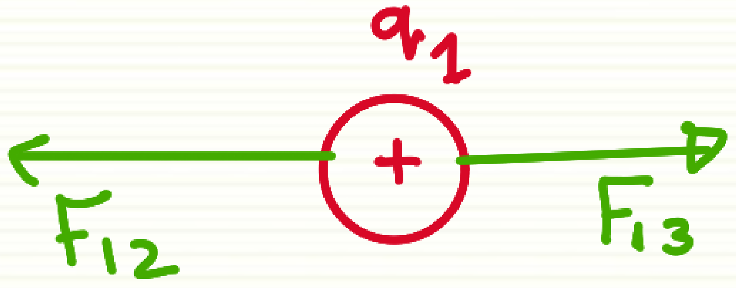
- (b) The particle 3 now lies on the x –axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19} C$ and is at a distance $\frac{3}{4}R$ from particle 1. what is the net electrostatic force $\vec{F}_{1,net}$ on particle 1 due to particles 2 and 3?





مامقدار القوه الكهربائيه الحاصله المؤثره على الشحنه q_1 (F_1 net)

والناتجه من الشحنه الثانيه والثالثه



$$\vec{F}_{12} = \frac{k\, q_1 q_2}{r^2} = \frac{8.99\times10^{9} \times 1.6\times10^{-19} \times 3.2\times10^{-19}}{(0.02)^2}$$

$$F_{12} = -1.15\times10^{-24}\,\hat{i} \quad N$$

$$F_{13} = \frac{k\, q_1 q_3}{r^2} = \frac{8.99\times10^{9} \times 1.6\times10^{-19} \times 3.2\times10^{-19}}{(\frac{3}{4}\times 0.02)^2}$$

$$F_{13} = +2.05\times10^{-24}\,\hat{i} \quad N$$

$$\bar{F}_{net} = F_{13} + F_{12}$$

$$= (-1.15\times10^{-24} + 2.05\times10^{-24})\,\hat{i}$$

$$= 0.9\times10^{-24}\,\hat{i}$$

$$= 9\times10^{-25}\,\hat{i} \quad N$$

Sample Problem 21.01

Finding the net force due to two other particles

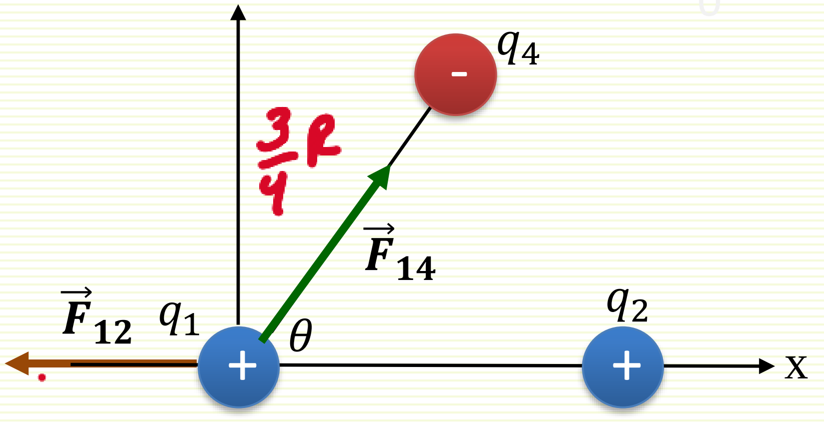
- (c) The particle 4 is now included, it has charge $q_4 = -3.20 \times 10^{-19} C$, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60^o$ with the $x-$axis. What is the net electrostatic force $\vec{F}_{1,net}$ on particle 1 due to particles 2 and 4?

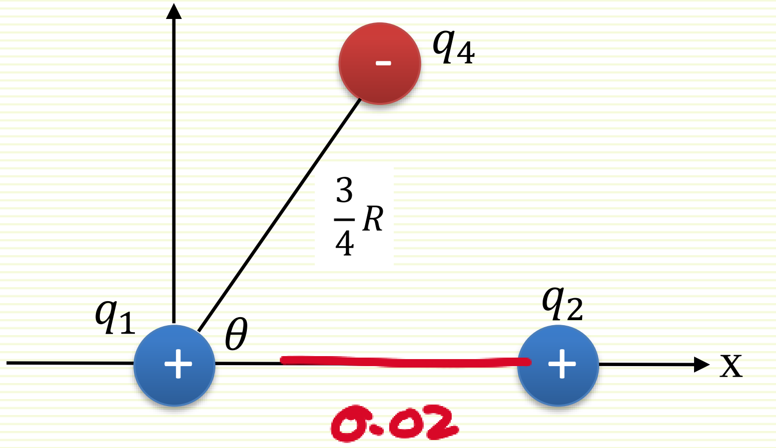
Solution:

- The net force $\vec{F}_{1,net}$ is the vector sum of $\vec{F}_{12}$ and a new force $\vec{F}_{14}$ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. so, force $\vec{F}_{14}$ on particle 1 is directed toward particle 4, at angle $\theta = 60^o$.

6 $$F_{14} = \frac{1}{4\pi\varepsilon_o}\frac{|q_1||q_2|}{\left(\frac{3}{4}R\right)^2} = \left(8.99 \times 10^9 N.\frac{m^2}{C^2}\right) \times \frac{(1.60\times10^{-19}C)(3.20\times10^{-19}C)}{\left(\frac{3}{4}\right)^2(0.0200\ m)^2}$$

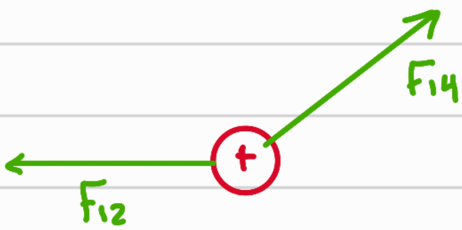
- $F_{14} = 2.05 \times 10^{-24} N$
- The net force $\vec{F}_{1,net}$ on particle 1 is
- $\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{14}$





المطلوب حساب القوى الكهروسكونية المحصلة المؤثرة في الجسم (q_1) والناتجة عن q_2 و q_4

رسم القوى

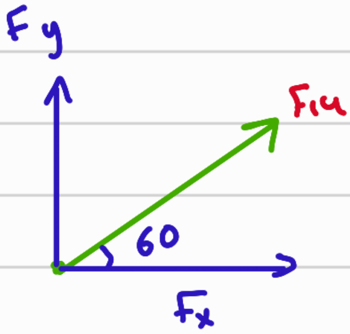


حساب كل قوة

$$\vec{F}_{12} = \frac{K q_1 q_2}{r^2} = \frac{8.99\times10^{9}\times1.6\times10^{-19}\times3.2\times10^{-19}}{(0.02)^2}$$

$$\boxed{F_{12} = -1.15\times10^{-24}\,\hat{i}}$$

$$F_{14} = \frac{K q_1 q_4}{(\frac{3}{4}R)^2} = \frac{8.99\times10^{9}\times1.6\times10^{-19}\times3.2\times10^{-19}}{(\frac{3}{4}\times0.02)^2}$$



$$F_{14} = 2.05\times10^{-24}\ N$$

$F_x = F\cos\theta$

$F_y = F\sin\theta$

$$F_x = F_{14}\cos 60 = 2.05\times10^{-24}\times\cos 60 = 1.025\times10^{-24}\,\hat{i}$$ ✓

$$F_y = F_{14}\sin 60 = 2.05\times10^{-24}\times\sin 60 = 1.775\times10^{-24}\,\hat{j}$$ ✓

$$\boxed{F_{14} = 1.025\times10^{-24}\,\hat{i} + 1.775\times10^{-24}\,\hat{j}}$$

القوة المحصلة = مجموع كل القوى

$$F_{1net} = F_{12} + F_{14}$$

$$= -1.15\times10^{-24}\,\hat{i} + 1.025\times10^{-24}\,\hat{i} + 1.775\times10^{-24}\,\hat{J}$$

$$\boxed{F_{1\,net} = -0.125\times10^{-24}\,\hat{i} + 1.78\times10^{-24}\,\hat{j}\ \ N}$$

Sample Problem 21.01

Finding the net force due to two other particles

- Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we cannot sum simply by combining their magnitudes. We use the summing in unit-vector notation

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{\imath} + (F_{14}\sin\theta)\hat{\jmath} = \left(2.05\times10^{-24}\cos 60^{o}\right)\hat{\imath} + \left(2.05\times10^{-24}\sin 60^{o}\right)\hat{\jmath}$$
$$= \left(1.025\times10^{-24}\,N\right)\hat{\imath} + \left(1.775\times10^{-24}\,N\right)\hat{\jmath}$$

- Then we sum:

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{14} = \left(-1.15\times10^{-24}N\right)\hat{\imath} + \left(1.025\times10^{-24}\,N\right)\hat{\imath} + \left(1.775\times10^{-24}\,N\right)\hat{\jmath}$$
$$= \left(-1.25\times10^{-25}N\right)\hat{\imath} + (1.78\times10^{-24}N)\hat{\jmath}$$

Sample Problem 21.02

Equilibrium of two forces on a particle

- Two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in equilibrium (the net force on it is zero)? Is that equilibrium stable or unstable? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?

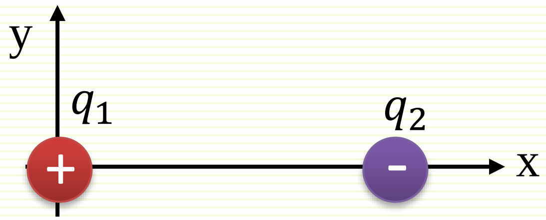


Fig. (a)

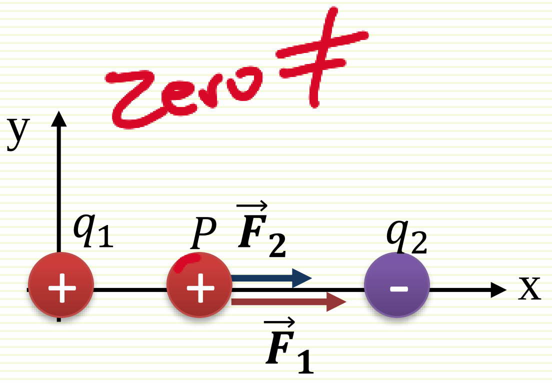


Fig. (b)

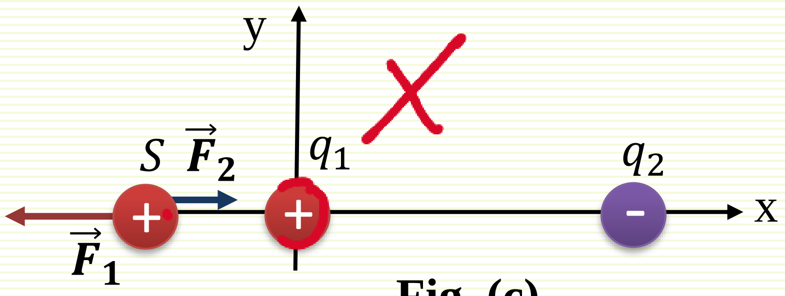


Fig. (c)

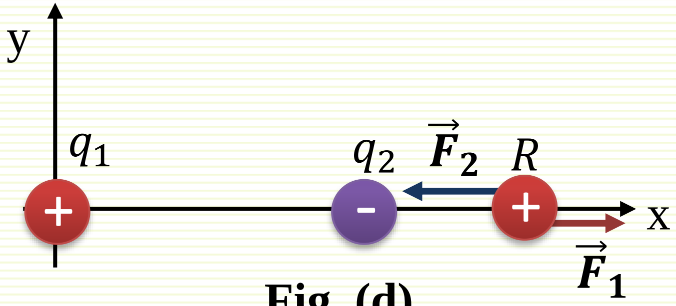
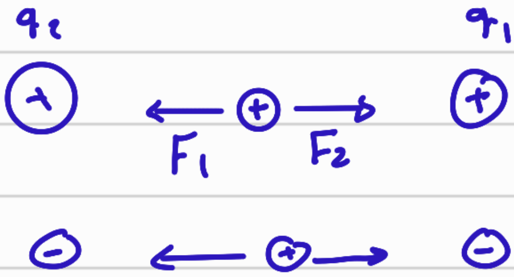
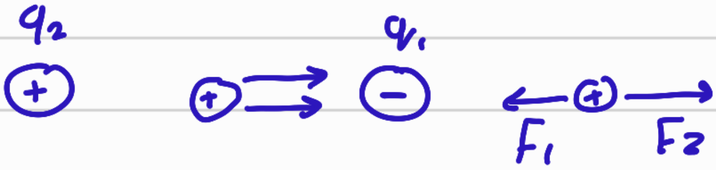


Fig. (d)

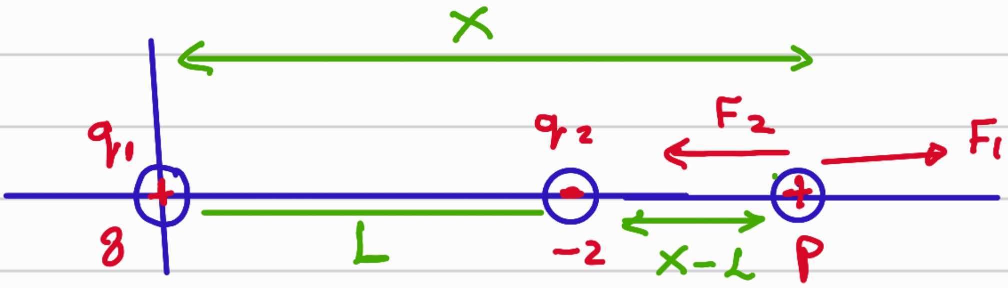
نقطة التعادل



اذا كانت الشحنتين لهما نفس الاشارة فان نقطة التعادل تقع بينهما و أقرب الى الشحنة الاصغر



اذا كانت الشحنات متعاكسة بالاشارة فان نقطة التعادل تقع خارجهما و أقرب الى الشحنة الاصغر



$$F_1 = F_2$$

$$\frac{k\,q_1\,q_p}{r_1^2} = \frac{k\,q_2\,q_p}{r_2^2}$$

$$\frac{\cancel{k}(8\cancel{q})\cancel{q_p}}{x^2} = \frac{\cancel{k}(2\cancel{q})\cancel{q_p}}{(x-L)^2}$$

$$\frac{8}{x} = \frac{2}{(x-L)^2}$$

$$\frac{8}{2} = \frac{(x)^2}{(x-L)}$$

$$\sqrt{4} = \sqrt{\frac{x^2}{(x-L)^2}}$$

$$2 = \frac{x}{x-L}$$

$$x = 2x - 2L$$

$$x - 2x = -2L$$

$$\boxed{x = 2L}$$

Sample Problem 21.02

Equilibrium of two forces on a particle

- If $\vec{F}_1$ is the force on the proton due to charge q_1 and $\vec{F}_2$ is the force on the proton due to charge q_2, then the point we seek is where

$$\vec{F}_1 + \vec{F}_2 = 0$$

- Thus,

$$\vec{F}_1 = -\vec{F}_2$$

- This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes, $F_1 = F_2$, and that the forces must have opposite directions.

Sample Problem 21.02

Equilibrium of two forces on a particle

- Because a proton has a positive charge, the proton and the particle of charge q_1 are of the same sign, and force $\vec{F}_1$ on the proton must point away from q_1.

- Also, the proton and the particle of charge q_2 are of opposite signs, so force $\vec{F}_2$ on the proton must point toward q_2. "away from q_1 and toward q_2" can be in opposite directions only if the proton is located on the x −axis.

- If the proton is on the x −axis at any point between q_1 and q_2, (Point P in Figure b), then $\vec{F}_1$ and $\vec{F}_2$ are in the same direction and not in opposite directions.

Sample Problem 21.02

Equilibrium of two forces on a particle

- If the proton is at any point on the x −axis to the left of q_1, (Point S in Figure c), then $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. But $\vec{F}_1$ and $\vec{F}_2$ cannot have equal magnitudes there: $\vec{F}_1$ must be greater than $\vec{F}_2$, because $\vec{F}_1$ is produced by a closer charge (with lesser r) of greater magnitude ($8q$ versus $2q$).

- Finally, if the proton is at any point on the x −axis to the right of q_2, (Point R in Figure d), then $\vec{F}_1$ and $\vec{F}_2$ are again opposite directions. However, because now the charge of greater magnitude (q_1) is farther away from the proton than the charge of lesser magnitude, there is a point at which $\vec{F}_1$ is equal to $\vec{F}_2$.

Sample Problem 21.02

Equilibrium of two forces on a particle

- Let x be the coordinate of this point, and let q_p be the charge of the proton. So

$$F_1 = F_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{(8q)(q_p)}{x^2} = \frac{1}{4\pi\varepsilon_o}\frac{(2q)(q_p)}{(x-L)^2}$$

$$\therefore \left(\frac{x}{x-L}\right)^2 = 4$$

- Taking the square roots of both sides, so

$$\left(\frac{x}{x-L}\right) = 2 \quad \Rightarrow \quad x = 2(x-L)$$

$$x = 2x - 2L \quad \Rightarrow \quad 2x - x = 2L$$

$$\therefore x = 2L$$

Sample Problem 21.02

Equilibrium of two forces on a particle

اتزان غير ثابت

- The equilibrium at $x = 2L$ is unstable; that is, if the proton is displaced leftward from the point R, then F_1 and F_2 both increase but F_2 increase more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward.

- If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decrease more, and a net force will then drive the proton farther rightward.

- In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

ويعتبر الاتزان غير مستقر لانه عند سحب الشحنه الى اليسار قليلاً تصبح قوه F_2 اكبر لتغير ويسحب البروتون نحو اليسار

المواد الموصلة
والمواد العازلة

Conductors and Insulators

- The materials are classified generally according to the ability of charge to move through them as:

تصنف المواد حسب قدرة الشحنات على الحركة

موصلات
1. Conductors: are materials which charge can move rather freely, for example: metals (such as copper in common lamp wire), the human body, and tap water.

غير موصلة
2. Nonconductors (Insulators): are materials which charge cannot move freely, for example: rubber, plastic, glass, and chemically pure water.

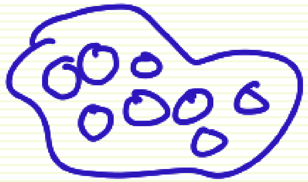
اشباه موصلات
3. Semiconductors: are materials that are intermediate between conductors and insulators, for example: silicon and germanium in computer chips.

4. Superconductors: are materials that are perfect conductors, allowing charge to move without any hindrance.

المواد فائقة الموصلية

عوائق

$12e^-$



الشحنة الكمية

Charge is quantized

محدده بقيم محددة.

- Electric charge is quantized (restricted to certain values). Any positive or negative charge q can be written as

شحنة الالكترون

عدد صحيح

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots.$$

- In which e, the elementary charge, has the approximate value

الشحنة الاساسية

$$e = 1.602 \times 10^{-19}\ C$$

- The electron and proton both have a charge of magnitude e.
- When a physical quantity such as charge is quantized. It is possible, for example, to find a particle that has no charge at all, or a charge of $+10e$ or $-6e$, but not a particle with a charge of $3.75e$.

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

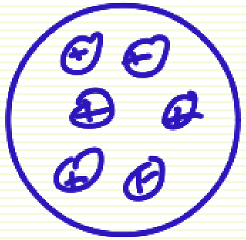
الكسور والاعشار لا يمكن ان تعبر عن الشحنة

Sample Problem 21.04

Mutual electric repulsion in a nucleus

- The nucleus in an iron atom has a radius of about $4.0 \times 10^{-15} m$ and contains 26 protons. what is the magnitude of the repulsive electrostatic force between two of the protons that are separately by $4.0 \times 10^{-15} m$?

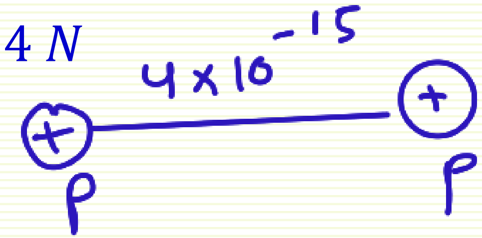
ما مقدار قوة التنافر بين بروتونين



- Solution

- The charge of a proton is $+e$, so

$$F = \frac{1}{4\pi\varepsilon_o}\frac{(e)(e)}{r^2} = \frac{(8.99 \times 10^9\, N.m^2/C^2)(1.602 \times 10^{-19} C)^2}{(4.0 \times 10^{-15} m)^2} = 14\, N$$



$$F = \frac{k q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times (1.602 \times 10^{-19})^2}{(4 \times 10^{-15})^2} = 14.3 N$$

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch 22

The Electric Field

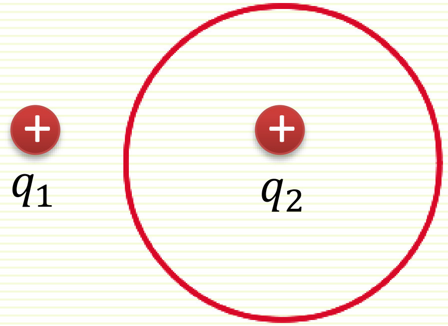
المجال الكهربائي
$\vec{E}$

Physics Department

College of Science

Contents

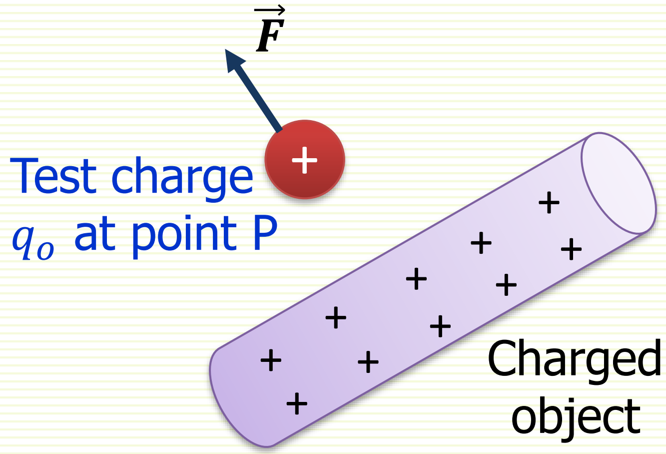
The Electric Field



- For two positively charged particles, we know that there is an electrostatic force acts on particle 1 due to the presence of particle 2, and we also know the force direction and its magnitude.
- We define the $\boxed{\text{electric field } \vec{E}}$ at a point, P near the charged object, as the ratio between the electrostatic force $\vec{F}$ (acting on a small positive charge q_o, called a test charge placed at P), and the value of the test charge q_o, that it is the electric field at that point is given by
- $\vec{E} = \frac{\vec{F}}{q_o}$ (N/C)

+ q

$\to E$
$\to F$
q_o



- we call $\boxed{q_o \text{ test charge}}$, because we use it to test the field.
- Because the test charge is $\boxed{\text{positive,}}$ the two vectors are in same direction, so the direction of $\vec{E}$ is the direction we measure for $\vec{F}$.
- The magnitude of electric field $\vec{E}$ at point P is $\boxed{F/q_o.}$
- The SI unit for the electric field is the $\boxed{\text{newton per coulomb (N/C).}}$

المجال الكهربائي $\vec{E}$

هو مقياس لتأثير القوة الكهربائية بشحنة اختبار صغيرة وضعت في منطقة المجال

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}} = \frac{\text{القوة}}{\text{الشحنة الاختبارية}}$$

وحدة القياس $= \frac{N}{C} = N/C$

اتجاه المجال يكون مع اتجاه القوة اذا كانت الشحنة الاختبارية موجبة (دائماً خارج من الشحنة الموجبة)

$\oplus \quad \overset{P}{\bullet} \rightarrow E$

اتجاه المجال يكون عكس اتجاه القوة اذا كانت الشحنة الاختبارية سالبة (دائماً داخل للشحنة السالبة)

$\ominus \quad E \leftarrow \overset{P}{\bullet}$

$E \leftarrow \underset{q_0}{\ominus} \rightarrow F$

$\oplus \rightarrow F$
$q_0 \rightarrow E$

$$\vec{E} = \frac{F}{q_0} = \frac{k\frac{q_1 \cancel{q_0}}{r^2}}{\cancel{q_0}}$$

$\overset{q_1}{\oplus}$ ——r^2—— q_0

$$\boxed{\vec{E} = \frac{k|q|}{r^2}\hat{r}}$$

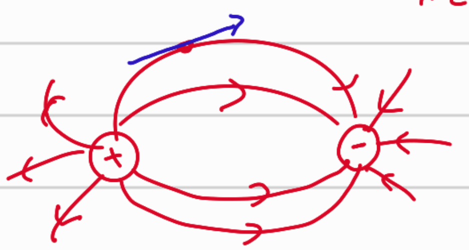
q الشحنة المنتجة للمجال

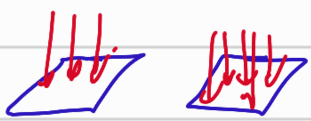
$\underset{5\mu C}{\oplus}$ ——$r = 1m$—— $\overset{P}{\bullet} \rightarrow \vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{E} = \frac{9\times10^{9}\times5\times10^{-6}}{1^2}$$

$$45\times10^{3}\ N/C$$





كلما كانت كثافة خطوط المجال اكبر كان المجال اكبر

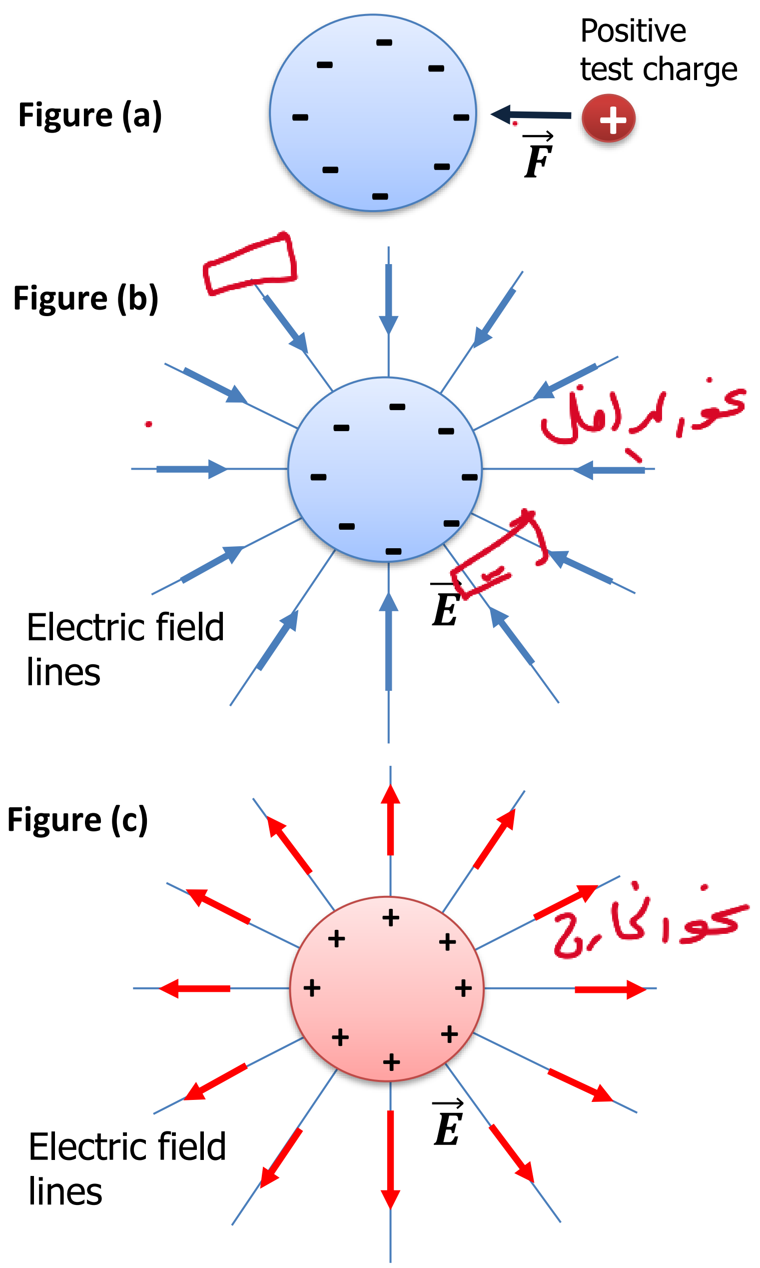
خطوط المجال

Electric Field Lines

8/16/2024

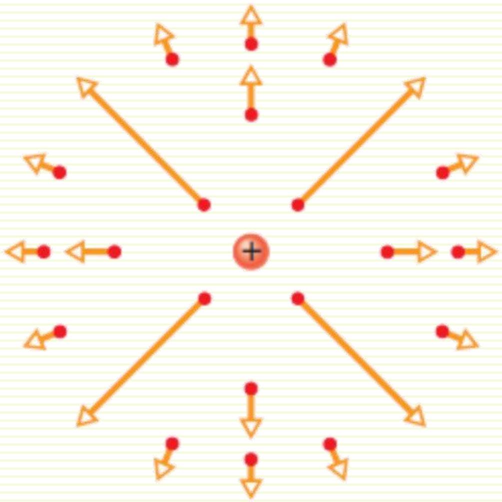
- A sphere is uniformly covered with negative charge. If we place a positive test charge at any point near the sphere (Figure a), we find that an electrostatic force pulls positive test charge toward the center of the sphere. So, at every point around the sphere, an electric field vector points radially inward toward the sphere.
  تجاه الداخل
- We can represent this electric field with electric field lines as in Figure (b). At any point, the direction of the field line through the point matches the direction of the electric vector at that point.
  قواعد رسم خطوط المجال
- The rules for drawing electric field lines are these:
- (1) At any point, the electric field vector must be tangent to the electric field line through that point and in the same direction.
  اتجاهه عبارة تمثيل المماس
- (2) In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.
  كثافة الخطوط تتناسب مع مقدار المجال
- Rule: Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate). So, a field line extending from a positive charge must end on a negative charge.

خط المجال الذي يخرج من موجب ينتهي في سالب



The Electric Field due to a Charged Particle (point charge)

- To find the electric field due to a charged particle q (a point charge), we place a positive test charge at any point near the particle, at distance r. From Coulomb's law, the force on the test charge q_o due to the particle with charge q is



$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q\,q_o}{r^2}\hat{r}$$

- The direction of $\vec{F}$ is directly away from the particle if q is positive (because q_o is positive) and directly toward it if q is negative.
- The electric field set up by the particle (at the location of the test charge) is

$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} \quad \text{(charged particle)}$$

$$E = k\frac{|q|}{r^2}$$

- The magnitude at any given distance r is given by

$$\boxed{E = \cancel{\frac{1}{4\pi\varepsilon_o}}\frac{|q|}{r^2}} \quad \text{(charged particle)}$$

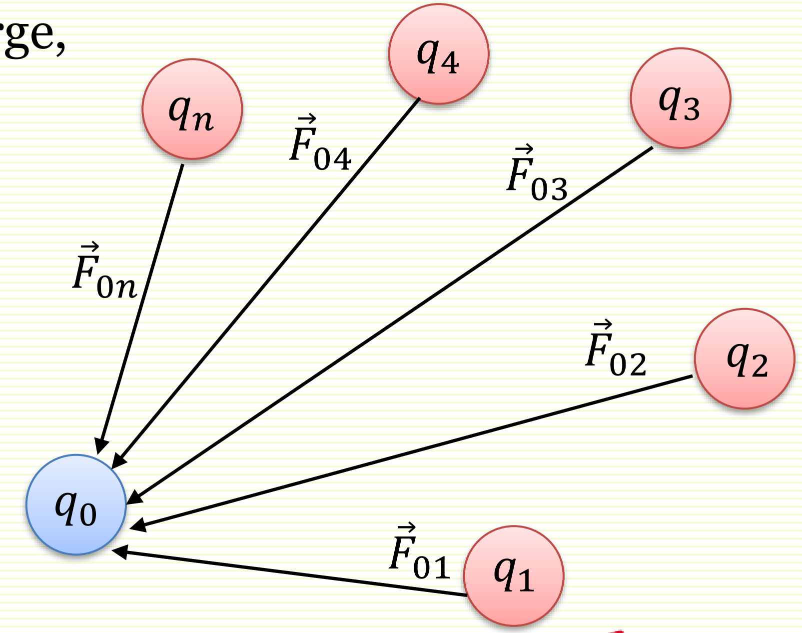
$$q_o \xrightarrow{E} \quad \xrightarrow{F}$$

$$q_o \xrightarrow{F} \quad \xleftarrow{E}$$

- The direction of $\vec{E}$ matches that of the force on the positive test charge: directly away from the point charge if q is positive and directly toward it if q is negative.

The Electric Field due to several Charged Particles

- If several electric fields are set up at a given point by several charged particles, we find the net field by placing a positive test particle, q_o, at the point and then writing out the force acting on the test charge, $\vec{F}_0$, due to each particle, such as $\vec{F}_{01}$ due to particle 1.



- Forces obey the principle of superposition, so

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}$$

$$\vec{E} = \frac{\vec{F}_0}{q_o} = \frac{\vec{F}_{01}}{q_o} + \frac{\vec{F}_{02}}{q_o} + \cdots + \frac{\vec{F}_{0n}}{q_o} = \boxed{\vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n}$$

- This tells us that **electric fields** also obey the principle of superposition.

اعتبار الكهربائي انو ينتج فكره
التداخل

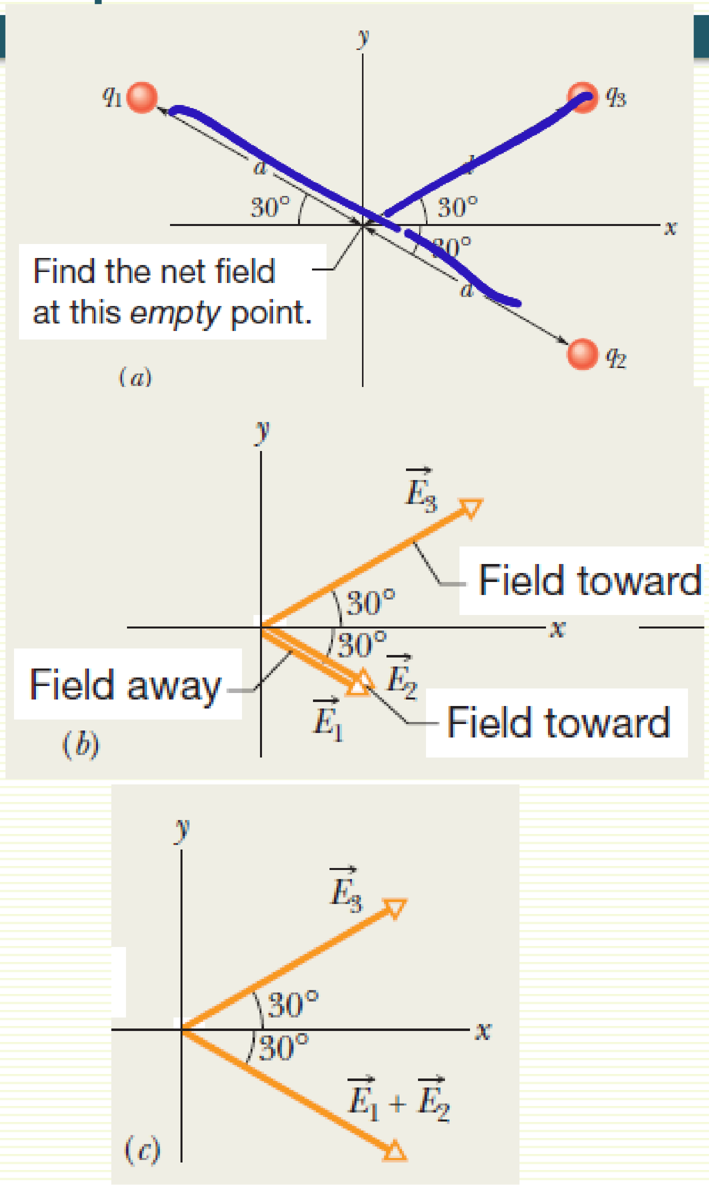
$$\frac{E}{q_0} = \frac{E_1}{q_0} + \frac{E_2}{q_0} + \frac{E_3}{q_0}$$

$$E = E_1 + E_2 + E_3$$

Sample Problem 22.01

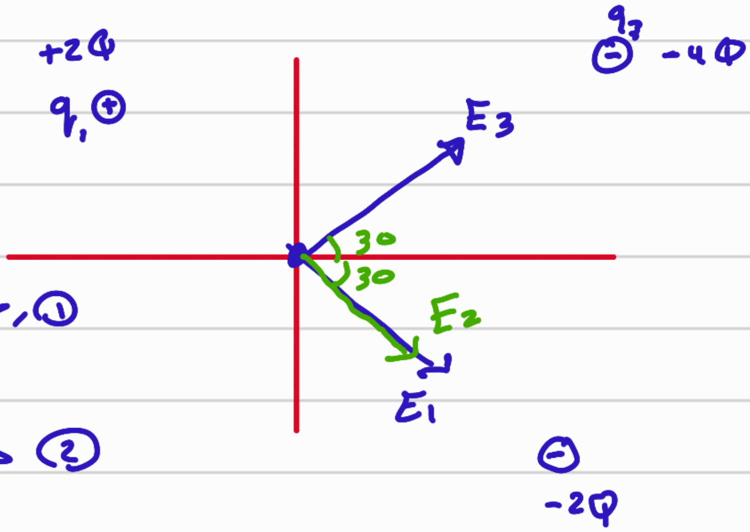
Net electric field due to three charged particles

- Three particles with charges $q_1 = +2Q, q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?



- Answer:
- Charges q_1, q_2, and q_3 produce electric vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$. The magnitude of $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ is
- $E_1 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{2Q}{d^2}$,
- $E_2 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{2Q}{d^2}$,
- $E_3 = \dfrac{1}{4\pi\varepsilon_o}\dfrac{4Q}{d^2}$

22.1



① رسم اتجاهات المجال

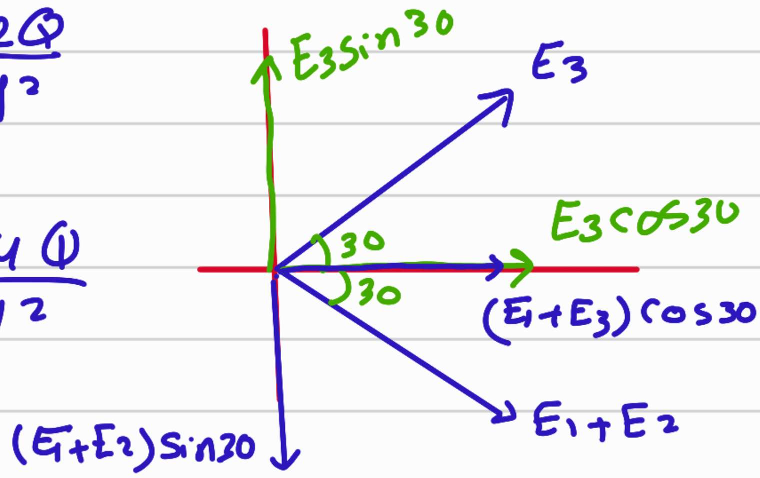
② حساب كل مجال لوحده

$$\bar{E}_1 = k\frac{q_1}{r_1^2} = \frac{k2Q}{d^2}$$

$$\bar{E}_1 + \bar{E}_2 = \frac{4kQ}{d^2}$$

$$\bar{E}_2 = \frac{kq_2}{r_2^2} = \frac{k2Q}{d^2}$$

$$\bar{E}_3 = \frac{kq_3}{r_3^2} = \frac{k4Q}{d^2}$$



③ كتابة المجالات على صورة متجهات (تحليل)

$$E_3 = E_3\cos30\,\hat{i} + E_3\sin30\,\hat{j}$$

$$\boxed{E_3 = \frac{4kQ}{d^2}(0.866)\,\hat{i} + \frac{4Qk}{d^2}(0.5)\,\hat{j}}$$

$$\boxed{E_1 + E_2 = \frac{4kQ}{d^2}(0.866)\,\hat{i} - \frac{4Qk}{d^2}(0.5)\,\hat{j}}$$

(4) جمع، متجهات

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$

$$\bar{E} = \frac{8Qk}{d^2}(0.866)\,i = \frac{8}{4\pi\epsilon_0 d^2}\,Q\,(0.866)$$

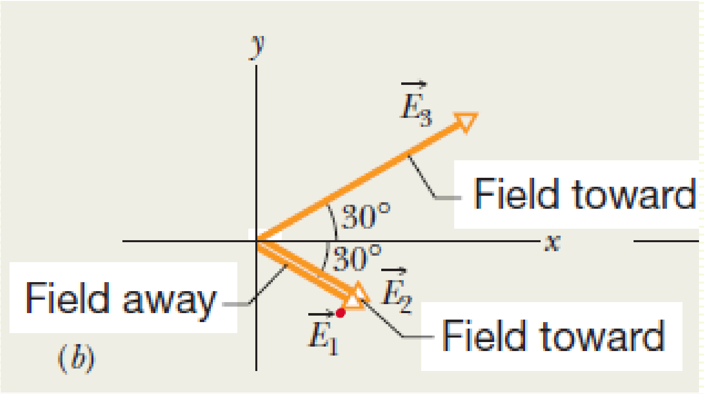
x axis

$$= \frac{6.93\,Q}{4\pi\epsilon_0 d^2} \; N/C$$

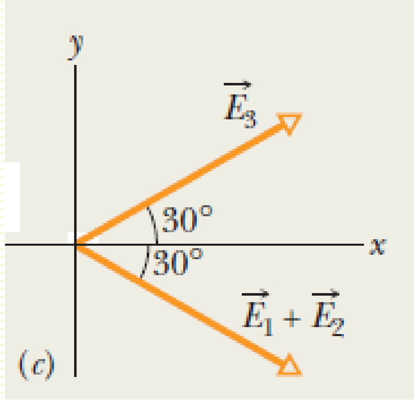
Sample Problem 22.01
Net electric field due to three charged particles

- The orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly away from it

- Because q_2 and q_3 are both negative, the field vectors they produce point directly toward each of them. So, the three electric fields produced at the origin by the three charged particles are oriented as in Figure (b).



- The electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction, so their vector sum has that direction and has the magnitude



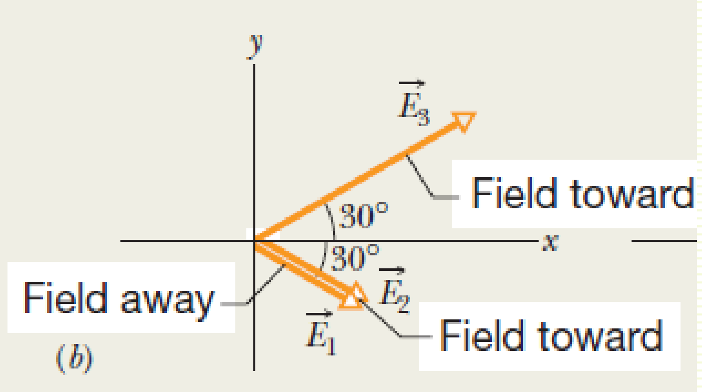
$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_o}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_o}\frac{4Q}{d^2}$$

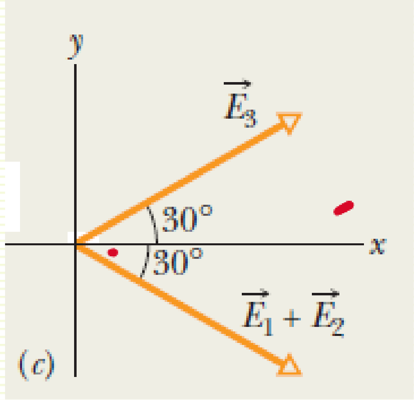
Sample Problem 22.01

Net electric field due to three charged particles

- From Figure (c), the equal y-components of two vectors $\vec{E}_3$ and $(\vec{E}_1 + \vec{E}_2)$ cancel (one is upward and the other is downward).
- Thus, the net electric field $\vec{E}$ at the origin is in the positive direction on the x –axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^o = (2)\left(\frac{1}{4\pi\varepsilon_o}\frac{4Q}{d^2}\right)(0.866) = \frac{6.93Q}{4\pi\varepsilon_o d^2}$$





A Point Charge in an Electric Field

- Suppose a charged particle is in an electric field set up by other stationary or slowly moving charges.

E

- The electrostatic force acts on the particle is given by

q

لو وضعنا شحنة داخل الحقل

$$\boxed{\vec{F} = q\,\vec{E}}$$

- In which q is the charge of the particle (including its sign) and $\vec{E}$ is the electric field that other charges have produced at the location of the particle.

مجال
خارجي

- The field acting on the particle is called the *external field*.
- The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative

اذا وضعنا شحنة في منطقة مجال فانها ستتأثر بقوة تكون مع اتجاه المجال اذا كانت الشحنة موجبة و عكس اذا كانت سالبة.

ايجاد المجال الكهربائي الناتج عن توزيعات من الشحنة

The Electric Field of Continuous Charge Distributions

كثافة الشحنة

- For an extended object, the charge is often conveyed in terms of a charge density rather than the total charge.

$\lambda = \frac{Q}{L} = C/m$

λ كثافة الشحنة الطولية

- For a line of charge, we use the *linear charge density* $\boxed{\lambda}$ (the charge per unit length), with the SI unit of (C/m). كولوم/متر

الشحنة الكلية لكل وحدة طول

الشحنة الكلية لكل وحدة مساحة

- For a charged surfaces, we use the surface charge density $\boxed{\sigma}$ (the charge per unit area), with the SI unit of (C/m^2).

$\sigma = \frac{Q}{A} = C/m^2$

σ كثافة الشحنة السطحية

- For a charged volumes, we use the volume charge density $\boxed{\rho}$ (the charge per unit volume), with the SI unit of (C/m^3).

ρ: كثافة الشحنة الحجمية

$\rho = \frac{Q}{V} = C/m^3$

The Electric Field of Continuous Charge Distributions

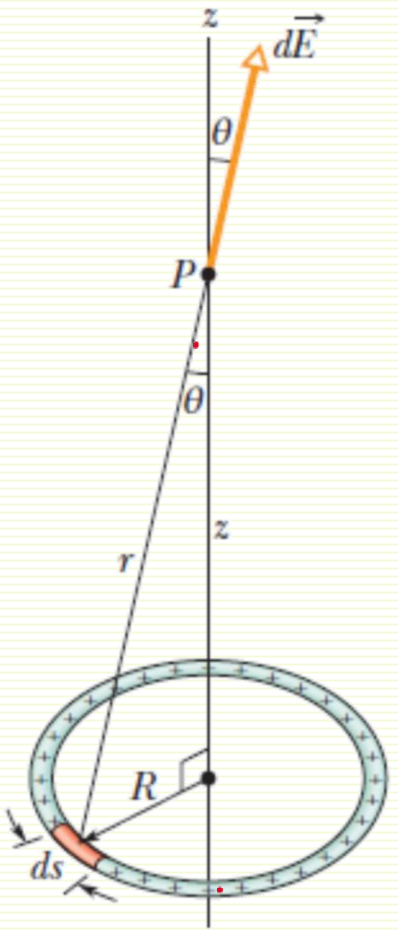
Electric field at a distance z from the center of a charged ring

- The electric field due to a line of charge:
- A thin ring of radius R with a uniform distribution of positive charge along its circumference. It is made of plastic, which means that the charge is fixed in place.
- The ring is surrounded by a pattern of electric field lines, but we calculate at point P that on the central axis at a distance z from the center point.
- In terms of the linear charge density λ, we have

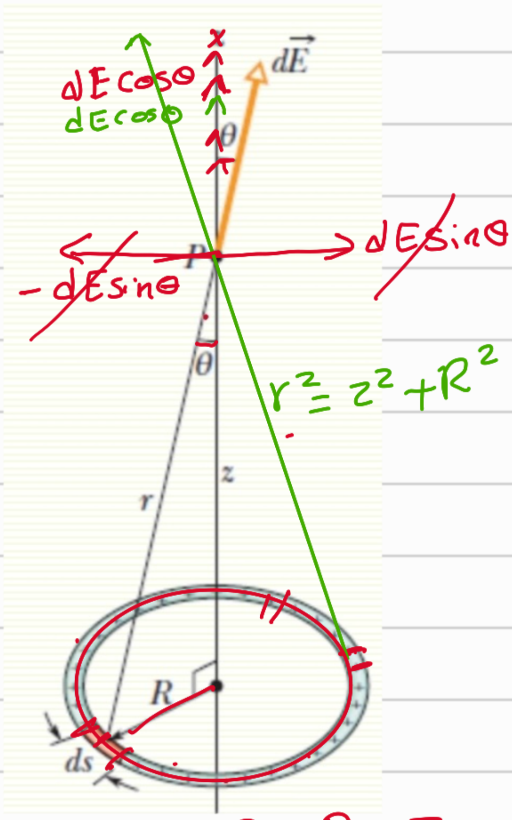
$$dq = \lambda\, ds$$

- This charge element sets up the differential electric field $d\vec{E}$ at P, at distance r from the element. The field element due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{\lambda ds}{(R^2+z^2)}$$



المطلوب حساب المجال الكهربائي عند النقطة



$$\lambda = \frac{Q}{L} = \frac{Q}{2\pi R} = \frac{Q}{S} = \frac{dq}{ds}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{ds\,\lambda}{z^2+R^2}$$

$$\cos\theta = \frac{z}{r}$$

المركبات الموازية لسطح الحلقة سوف تلغي بعضها $0 = \Sigma\, dE\sin\theta$

النتيجة هي $dE\cos\theta$

$$E = \sum dE\cos\theta = \int dE\cos\theta$$

$$E = \int \frac{1}{4\pi\epsilon}\frac{\lambda\, ds}{(R^2+z^2)}\cos\theta = \int \frac{1}{4\pi\epsilon}\frac{\lambda\, ds}{(R^2+z^2)}\frac{z}{r}$$

$$E = \frac{1}{4\pi\epsilon}\lambda \cdot \int \frac{ds\; z}{(R^2+z^2)^1 (R^2+z^2)^{1/2}}$$

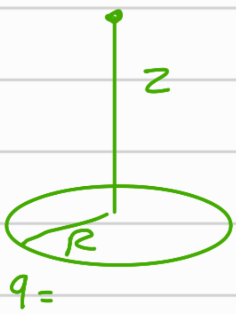
$$E = \frac{1}{4\pi\epsilon}\frac{\lambda z}{(R^2+z^2)^{3/2}}\int_0^{2\pi R} ds$$

$$E = \frac{1}{4\pi\epsilon}\frac{\lambda z}{(R^2+z^2)^{3/2}}(2\pi R)$$

$$E = \frac{1}{4\pi\epsilon} \frac{\frac{q}{\cancel{2\pi R}} z}{(R^2+z^2)^{3/2}} (\cancel{2\pi R})$$

محيط

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{qz}{(R^2+z^2)^{3/2}}}$$



حالة خاصة $z \gg R$

عندما تكون النقطه P بعيده جداً عن الحلقه

$$\cancel{R^2}^{\,0} + z^2 = z^2$$

$$E = \frac{1}{4\pi\epsilon} \frac{qz}{(z^2)^{3/2}} = \frac{1}{4\pi\epsilon} \frac{qz}{z^3}$$

$$\boxed{E = \frac{1}{4\pi\epsilon} \frac{q}{z^2}} \qquad z \gg R$$

حالة خاصة , عندما في مركز الحلقة

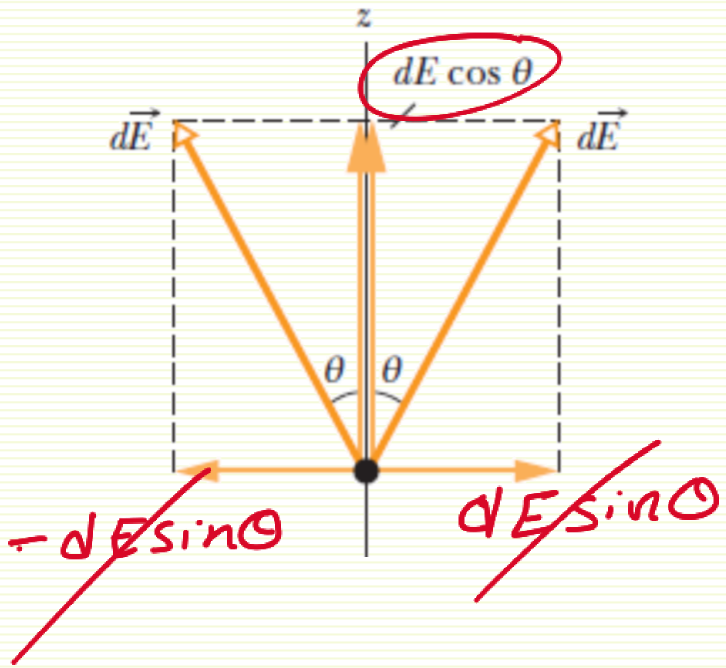
$$z = 0$$

$$\boxed{E = 0}$$ في مركز الحلقة

The Electric Field of Continuous Charge Distributions

Electric field at a distance z from the center of a charged ring

- Now, consider the charge element on the opposite side of the ring, it too contributes the field magnitude dE but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as in the following figure.
- Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its symmetric partner on the opposite side of the ring, so we can neglect all the perpendicular components.
- All the remaining components are in the positive direction of the z-axis.

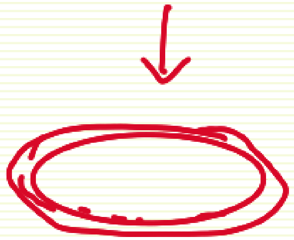


The Electric Field of Continuous Charge Distributions

Electric field at a distance z from the center of a charged ring

- The direction of the net electric field at P is directly away from the ring. The net electric field at P is
- $E = \int dE \cos\theta = \int \frac{1}{4\pi\varepsilon_o} \frac{\lambda ds}{(R^2+z^2)} \cos\theta = \int \frac{1}{4\pi\varepsilon_o} \frac{\lambda ds}{(R^2+z^2)} \frac{z}{r} = \int \frac{1}{4\pi\varepsilon_o} \frac{z\lambda}{(R^2+z^2)^{3/2}}\, ds$
- $= \frac{1}{4\pi\varepsilon_o} \frac{z\lambda}{(R^2+z^2)^{3/2}} \int_0^{2\pi R} ds = \frac{1}{4\pi\varepsilon_o} \frac{z\lambda}{(R^2+z^2)^{3/2}} (2\pi R) = \frac{1}{4\pi\varepsilon_o} \frac{z\,(q/2\pi R)}{(R^2+z^2)^{3/2}} (2\pi R) = \boxed{\frac{qz}{4\pi\varepsilon_o (R^2+z^2)^{3/2}}}$
- Where the line charge density λ is
- $\lambda = \frac{q}{2\pi R}$
- If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still as given, but the electric field vector points toward the ring instead of away from it.
- If $z \gg R$, so
- $z^2 + R^2 \cong z^2$

ازا كانت شحنة الحلقة سالبه
اذا المجال يكون نحوها



The Electric Field of Continuous Charge Distributions

Electric field at a distance z from the center of a charged ring

- Then
- $E \cong \frac{1}{4\pi\varepsilon_o}\frac{qz}{(z^2)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\frac{qz}{z^3} = \boxed{\frac{1}{4\pi\varepsilon_o}\frac{q}{z^2}}$ $z \gg R$
- The electric field at a point at the center of the ring – that is, for $z = 0$.
- $E = \frac{qz}{4\pi\varepsilon_o(R^2+z^2)^{3/2}} = 0$
- This is a reasonable result because if we were to place a test charge at the center of the ring, there would be no net electrostatic force acting on it; the force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring. If the force at the center of the ring were zero, the electric field there would also have to be zero.

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineers

Code: PHY1119-4

$E = k\frac{q}{r^2}$

$E = \int dE$

Ch 23

Gauss' Law

قانون غاوس

Physics Department

College of Science

Contents

Introduction

Carl Friedrich Gauss
(1777–1855)

- In this chapter we discuss an important relationship between charge and electric field that allows us, in certain symmetric situations, to find the electric field of an extended charged object with a few lines of algebra. The relationship is called Gauss' law, which was developed by German mathematician and physicist Carl Friedrich Gauss (1777-1855). السطح الغاوسي :. هو سطح تخيلي يحيط بالجسم المشحون
- Figure 1 shows a particle with charge $+Q$ that is surrounding by an imaginary concentric sphere (said to be a Gaussian surface).
- At points on the sphere, the electric field vectors have a magnitude ($E = kQ/r^2$) and point radially away from the particle (because it is positively charged). The electric field lines are also outward and have a moderate density.
- We say that the field vectors and the field lines *pierce* تخترق the surface.

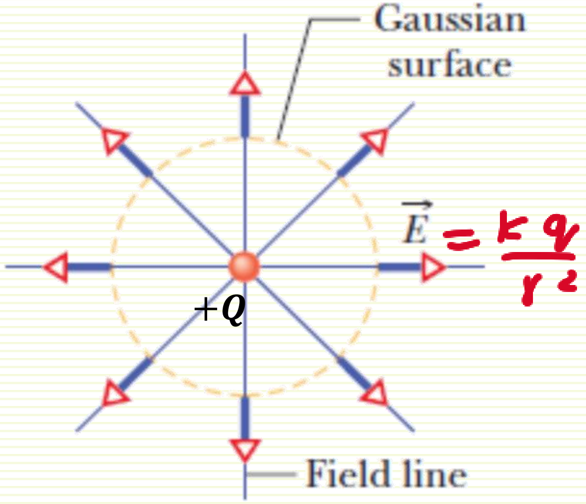


Figure 1

Introduction

سوف يتضاعف عدد الخطوط المخترقة للسطح

8/16/2024

- Figure 2 is similar except that the enclosed particle has charge $+2Q$. Because the enclosed charge now is twice as much, the magnitude of the field vectors piercing outward through the same Gaussian surface is twice as much as in Figure 1, and the density of the field lines is also twice as much.

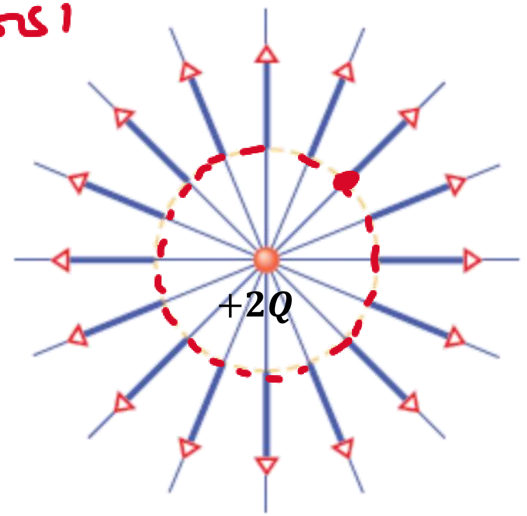


Figure 2

- Gauss' law relates the electric field at points on a closed Gaussian surface to the net charge enclosed by that surface.

E

قانون غاوس يحسب المجال عن نقطة كل السطح
وعلاقتها بالشحنة المحصورة

- Example: let us consider a particle that is enclosed by the same spherical Gaussian surface as shown in Figure 3.
- What is the amount and sign of the enclosed charge?
- From the inward piercing, we see immediately that the charge must negative.
- From the fact that the density of field lines is half that of Figure 1, we also see that the charge must be ($0.5\ Q$).

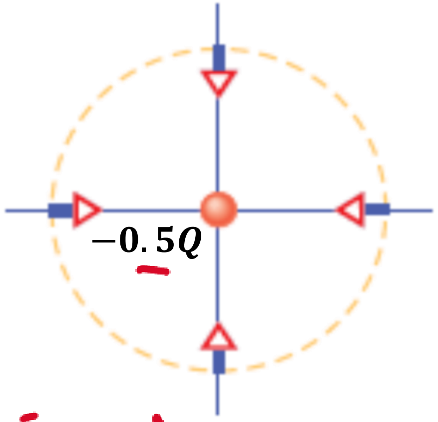


Figure 3

اصبحت خطوط
الاتجاه نصف ما كانت عليه في الشكل الاول لان الشحنة اصبحت 0.5Q

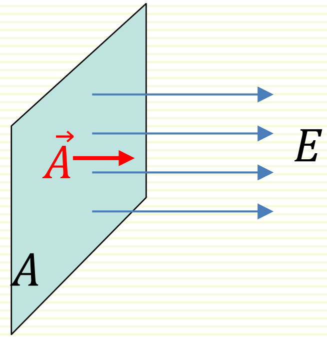
التدفق الكهربائي Electric Flux Φ N/cm^2

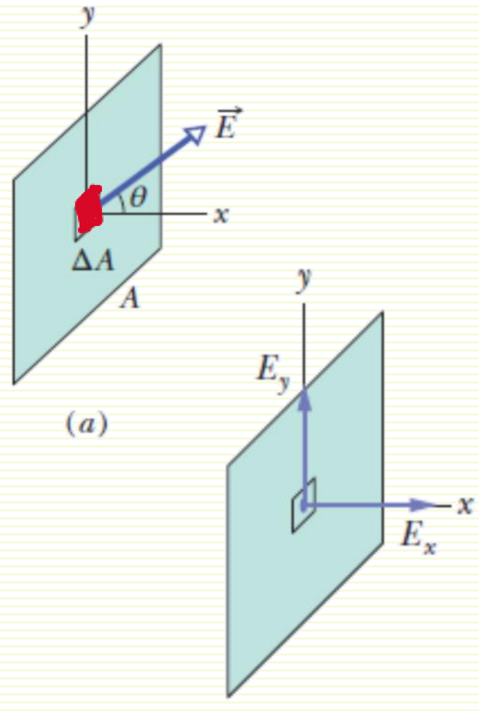
- **The electric flux** through a surface is the number of electric field lines penetrating normally the surface.
- The electric field can be expressed in terms of the number of electric field lines per unit area, that is the electric field is the density of the electric lines.
- For a Flat Surface with area A, exposed to a Uniform Field, $\vec{E}$ normal to the surface, then the electric flux is given by
- $\Phi = EA$
- Since the electric field is a vector, then we can write the last equation as
- $\Phi = \vec{E} \cdot \vec{A}$ متجه المساحة
- Where $\vec{A}$ is the area vector of the surface.
- If the field is not normal but making an angle θ with the normal as in Figure (a) in which the electric field vectors $\vec{E}$ piercing a small square patch with area ΔA, only the x-component (the normal component with magnitude $E_x = E\cos\theta$) as in Figure (b) will be considered to calculate the electric flux. The y-component does not piercing the surface and does not contribute in Gauss' law.
- The amount of electric field piercing the patch is defined to be the electric flux $\Delta\Phi$ through it:

$$\Delta\Phi = (E\cos\theta)\,\Delta A$$

$= E\Delta A\cos\theta$

$E \cdot \Delta A$



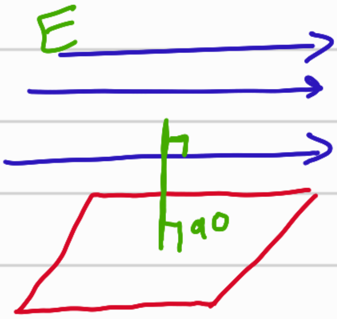


(a) (b)

Φ (Flux) التدفق الكهربائي

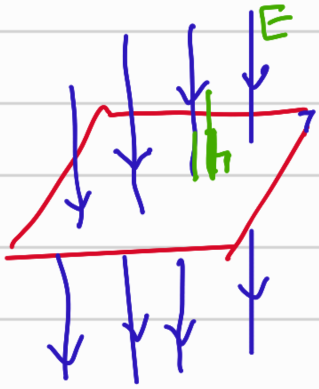
عدد خطوط المجال التي تخترق وحدة المساحة عمودياً

$\Phi = max$ $\quad\quad$ $0 < \Phi < \Phi_{max}$



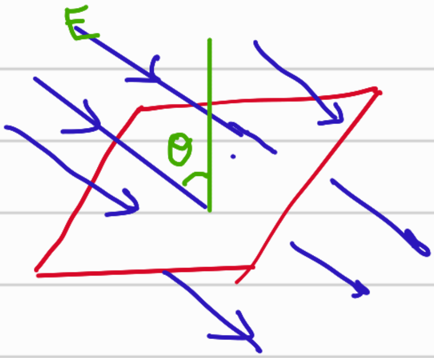
$\Phi = 0$

$\theta = 90$



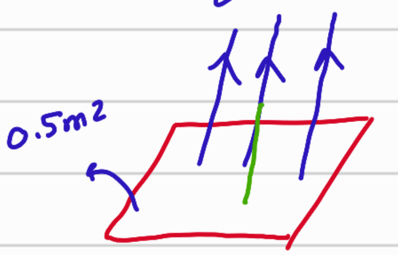
$\theta = 0$ خارجه من السطح

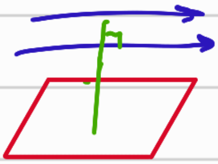
$\theta = 180$ داخله في السطح



$$\Phi = \vec{E} \cdot \vec{A}$$

$$\phi = E \cdot A \cos\theta$$





$\phi = EA\cos\theta$

$= 200 \times 0.5 \cos 0$

$= 100 \; N/cm^2$

$\Phi = AE\cos\theta$

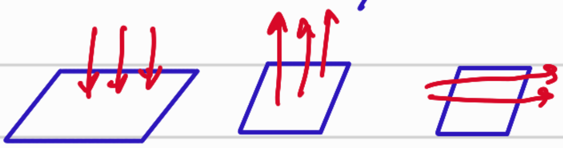
$= 200 \times 0.5 \times \cos 90$

$= 0$

$\Phi = EA\cos\theta$

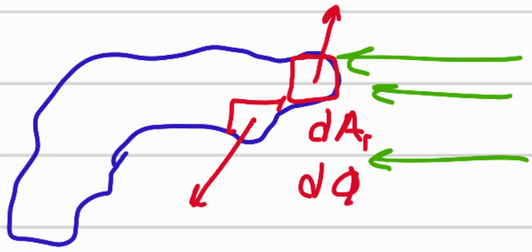
$= 200 \times 0.5 \times 0.5$

$= 50$



$\Phi = -EA$ $\quad\quad$ $\Phi = EA$ $\quad\quad$ $\Phi = 0$

في حالة السطوح الغير منتظمة



$$d\Phi = \bar{E} \cdot d\bar{A}$$

أو

$$\Delta\Phi = E \cdot \Delta\bar{A}$$

$$\Phi = \sum \Delta\Phi = \int \vec{E} \cdot d\bar{A}$$

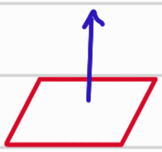
$$\Phi = \int E \cdot dA$$

الضرب القياسي

$$\Phi = E \cdot A$$

$$\vec{E} = 5i + 6j + 7k \qquad \bar{A} = 2\hat{j}$$

2m



$$\bar{E} \cdot \vec{A} = (5 \times 0) + (6 \times 2) + \cancel{(7 \times 0)}$$
$$= 12$$

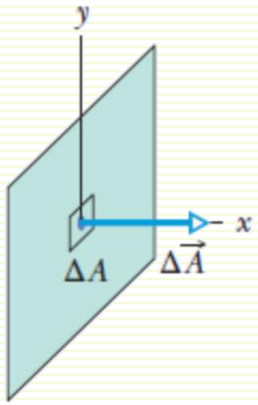
Electric Flux

- In other words: We define an area vector $\Delta\vec{A}$ that is perpendicular to the patch and that has a magnitude equal to the area ΔA of the patch (Figure *c*). So

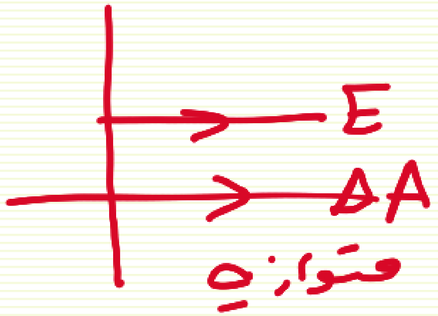
$$\Delta\Phi = \vec{E}.\Delta\vec{A}$$

- The dot product gives us the component of $\vec{E}$ that is parallel to $\Delta\vec{A}$ and thus piercing the patch.
- The total flux Φ through the surface is التدفق الكلي
- $\Phi = \sum \Delta\Phi = \sum \vec{E}.\Delta\vec{A}$
- If the area of the patch is very small dA, then the element of the flux through an area element dA is given by
- $d\Phi = \vec{E}.d\vec{A}$
- The total flux Φ through the surface is

$$\Phi = \int d\Phi = \int \vec{E}.d\vec{A}$$



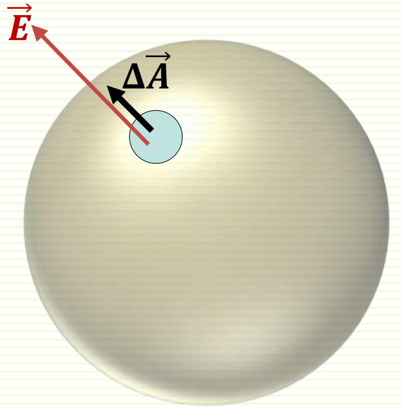
(*c*)



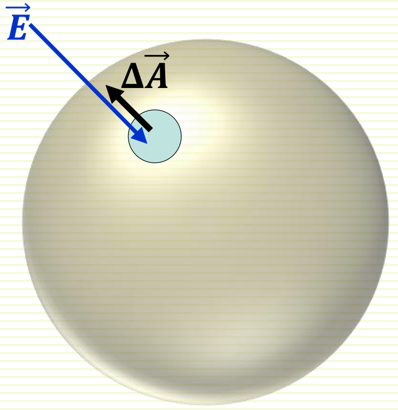
Electric Flux

- **Directions:** الاتجاهات
- We use an area vector $\Delta\vec{A}$ that is perpendicular to a patch and always draw it pointing outward from the surface (away from the interior).
- Then if a field vector pierces outward, so the area vector and electric field vector are in the same direction, the angle is $\theta = 0$ and $\cos\theta = 1$. Thus, the dot product $\vec{E}.\Delta\vec{A}$ (the electric flux) is positive. EA
- Conversely, if a field vector pierces inward, the angle is $\theta = 180^o$ and $\cos\theta = -1$. Thus, the dot product $\vec{E}.\Delta\vec{A}$ (the electric flux) is negative.
- If a field vector skims the surface (no piercing), the electric flux is zero (because $\cos 90^o = 0$). ادا قل صلس
- An inward piercing field is negative flux. An outward piercing field is positive. A skimming field is zero flux.

الخارج موجب



$\vec{E}.\Delta\vec{A}$ ***is positive***
Flux is positive

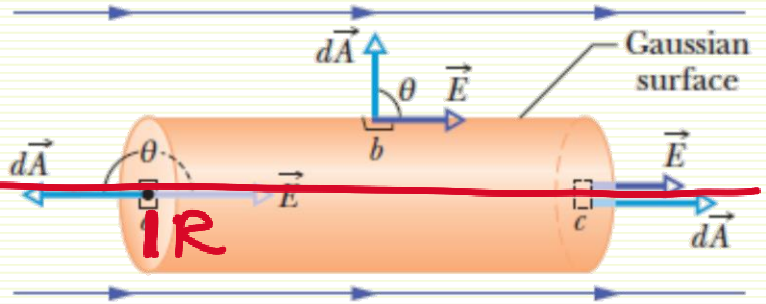


$\vec{E}.\Delta\vec{A}$ ***is negative***
Flux is negative

Sample Problem 23.01

Flux through a closed cylinder, uniform field

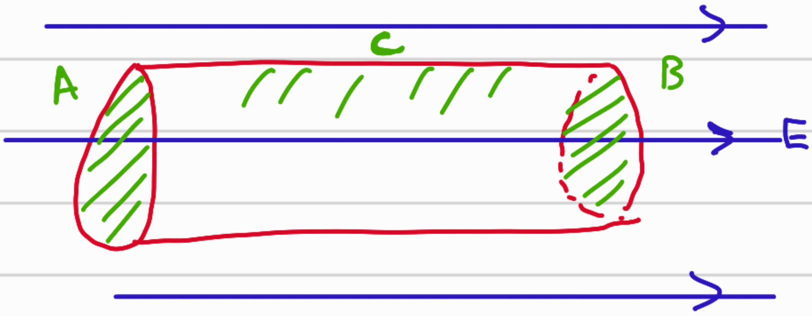
- A Gaussian surface in the form of a closed cylinder of radius R is shown in the following Figure. It lies in a uniform electric field $\vec{E}$ with the cylinder's central axis (along the length of the cylinder) parallel to the field. What is the net flux Φ of the electric field through the cylinder?



- Solution: احسب التدفق الكلي عن سطح الاسطوانة
- The cylinder's surface can be divided into three surfaces: the circular end a, the cylindrical surface b, and the circular end c.
- The electric flux is

$$\Phi = \oint \vec{E}.d\vec{A} = \int_a \vec{E}.d\vec{A} + \int_b \vec{E}.d\vec{A} + \int_c \vec{E}.d\vec{A}$$

المطلوب حساب التدفق الكلي



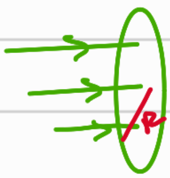
$$\Phi = \oint E \cdot dA$$

$$= \int_A E \cdot dA + \int_B E \cdot dA + \int_C E \cdot dA$$

$$= -EA + EA + 0 = \text{Zero}$$

داخلة

$\theta = 180$



$$\Phi_A = \int E \, dA \cos 180$$

$$= -\int E \, dA$$

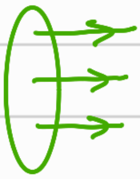
$$= -E \int dA_A$$

$$= -E A_A$$

$$= -E(\pi R^2)$$

خارجة

$\theta = 0$



$$\Phi_B = \int E \, dA \cos 0$$

$$= E \int dA$$

$$= EA$$

$$= E(\pi R^2)$$

مماسة

لا يوجد تدفق

لأن الخطوط

لا تمر

من جانب السطح

$$\Phi_C = \int E \, dA \cos 90$$

$$\Phi_C = 0$$

Sample Problem 23.01
Flux through a closed cylinder, uniform field

- The flux through the circular end a is

$$\int_a \vec{E}.d\vec{A} = \int E(\cos 180^o)\,dA = -E\int dA = -EA = -E(\pi R^2)$$

- The flux through the circular end c is

$$\int_c \vec{E}.d\vec{A} = \int E(\cos 0^o)\,dA = E\int dA = EA = E(\pi R^2)$$

- The flux through the cylindrical surface b is

$$\int_b \vec{E}.d\vec{A} = \int E(\cos 90^o)\,dA = 0$$

- So, the net flux is

$$\Phi = -EA + 0 + EA = 0$$

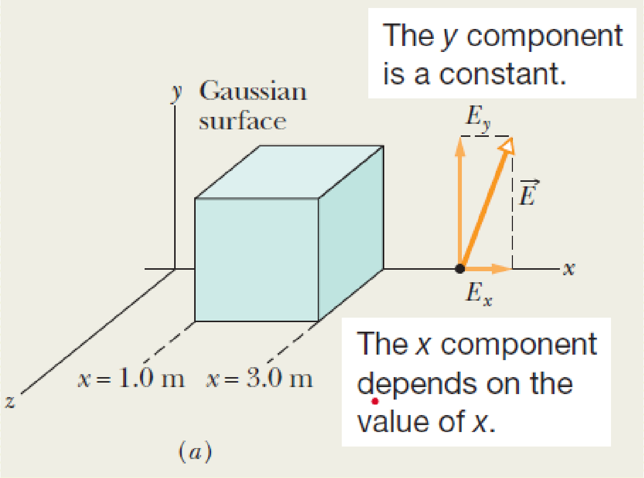
التدفق سيصير
صفر لأن اللي دخل نفس
اللي خرج.

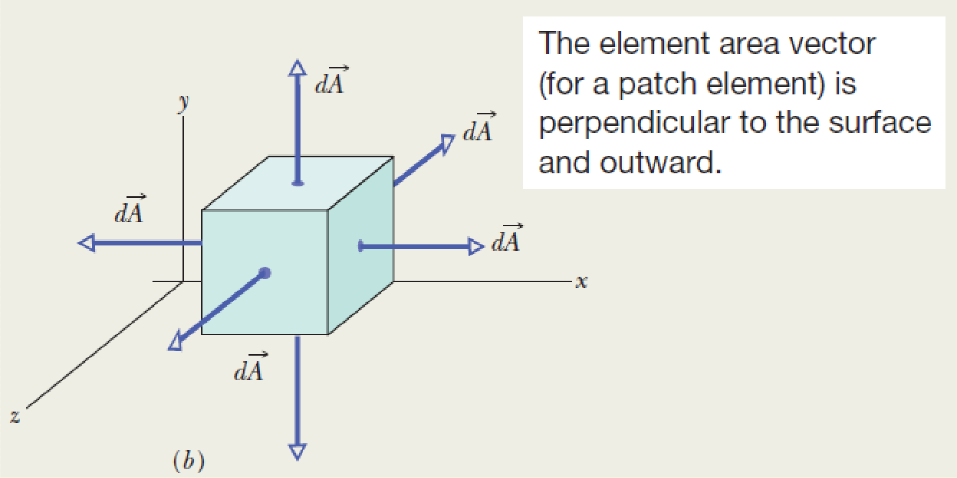
- The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

Sample Problem 23.02

Flux through a closed cube, nonuniform field

- A nonuniform electric field given by $\vec{E} = 3.0x\,\hat{\imath} + 4.0\,\hat{\jmath}$ pierces the Gaussian cube shown in the following Figure. (E is in newtons per coulomb and x is in meters). What is the electric flux through the right face, the left face, and the top face?





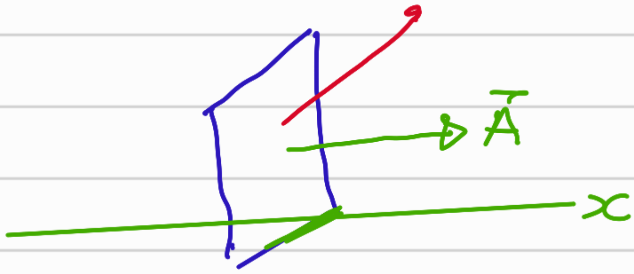
- **Right face**: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any patch element on the right face of the cube must point in the positive direction of the x- axis.

$$d\vec{A} = dA\,\hat{\imath}$$

المطلوب حساب التدفق عبر السطح الايمن و الايسر
و العلوي

مساحه كل سطح = $A = 4m$

المجال الكهربائي $\vec{E} = 3x\hat{i} + 4\hat{j}$



<u>السطح الايمن</u>

$d\vec{A} = dA\hat{i}$ $\qquad E = \underline{3x\hat{i}} + \cancel{4\hat{j}} \qquad A = \underline{4\hat{i}}$

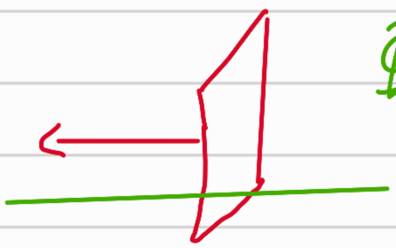
$$\Phi = \int E \cdot dA$$

$$= \int (3x\hat{i} + 4\hat{j}) \cdot (dA\hat{i})$$

$$= \int 3x\, dA$$

$$= 3(3) \int dA$$

$$= 3(3)4 = 36 \ N\,m^2/C$$



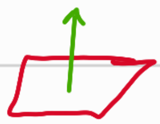
$A = -4\hat{i}$

$\Phi = E \cdot A$

$= (3x\hat{i} + 4\hat{j}) \cdot (-4\hat{i})$

$3x(-4)$

$= -12 \ Nm^2/C$



$A = 4\hat{j}$

$\Phi = E \cdot A = (3x\hat{i} + 4\hat{j}) \cdot (4\hat{j})$

$16 \ N/m^2$

Sample Problem 23.02

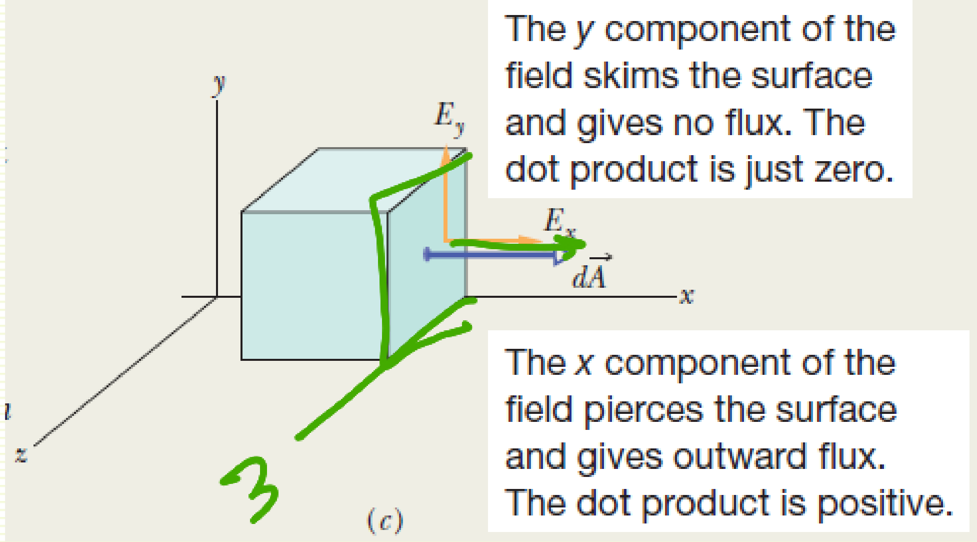
Flux through a closed cube, nonuniform field

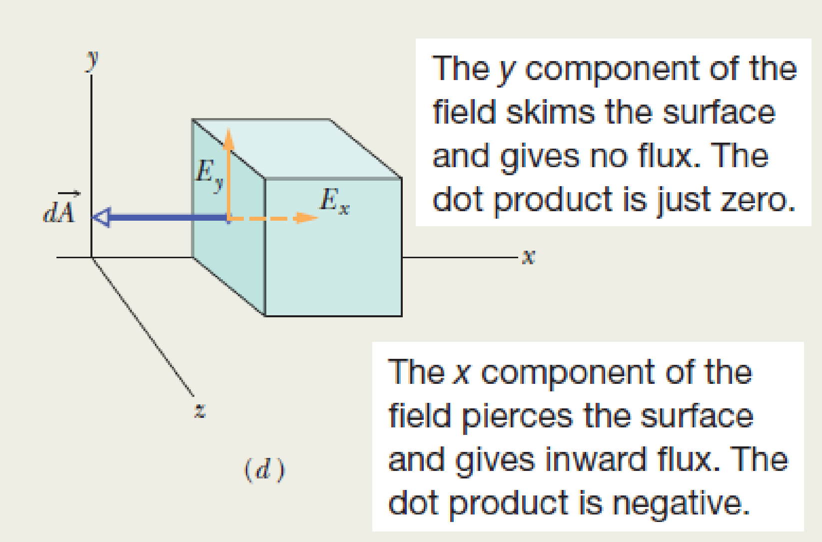
- So, the flux Φ_r through the right face is

$$\Phi_r = \int \vec{E}.d\vec{A} = \int (\underbrace{3.0\,x\,\hat{\imath} + 4.0\,\hat{\jmath}}_{E}).(dA\,\hat{\imath}) = \int ((3.0x)(dA)\,\cancel{\hat{\imath}.\hat{\imath}} + \cancel{(4.0)(dA)\hat{\jmath}.\hat{\imath}}) = \int (3.0x\,dA + 0)$$

$$= 3.0 \int x\,dA = 3.0 \int (3.0)\,dA = 9.0\,A = 9.0\,(4.0) = 36\,N.m^2/C$$

- where the are $A = 4.0\,m^2$ of the right face.





Sample Problem 23.02

Flux through a closed cube, nonuniform field

- **Left face:** We repeat this procedure for the left face. However, two factors change.

  (1) The element area vector $d\vec{A}$ points in the negative direction of the x-axis, and thus $d\vec{A} = -\,dA\,\hat{\imath}$.

  (2) On the left face, $x = 1.0\ m$. The flux Φ_l through the left face is

$$\Phi_l = \int \vec{E}.\,d\vec{A} = \int (3.0\,x\,\hat{\imath} + 4.0\,\hat{\jmath}).\,(-dA\,\hat{\imath}) = -\int \left((3.0x)(dA)\,\hat{\imath}.\hat{\imath} + (4.0)\cancel{(dA)}\hat{\jmath}.\hat{\imath}\right)$$

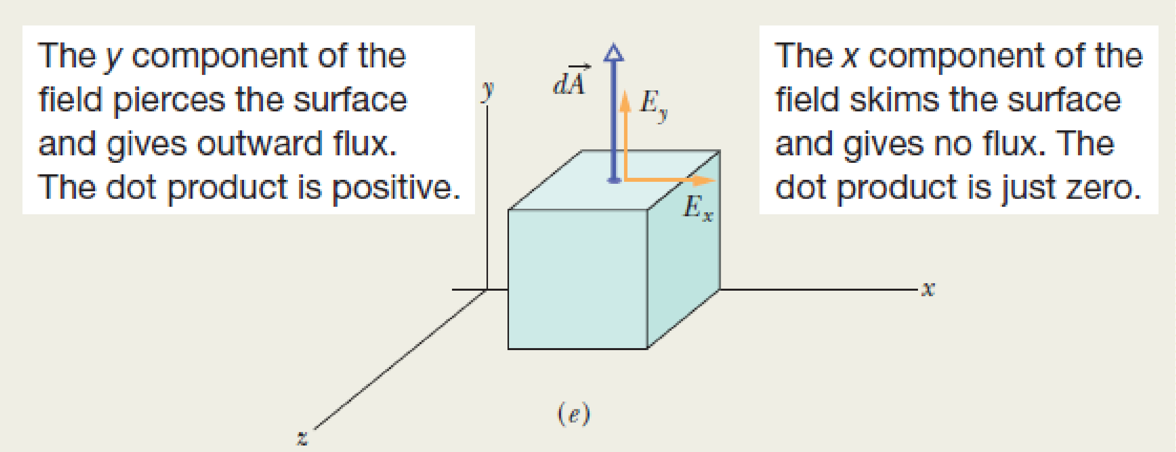
$$= -\int (3.0x\,dA + \cancel{0}) = -3.0\int x\,dA = -3.0\int (1.0)\,dA = -3.0\,A = -3.0\,(4.0) = -12\ N.m^2/C$$

Sample Problem 23.02
Flux through a closed cube, nonuniform field

- **Top face**: $d\vec{A}$ points in the positive direction of the y-axis, so, $d\vec{A} = dA\,\hat{j}$. The flux Φ_t is

$$\Phi_t = \int \vec{E}.d\vec{A} = \int (3.0\,x\,\hat{\imath} + 4.0\,\hat{j}).(dA\,\hat{j}) = \int \big((3.0x)(dA)\,\hat{\imath}.\hat{j} + (4.0)(dA)\hat{j}.\hat{j}\big) = \int (0 + 4.0\,dA)$$

$$= 4.0 \int dA = 4.0\,(4.0) = 16\,N.m^2/C$$



Gauss’ Law

- Gauss’ law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is enclosed by that surface.

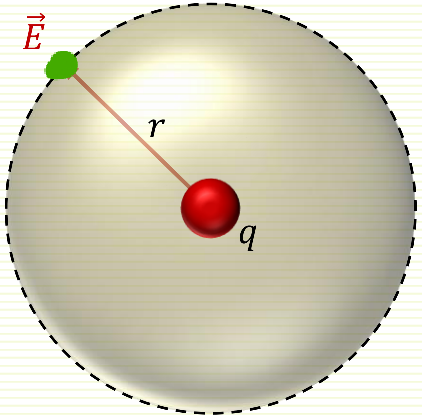
الشحنه الداخليه

$$\varepsilon_o \Phi = q_{enc} \qquad \text{(Gauss'law)}$$

- From the definition of flux, we can also write Gauss’ law as

$$\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enc} \qquad \text{(Gauss'law)}$$

- The net charge q_{enc} is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero. We include the sign, rather than just use the magnitude of the enclosed charge, because the sign tells us: if q_{enc} is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward.



$\vec{E}$. at a distance r from point charge q is

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

The electric flux through a sphere of radius r is given by

$$\Phi = \int \vec{E}.d\vec{A} = \int \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}.dA$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\int dA = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}(4\pi r^2)$$

$$= \frac{q}{\varepsilon_o} \Rightarrow$$

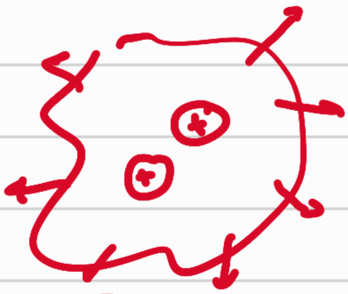
$$\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enc} \quad \text{(Gauss'law)}$$

قانون غاوس

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$q_{enc} = 0$

$\Phi = 0$



$$\Phi = \frac{\Sigma q_{en}}{\varepsilon_0}$$

خطوات استخدام قانون غاوس

① اختيار سطح غاوسي متناظر مع للجسم المشحون بحيث تكون النقطة المراد حساب المجال عندها على السطح

② تطبيق القانون

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon}$$

سلك $dq = \lambda \, dL$

صفيحة $dq = \sigma \, dA$

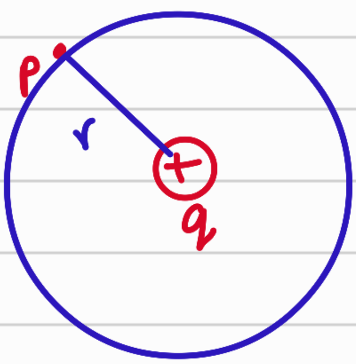
مجسم $dq = \rho \, dV$

③ نكامل مساحة السطح الغاوسي

④ نحسب بدلالة E

$$E = \frac{1}{4\pi\varepsilon} \frac{q}{r^2}$$

استخدم قانون غاوس لاشتقاق علاقة حساب المجال الناتج عن شحنة نقطية



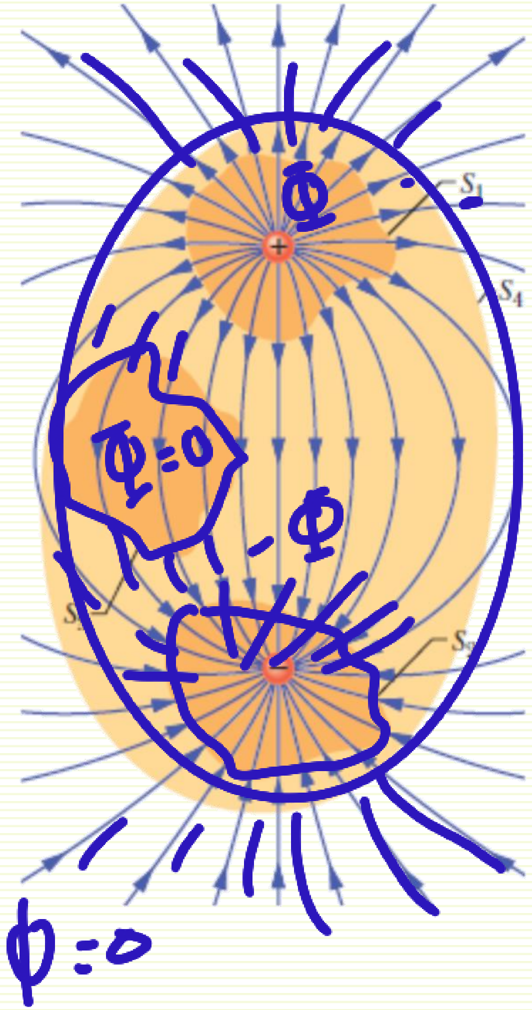
$$\oint E \, dA = \frac{q}{\varepsilon_0}$$

$$E \oint dA = \frac{q}{\varepsilon_0}$$

$$E \, 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad E = \frac{q}{4\pi\varepsilon r^2}$$

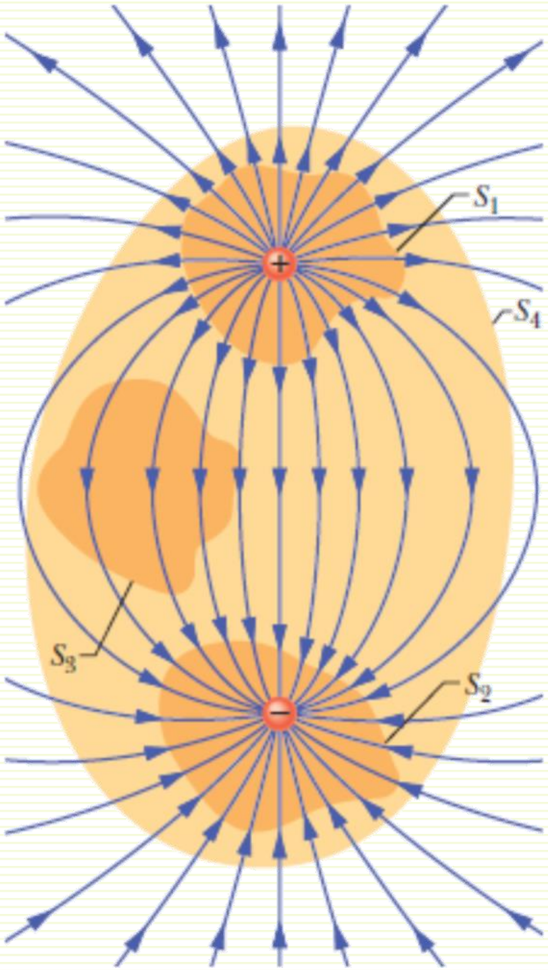
Gauss’ law

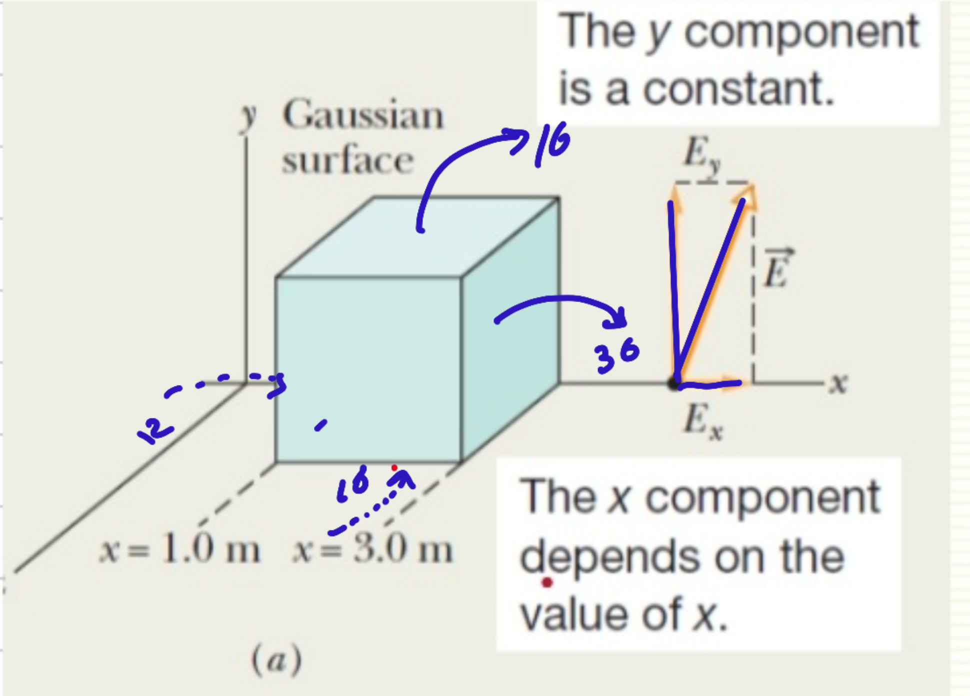
- In applying Gauss’ law, we draw a suitable surface (called Gaussian surface) surrounding the charge, then we begin to apply Gauss’ law to calculate the electric field.
- The electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface because as many field lines due to that charge enter the surface as leave it.
- Let us apply these ideas to the following Figure, which shows two particles, with charges equal in magnitude but opposite in sign, and the field lines describing the electric fields the particles set up in the surrounding space.
- Four Gaussian surfaces are also shown in cross section.



Gauss' law

- **Surface S1**. The electric field is outward for all points on this surface. So, the flux of the electric field through this surface is positive, and so is the net charge within the surface is positive (q_{enc}), as Gauss' law requires.
- **Surface S2**. The electric field is inward for all points on this surface. So, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.
- **Surface S3**. this surface encloses no charge and thus $q_{enc} = 0$. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.
- **Surface S4**. This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S4 as entering it.





Φ الايمن $= 36 \ N m^2/C$

Φ الايسر $= -12 \ N m^2/C$

Φ العلوي $= 16 N m^2/C$

Φ الامامي $= 0$

Φ الخلفي $= 0$

Φ الاسفل $= \oint E \cdot dA = E \cdot A$

$= (3x i + 4j) \cdot -4j$

$= -16 \ N m^2/C$

$\Phi = 36 - 12 + 16 - 16 + 0 + 0 = 24 \ N m^2/C$

ما مقدار الشحنة الداخلية في المكعب

$q_{enc} = \Phi \varepsilon$

$= 24 \times 8.85 \times 10^{-12} = 2.1 \times 10^{-12} \ C$

Sample Problem 23.03
Using Gauss' law to find the electric field

- What is the net charge enclosed by the Gaussian cube in Example 2?
- Solution:
- We need to know the flux through all six faces of the cube. We know the flux through the right face ($\Phi_r = 36\ N.m^2/C$), the left face ($\Phi_l = -12\ N.m^2/C$), and the top face ($\Phi_t = 16\ N.m^2/C$).
- For the bottom face, our calculation is like that of the top face except that the element area vector is directed downward along the y-axis, the flux is ($\Phi_b = -16\ N.m^2/C$).

Sample Problem 23.03

Using Gauss' law to find the electric field

- For the front face we have $d\vec{A} = dA\,\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\,\hat{\imath} + 4.0\,\hat{\jmath}$ with either of these expressions for $d\vec{A}$, so there is no flux through those faces.

- The total flux through the six sides of the cube is

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\ N.m^2/C \ = 24\ N.m^2/C$$

- The enclosed charge is:

الشحنة
الداخليه

$$q_{enc} = \varepsilon_o \Phi = \left(8.85 \times 10^{-12}\ C^2/Nm^2\right)\left(24\ N.m^2/C\right) = 2.1 \times 10^{-12} C$$

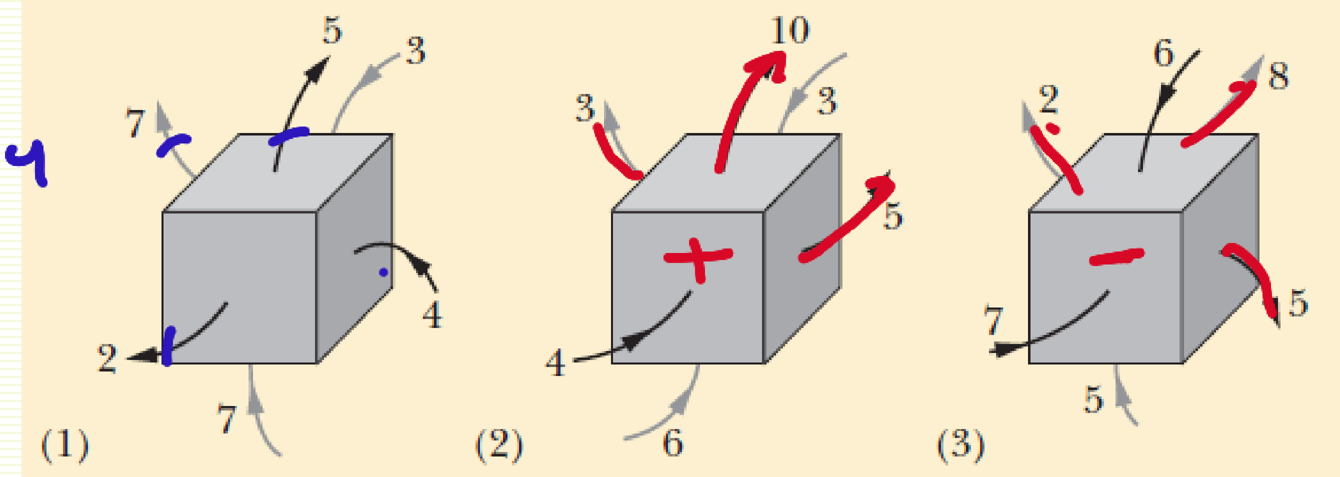
Sample Problem 23.04

Using Gauss' law to find the enclosed charge

- The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $N.m^2/C$) of the flux through the six sides of each cube. (the lighter arrows are for the hidden faces). In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?

$5+7+2-3-7-4$

$= 0$



$2+8+5-7-6-5$

$15-18$

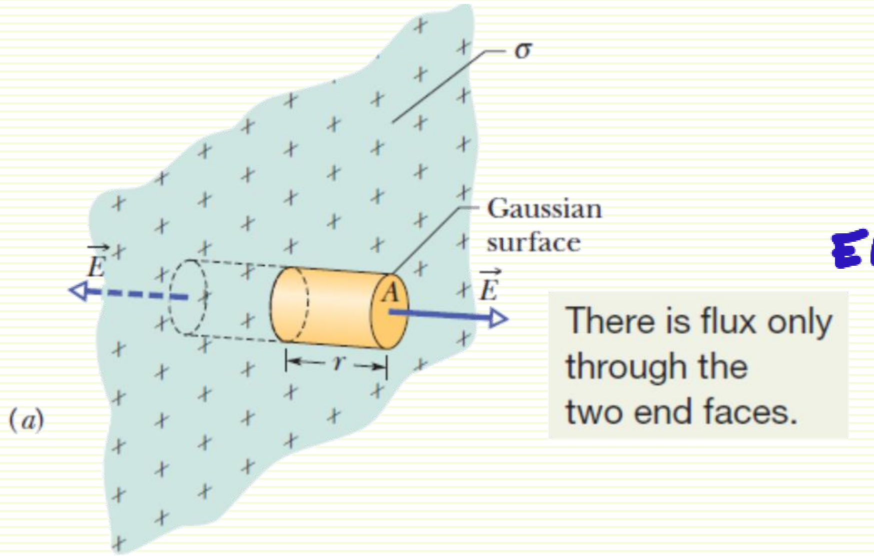
-3

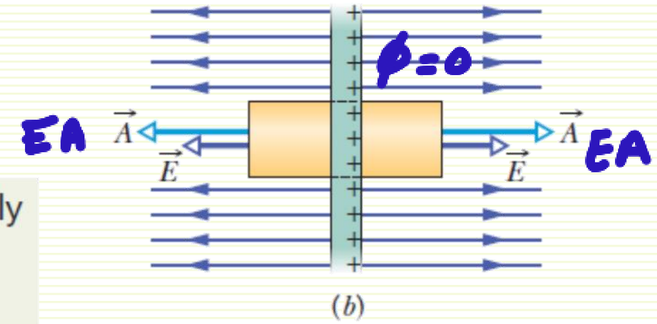
$3+10+5-4-6-3$

$+5$

Applying Gauss' Law: Planar Symmetry

- (1) Nonconducting Sheet:
- A portion of a thin, infinite, nonconduction sheet with a uniform (positive) surface charge density σ. A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field $\vec{E}$ a distance r in front of the sheet.
- A useful Gaussian surface is a closed cylinder with end caps of area A, arranged to pierce the sheet and hence to the end caps.

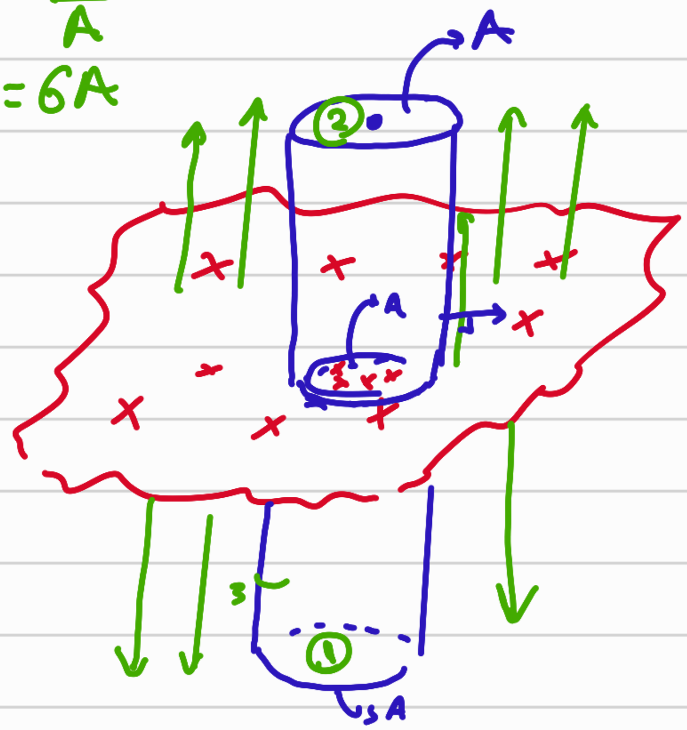




احسب المجال الكهربائي الناتج عن صفيحة واسعة غير موصلة لانهائية عليها كثافة شحنة مقدارها σ

$\sigma = \frac{q}{A}$

$q = \sigma A$



✓ اختيار سطح غاوس مناسب

✓ نطبق قانون غاوس

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon}$$

$$\oint E \cdot dA = \frac{\sigma A}{\varepsilon_0}$$

$$EA + EA = \frac{\sigma A}{\varepsilon}$$

$$2EA = \frac{\sigma A}{\varepsilon}$$

$$E = \frac{\sigma \cancel{A}}{2\cancel{A}\varepsilon}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon}}$$

قانون المجال الناتج عن الصفيحة

نقسم المساحة 3 قطع

① $EA\cancel{\cos 0}$ 1

② EA

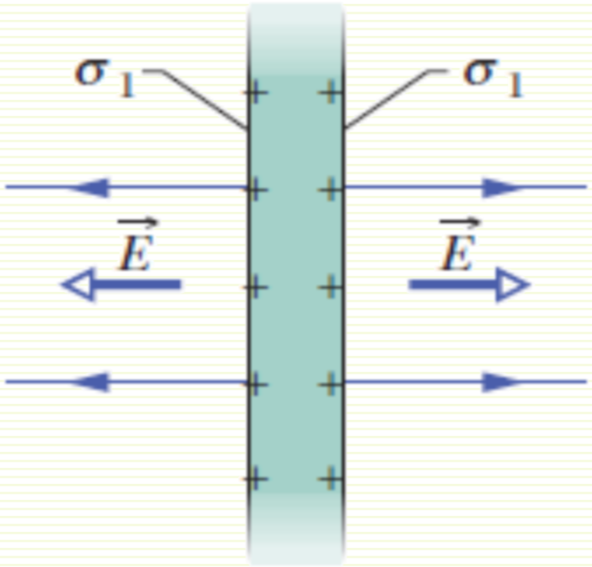
③ $\Phi = 0 \quad E \cdot A_3 = 0$


Applying Gauss' Law: Planar Symmetry

- From symmetry, $\vec{E}$ must be perpendicular to the sheet and hence to the end caps. Since the charge is positive, $\vec{E}$ is directed away from the sheet, and thus the electric field lines pierce the two Gaussian end caps in an outward direction. Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface.
- Thus,
- $\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enc}$
- $\varepsilon_o(EA + EA) = \sigma A$
- $2\varepsilon_o EA = \sigma A$
- The electric field is
- $E = \frac{\sigma}{2\varepsilon_o}$

Applying Gauss' Law: Planar Symmetry

- (2) Two Conducting Plates الصفيحة كطلة
- A cross section of a thin, infinite conducting plate with excess positive charge is shown in following Figure (a). We know that this excess charge lie on the surface of the plate. Since the plate is thin and very large, we can assume that essentially all the excess charge is on the two large faces of the plate.
- If there is no external field to force the positive charge into some particular distribution, it will spread out on the two faces with a uniform surface charge density of magnitude σ_1.
- We know that just outside the plate this charge sets up an electric field of magnitude $E_1 = \sigma_1/\varepsilon_o$. Because the excess charge is positive, the field is directed away from the plate.



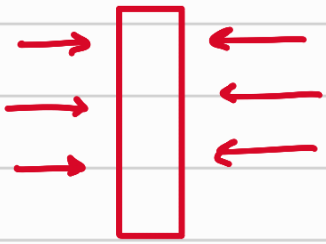
(a)

الصفيحة الموصلة

في حال عدم وجود مجال مؤثر خارجي فان الشحنات الاضافية سوف تتوزع على كلا السطحين

$$E = \frac{\sigma}{2\varepsilon}$$

سوف تتحمل كل سطح مجال كهربائي مقود اذا شكونه الصف الحالة التي تكون فيها الصفيحة معزولة

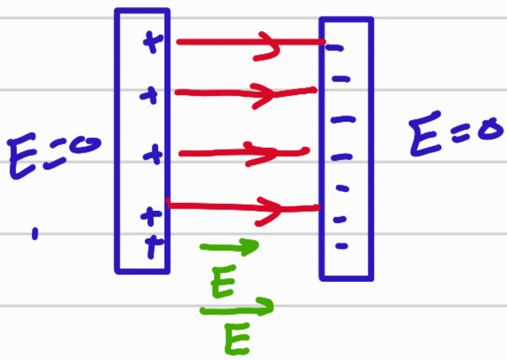


$$E = 2\frac{\sigma}{2\varepsilon}$$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$ *

اعتبار انه تأثر الصفيحة موصلة

صفيحتين متوازيتين



$$E = \frac{\sigma}{\varepsilon} + \frac{\sigma}{\varepsilon}$$

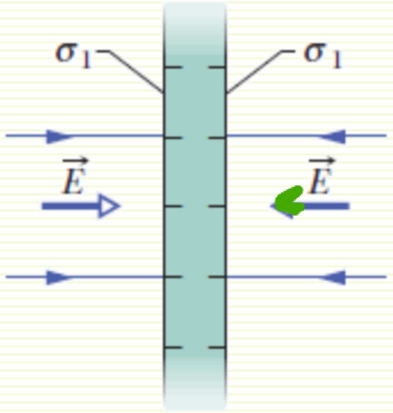
$$\boxed{E = \frac{2\sigma}{\varepsilon}}$$ *

الصفر

Applying Gauss' Law: Planar Symmetry

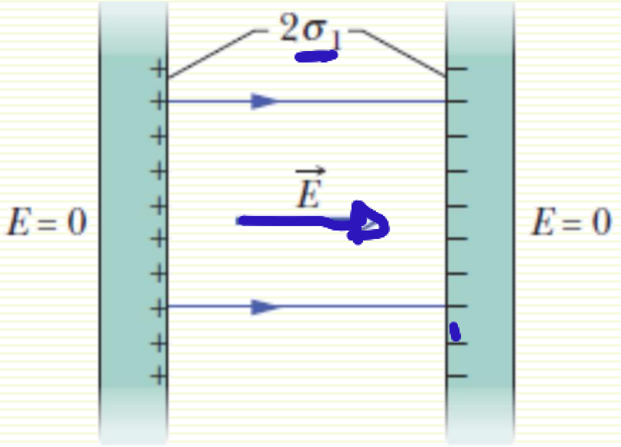
- Figure (b) shows an identical plate with excess negative charge having the same magnitude of surface charge density σ_1. The only difference is that now the electric field is directed toward the plate.

$E = \frac{\sigma}{\varepsilon}$



(b)

- Suppose we arrange for the plates of Figures (a) and (b) to be close to each other and parallel (Figure c). Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates as in Figure (c).



(c)

Applying Gauss' Law: Planar Symmetry

- With twice as much charge now on each inner face, the new surface charge density (σ) on each inner face is twice σ_1. Thus, the electric field at any point between the plates has the magnitude

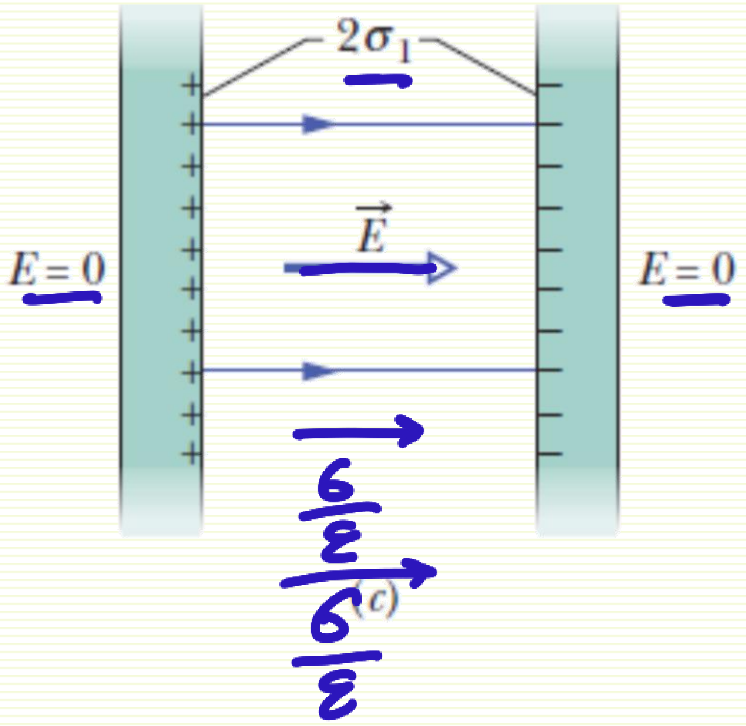
$\sigma = 2\sigma_1$

كثافة جديدة

قديمة

$$E = \frac{2\sigma_1}{\varepsilon_o} = \frac{\sigma}{\varepsilon_o};$$

- This field is directed away from the positively charged plate and toward the negatively charged plate. Since no excess charge is left on the outer faces, the electric field to the left and right of the plates is zero.



Sample Problem 23.06

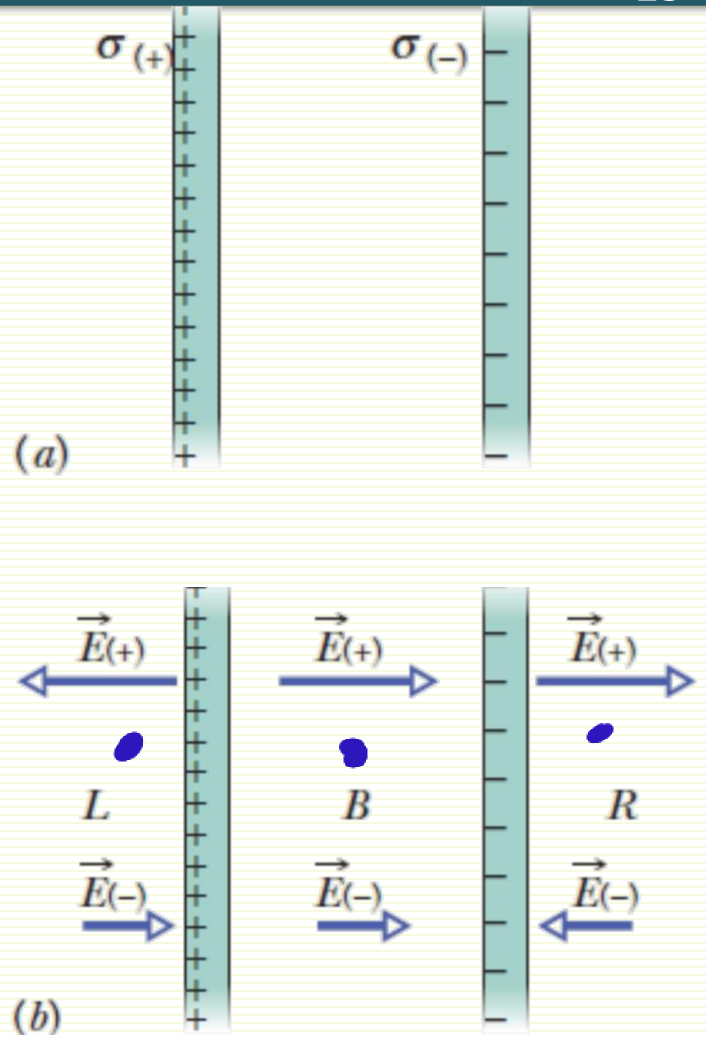
Gauss’ law and an upward streamer in a lightning storm

- Portions of two large, parallel, non conducting sheets, each with a fixed uniform charge on one side are shown in the following Figure. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu C/m^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu C/m^2$ for the negatively charged sheet.
- Find the electric field $\vec{E}$ (a) to the left of the sheets (b) between the sheets, and (c) to the right of the sheets.
- Solution:
- At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed away from the sheet and has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_o} = \frac{6.8 \times 10^{-6}\ C/m^2}{(2)(8.85 \times 10^{-12} C^2/Nm^2)} = 3.84 \times 10^5 N/C$$

- Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed toward that sheet and has the magnitude

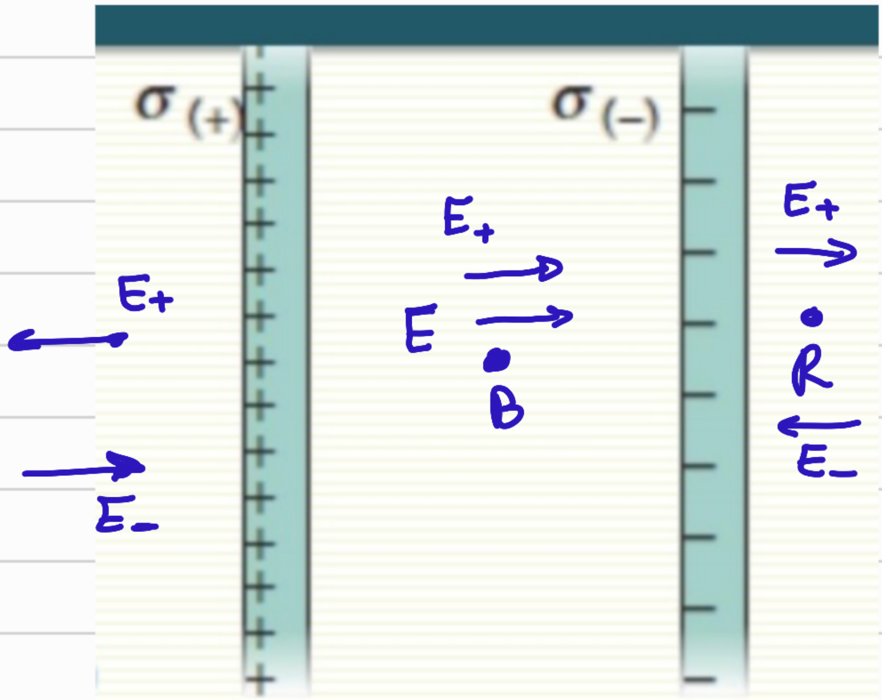
$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_o} = \frac{4.3 \times 10^{-6}\ C/m^2}{(2)(8.85 \times 10^{-12} C^2/Nm^2)} = 2.43 \times 10^5 N/C$$



$$E_+ = \frac{\sigma}{2\varepsilon}$$

$$= \frac{6.8\times10^{-6}}{2\times 8.85\times10^{-12}}$$

$$= 3.84\times10^{5}\ N/C$$



$$E_2 = \frac{4.3\times10^{-6}}{2\times8.85\times10^{-12}}$$

$$= 2.43\times10^{5}\ N/C$$

في الوسط

$$E_B = E_+ + E_-$$

$$E_B = 3.84\times10^{5} + 2.43\times10^{5}$$

$$= 6.3\times10^{5}\ N/C$$

يمين

على اليمين

$$E_R = E_L = (3.84 - 2.43)\times10^{5}$$

$$= 1.4\times10^{5}\ N/C$$

$E_R =$ Right $\qquad E_L =$ left

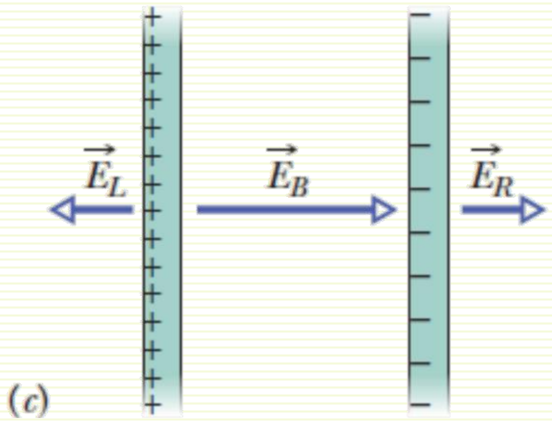
Sample Problem 23.06

Gauss' law and an upward streamer in a lightning storm

- Figure (b) shows that the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).
- The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$E_L = E_{(+)} - E_{(-)} = 3.84 \times 10^5 N/C - 2.43 \times 10^5 N/C = 1.4 \times 10^5\ N/C$$

- Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as in Figure (c) shows. To the right of the sheets, the net electric field has the same magnitude but is directed to the right, as in Figure (c) shows. Between the sheets, the two fields add and we have



$$E_B = E_{(+)} + E_{(-)} = 3.84 \times 10^5\ N/C + 2.43 \times 10^5 N/C = 6.3 \times 10^5\ N/C$$

- The electric field $\vec{E}_B$ is directed to the right.

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineers

Code: PHY1119-4

Ch 24

Electric Potential

V

كمية قياسية

فولت

Physics Department

College of Science

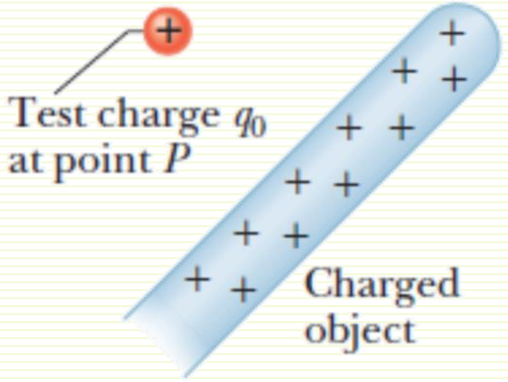
Contents

طاقة الجهد الكهربائية U

Electric Potential and Electric Potential Energy

الشغل اللازم لنقل شحنة (1 كولوم) من (∞) الى الموضع المطلوب ضد المجال

- The electric potential V, at a point, is the work done required to move a unit charge from infinity (where the potential is zero) to that point against the field.
- In Figure (a), we want to find the potential energy U associated with a positive test charge q_o located at point P in the electric field of a charged rod.
- First, we need a reference configuration for which $U = 0$. A reasonable choice is for the test charge to be infinitely far from the rod, because then there is no interaction with the rod.
- Next, we bring the test charge in from infinity to point P to form the configuration in Figure (a). We calculate the work done by the electric force on the test charge.
- The potential energy of the final configuration is given by
- $U = -W$ (potential energy)
- where W is the work done by the electric force. Let's use the notation W_∞ to emphasize that the test charge is brought in from infinity.
- The work and the potential energy can be positive or negative depending on the sign of the rod's charge.



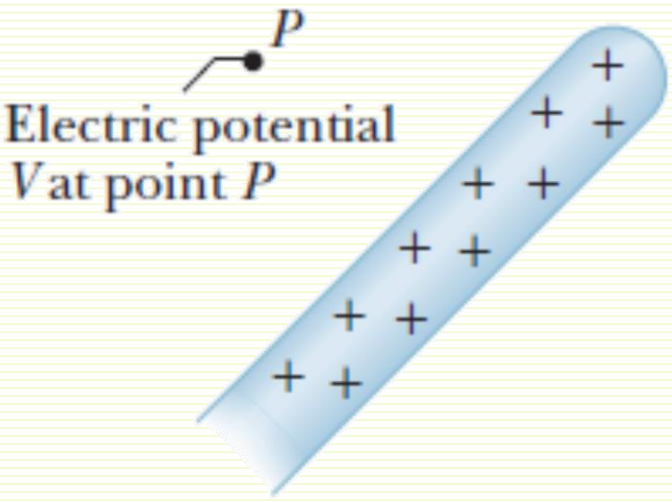
The rod sets up an electric potential, which determines the potential energy.

(a)

الشغل المبذول = - الطاقة المخزنة

Electric Potential and Electric Potential Energy

- Next, we define the electric potential V at P in terms of the work done by the electric force and the resulting potential energy:
- $\boxed{V = -\frac{W_\infty}{q_o} = \frac{U}{q_o}}$ (electric potential)
- That is, the electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity.
- The rod sets up this potential V at P regardless of whether the test charge happens to be there (Figure b).
- The potential V is a scalar quantity (because there is no direction associated with potential energy or charge) and can be positive or negative (because potential energy and charge have signs).



(b)

$V = \frac{\text{شغل}}{\text{الشحنة}}$

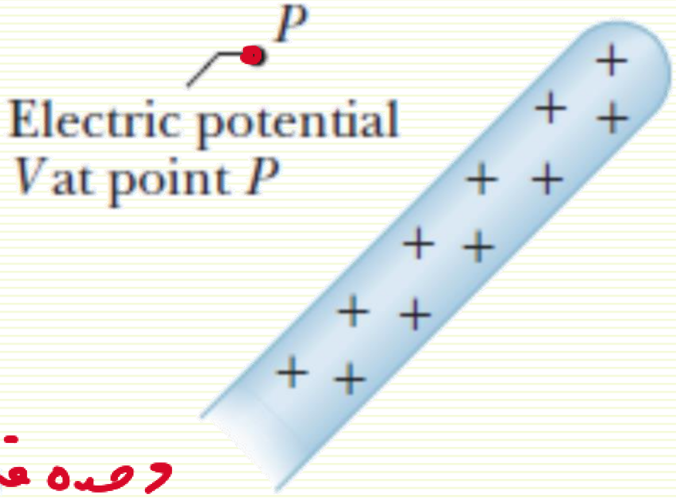
$V = \frac{-W}{q} = \frac{U}{q}$

Electric Potential and Electric Potential Energy

- electric potential energy = (particle′s charge)(electric potential)
- $U = qV$

الطاقة الكهربائية = الشحنة x الجهد

- where q can be positive or negative.



(b)

- Units: The SI unit for potential is *Joule/coulomb*, or *volt*
- $1\ volt = 1\ Joule/coulomb$, or

وحدة قياس الجهد = جول/كولوم = فولت

- $1\ V = \frac{J}{C} = \frac{N.m}{C}$

- Since $1\ N = J/m$, we can now switch the unit for electric field from newtons per coulomb (N/C) to volt per meter (V/m)

$V = \frac{U}{q}$

- $\frac{N}{C} = \frac{J/m}{C} = \frac{J}{Cm} = \frac{J/C}{m} = \frac{V}{m}$

$U = Vq$

ممكن استخدام وحده N/C بدلا من V/m كما بالمجال الكهربائي

ضروري جداً نعوض بالشحنه

Electric Potential and Electric Potential Energy

Motion Through an Electric Field

- (1) Change in Electrical Potential: if we move from an initial point i to a final point f in the electric field of a charged object, the electric potential changes by
- $\Delta V = V_f - V_i$

فرق الجهد ΔV = الجهد النهائي - الجهد الابتدائي

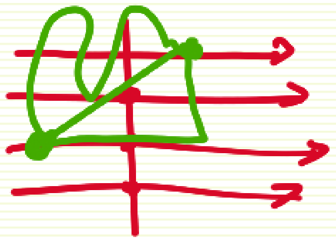
- If we move a particle with charge q from i to f, the potential energy of the system changes by
- $\Delta U = q\,\Delta V = q(V_f - V_i)$

$q\Delta V = qV_f - qV_i = \Delta U$

- The change can be positive or negative, depending on the signs of q and ΔV. It can also be zero, if there is no change in potential from i to f (the points have the same value of potential).
- Because the electric force is conservative, the change in potential energy ΔU between i and f is the same for all paths between those points (it is *path independent*).

فرق الجهد (فرق الطاقة لا يعتمد على المسار)

ΔV ، ΔU لجميع المسارات هو نفسه



Electric potential V (volt)

Potential energy U (Joul)

الجهد و الطاقة عند $\infty =$ صفر

$W_\infty =$ الشغل المبذول لنقل شحنة من ∞ الى p

$-W = U$

الشغل والطاقة كمية قياسية يمكن ان تكون موجب او سالب

$$V = \frac{-W_\infty}{q} = \frac{U}{q}$$

V كمية قياسية ليس لها اتجاه ونعوض فيها اشارة الشحنة

$1V = 1J/C$

$$U = Vq$$

الجهد $V =$ فولت $=$ جول/كولوم

* اثبت ان المجال الكهربائي E يمكن حسابه باحدى الوحدتين

$$V/m = N/C$$

$$\frac{N}{C} = \frac{J/m}{C} = \frac{J}{mC} = \frac{V}{m}$$

فرق الجهد ΔV

عند نقل شحنة q من مكان الى اخر في مجال كهربائي

$$\Delta V = V_f - V_f$$

$$\Delta U = q(V_f - V_i)$$

الطاقة يمكن ان تكون موجب او سالبه ولا تعتمد على المسار

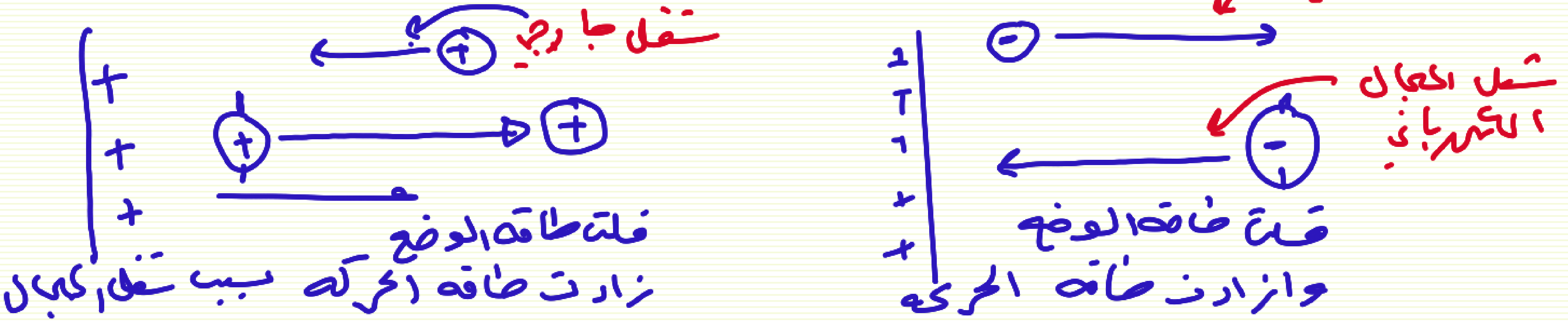
Electric Potential and Electric Potential Energy

Motion Through an Electric Field

- (2) Work by the Field: we can relate the potential energy change ΔU to the work W done by the electric force as the particle moves from i to f by applying the general relation for a conservative force:
- $W = -\Delta U$ (work, conservative force)

W القوة الخارجية $= -W$ المجال

- Next, we can relate that work to the change in the potential
- $W = -\Delta U = -q\,\Delta V = \boxed{-q\,(V_f - V_i)}$
- The W is the work done on the particle by the electric field (because it, of course, produces the force). The work can be positive, negative, or zero. Because ΔU between any two points in path independent, so is the work W done by the field.



Electric Potential and Electric Potential Energy

Motion Through an Electric Field

- (3) Conservation of Energy: If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved. Let us assume that we can assign the electric potential energy to the particle alone. Then we can write the conservation of mechanical energy of the particle that moves from point i to point f as
- $U_i + K_i = U_f + K_f$
- $K_f - K_i = U_i - U_f = -(U_f - U_i)$
- $\Delta K = -\Delta U$
- So, the work done
- $W = \Delta K = -\Delta U = -q\,\Delta V = -q(V_f - V_i)$

$K_i \quad U_i \qquad\qquad K_f \quad U_f$

مجموع طاقة الوضع وطاقة الحركة يبقى
ثابت عند اي نقطة

$K_i + U_i = K_f + U_f$

$-(U_f - U_i) = K_f - K_i$

$-\Delta U = +\Delta K$

Electric Potential and Electric Potential Energy

Motion Through an Electric Field

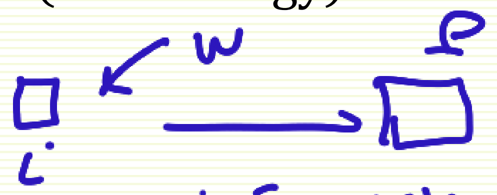
شغل القوة الخارجية

- (4) Work by an Applied Force: If some force in addition to the electric force acts on the particle, we say that the additional force is *an applied force or external force*, which is often attributed to an *external agent*. مساعده خارجيه

قوة خارجية

(initial energy) + (work by applied force) = (final energy)

- $U_i + K_i + W_{app} = U_f + K_f$



- $K_f - K_i = U_i - U_f + W_{app} = -(U_f - U_i) + W_{app}$
- $\Delta K = -\Delta U + W_{app} = -q\,\Delta V + W_{app}$

الطاقة النهائية = الطاقة الابتدائية + الشغل المبذول

- The work by the applied force can be positive, negative, or zero, and thus the energy of the system can increase, decrease, or remain the same.
- In special case, where the particle is stationary before and after the move, the kinetic energy are zero, and we have
- $\boxed{W_{app} = q\,\Delta V}$ (for $K_i = K_f$)

السرعة صفر

السرعة صفر

$\Delta K = 0$

الشغل المبذول من المجال الكهربائي

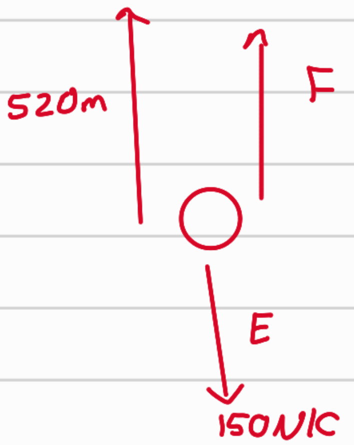
$$W = -\Delta U = -q(V_f - V_i)$$

عندما لا يوجد مؤثر خارجي $\quad \Delta K = -\Delta U = -q(V_f - V_i)$

عندما يوجد مؤثر خارجي $\quad \Delta K = -\Delta U + W_{app}$

$$= -q(V_f - V_i) + W_{app}$$

24.01



$$\Delta U = -W$$

$$W = F \cdot d = F\, d \cos\theta$$

$$= q\underline{E\, d} \cos\theta$$

$\theta = 180$

$E \quad d$

$$= qEd \cos 180 \xrightarrow{} -1$$

$$= -qEd$$

$$= -(1.6\times10^{-19})(150)(520)$$

$$= 1.2\times10^{-14}\, J$$

$$\Delta U = -W = -1.2\times10^{-14}$$

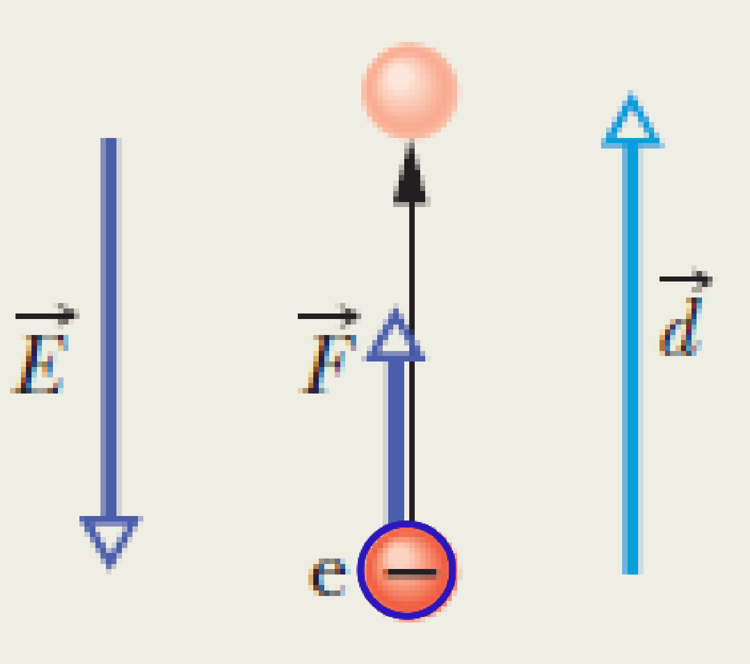
$$\Delta V = \frac{\Delta U}{q} = \frac{1.2\times10^{-14}}{1.6\times10^{-19}} = 7.5\times10^{4}\, V$$

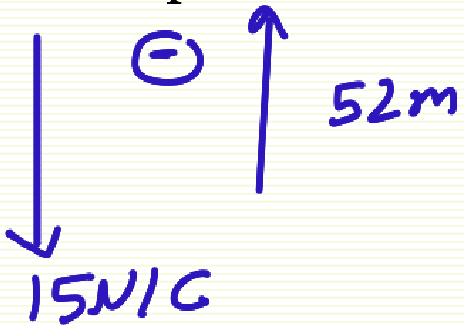
$75\, kV$

Sample Problem 24.01

Work and potential energy in an electric field

- Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once, released, each electron experiences an electric force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150\ N/C$ and is directed downward.
- What is the change in the electric potential energy dU of a released electron when the electric force cause it to move vertically upward through a distance $d = 520\ m$? Through what potential change does the electron move?





Sample Problem 24.01
Work and potential energy in an electric field

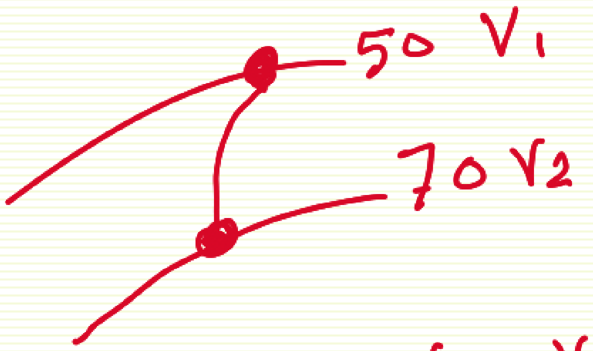
- The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is
- $W = \vec{F}.\vec{d} = \left(q\vec{E}\right).\vec{d} = qEd\cos\theta = (-1.6\times10^{-19}\,C)(150\,N/C)(520\,m)(\cos 180^{o}) = 1.2\times 10^{-14}\,J$
- Where the θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement is directed $\vec{d}$ upward.
- $\Delta U = -W = -1.2\times10^{-14} J$
- This result tell us that during the 520 m ascent, the electric potential energy of the electron decrease by $1.2\times10^{-14}J$. The change in electric potential is
- $\Delta V = \frac{\Delta U}{q} = \frac{-1.2\times10^{-14}\,J}{-1.6\times10^{-19}C} = 7.5\times10^{4}\ V = 75\ kV$
- This tells us that the electric force does work to move the electron to a higher potential.

اذا كانت فرق الجهد موجب معنى ان الشحنة تتحرك نحو الجهد الاعلى

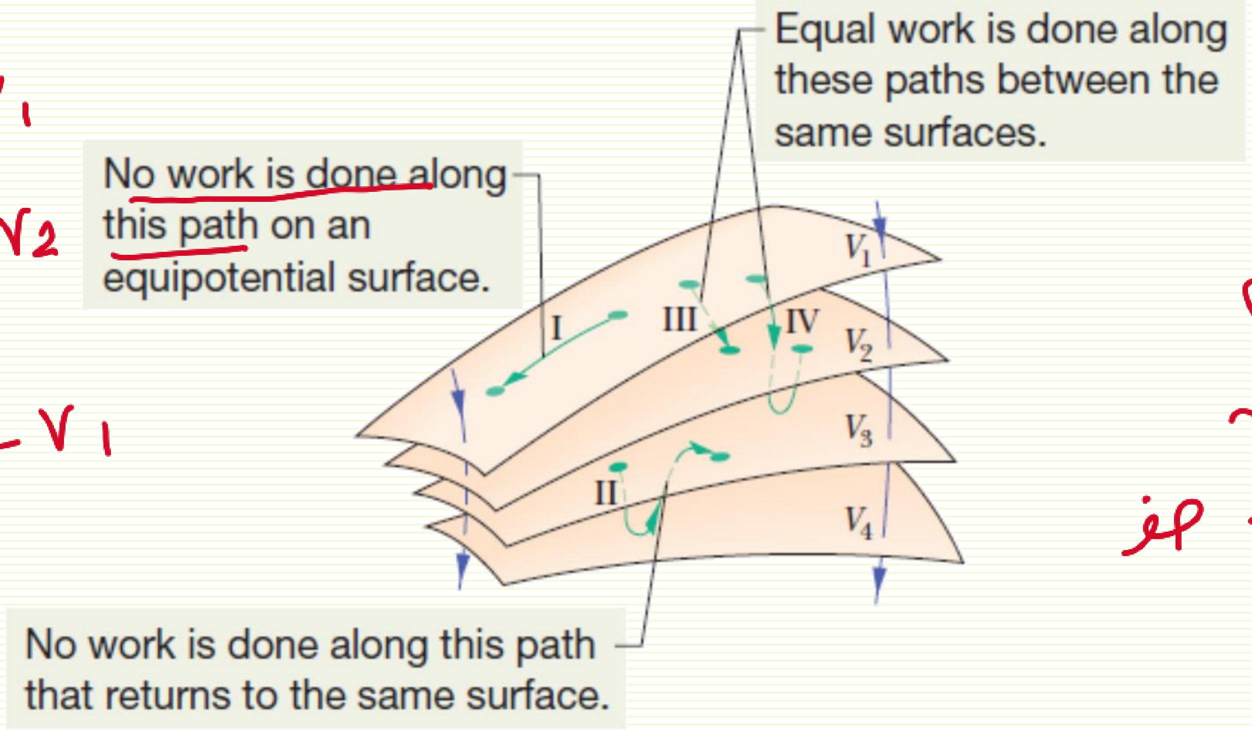
سطوح تساوي

Equipotential Surfaces and The Electric Field

- A adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface.
- No net work W is done on a charge particle by an electric field with the particle moves between two points i and f on the same equipotential surface.



$\Delta V = V_2 - V_1$

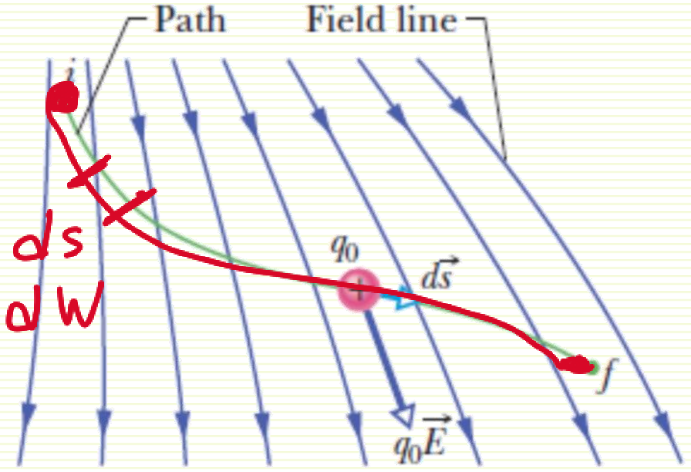


لأن جميع النقاط
لها نفس الجهد
اذا الشغل اللازم
لنقل الشحنة من
نقطة الى اخرى = صفر

Calculating the Potential from the Field

W = قوه x مسافه

- We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector $\vec{E}$ all along any path connecting those points.
- We find the work done on a positive test charge by the field as the charge moves from i to f, and then use the Eq.
- $W = -\Delta U = -q\,\Delta V = -q\left(V_f - V_i\right)$



- Consider an arbitrary electric field, represented by the field lines in the following Figure, and a positive test charge q_o that moves along the path shown from point i to point f. At any point on the path, an electric force $\vec{F}$ acts on the charge as it moves through a differential displacement $d\vec{s}$.

$W = -U = -q(V_f - V_i)$

- $\vec{F} = q_o\,\vec{E}$
- The differential work dW done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is given by

$dW = F \cdot ds$

- $dW = \vec{F}.d\vec{s} = q_o\vec{E}.d\vec{s}$

$W = \int dW = \int_i q\,\vec{E} \cdot \vec{ds}$

Calculating the Potential from the Field

$W = q \int E \cdot ds$

- The total work W done on the particle by the field is

$-q(V_f - V_i) = q \int E \cdot ds$

- $W = \int_i^f dW = q_o \int_i^f \vec{E}.d\vec{s}$

$\boxed{\Delta V = V_f - V_i = -\int E \cdot ds}$

- $\because W = -q_o\left(V_f - V_i\right) = q_o \int_i^f \vec{E}.d\vec{s}$
- $\therefore \left(V_f - V_i\right) = -\int_i^f \vec{E}.d\vec{s}$
- Thus, the potential difference $(V_f - V_i)$ between two points i and f in an electric field is equal to the negative of the line integral of $\vec{E}.d\vec{s}$ from i to f.
- Because the electric force is conservative, so all paths yield the same result.
- We can calculate the difference in potential between any two points in the field, if we set potential $V_i = 0$, then

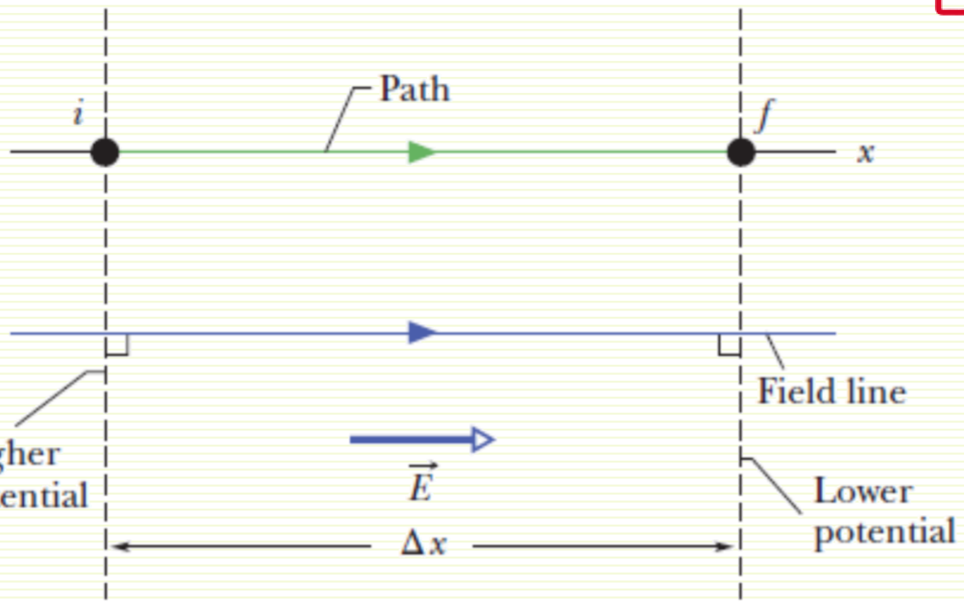
$V = \int \vec{E} \cdot d\vec{s}$

- $V = -\int_i^f \vec{E}.d\vec{s}$
- This equation gives us the potential V at any point f in the electric field relative to the zero potential at point i. If we let point i be at infinity, then this equation gives us the potential V at any point f relative to the zero potential at infinity.

Calculating the Potential from the Field

- Uniform Field: حاله خاصه عندما تكون المجال منتظم
- If we start at point i on an equipotential line with potential V_i and move to point f on an equipotential line with a lower potential V_f. The separation between the two equipotential lines is Δx.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E\, ds \cos\theta = -\int_i^f E\, ds \cos 0 = \boxed{-E \int_i^f ds}$$



$\Delta V = -E \int ds \cancel{\cos\theta}$

نتحرك مع خطوط

$\theta = 0 \quad \cos\theta = 1$

Calculating the Potential from the Field

- The integral is simply an instruction for us to add all the displacement element ds from i to f, it is the length Δx. Thus, we can write the change in potential $V_f - V_i$ in this uniform field as

$$\boxed{\Delta V = (V_f - V_i) = -E\,\Delta x}$$

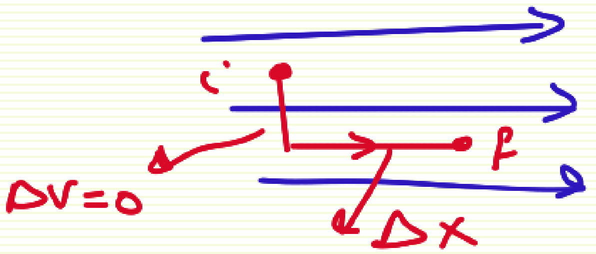
- If we move in the direction of the field by distance Δx, the potential decreases. In the opposite direction, it increases.

اذا تحركنا مع خطوط المجال الجهد يقل
وعكس خطوط المجال الجهد يزداد

- The electric field vector points from higher potential toward lower potential.

$\Delta V = -E\ \Delta x$

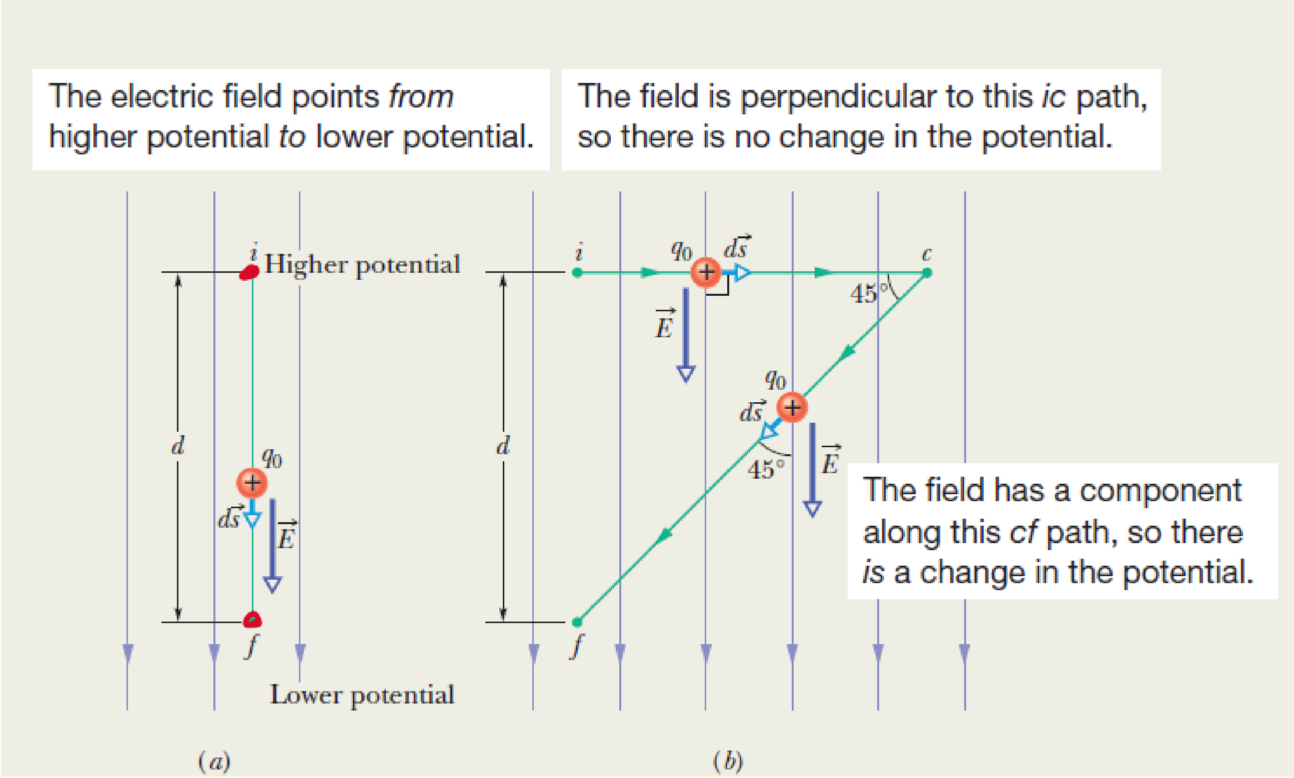
خطوط المجال تتجه من الجهد
العالي الى الجهد المنخفض



Sample Problem 24.02

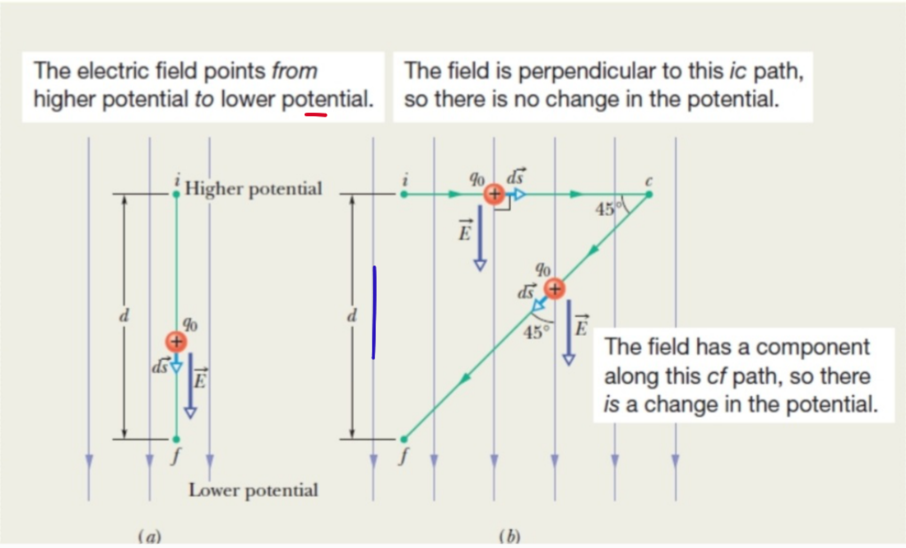
Finding the potential change from the electric field

- Two points i and f in a uniform electric field $\vec{E}$ are shown in the following Figure. The points lie on the same electric field line (not shown) and are separated by a distance d. Find the potential difference $V_f - V_i$ by moving a positive test charge q_o from i to f along the path shown, which is parallel to the field direction.



$\Delta V = -E \Delta x$

مع خطوط ، كبال



a) شحنة موجبة تتحرك مع خطوط المجال

$$\Delta V = -E\,d$$

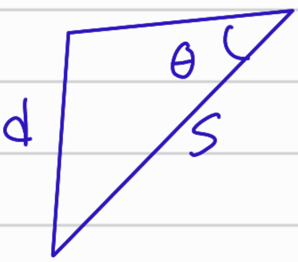
b) نفس الحركة لكن بمسار مختلف

$$\Delta V = \overbrace{\cancel{V_{i \to c}}}^{zero} + V_{c \to f}$$

$$\Delta V = -\int_c^f E\, ds \cos\theta$$

θ بين المجال والمسافة

$$-E\cos 45 \int_c^f ds$$



$$= -E\cos 45 \int_c^f \frac{d}{\sin 45}$$

$$\sin\theta = \frac{d}{s}$$

$$= -E\,\cancel{\frac{\cos 45}{\sin 45}}\,d = -Ed$$

Sample Problem 24.02

Finding the potential change from the electric field

- Solution:
- In Figure (a): The potential difference is

$$V_f - V_i = -E.d = -E\ d$$

- In Figure (b): the potential difference $V_f - V_i$ by moving the positive test charge q_o from i to f along the path icf as shown in Figure (b) is:
- We have to lines ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90^o and the dot product $\vec{E}.d\vec{s} = 0$. So, the points i and c are at the same potential $V_c - V_i = 0$. The points are on the same equipotential surface, which is perpendicular to the electric field lines.
- For line cf, we have $\theta = 45^o$, so

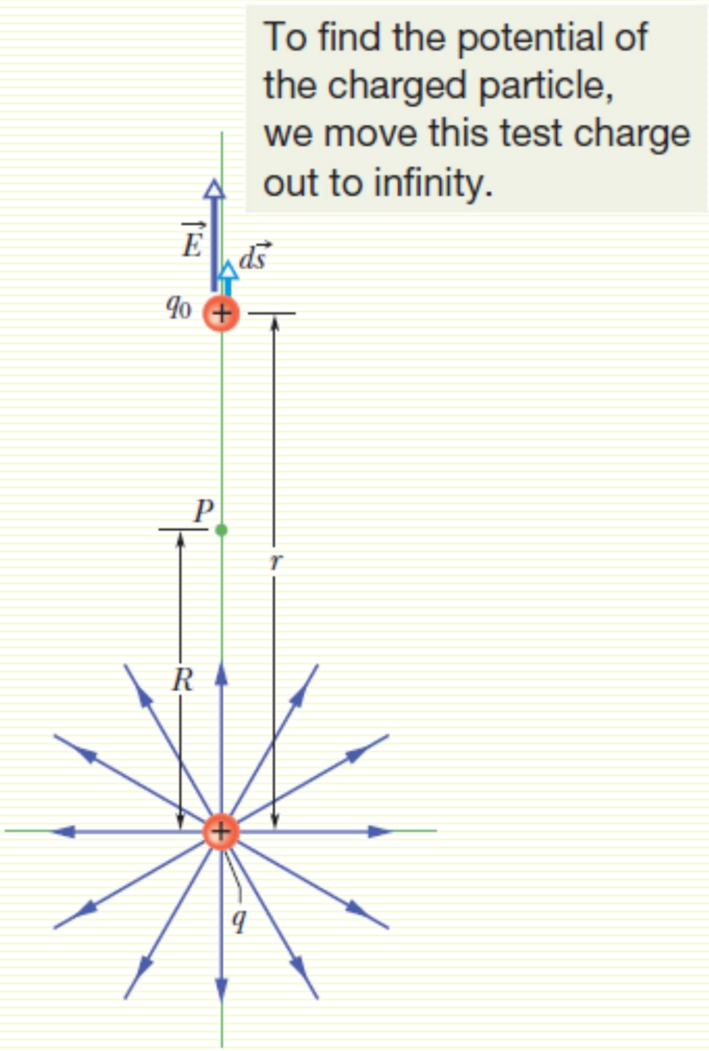
$$V_f - V_i = -\int_c^f \vec{E}.d\vec{s} = -\int_c^f E(\cos 45^o)\,ds = -E(\cos 45^o)\int_c^f ds = -E(\cos 45^o)\frac{d}{\sin 45^o} = -Ed$$

- The integral in the equation is just the length of line c. This is the same result we obtained in (a).

الجهد الكهربائي الناتج نقطية

Potential due to a Charged Particle

- Consider a point P at distance R from a fixed particle of positive charge q, we imagine that we move a positive test charge q_o from point P to infinity. Because the path we take does not matter, we take a line that extends radially from the fixed particle through P to infinity.
- $\vec{E}.d\vec{s} = E\cos\theta\ ds$
- The electric field $\vec{E}$ is directly radially outward from the fixed particle. So, the differential displacement $d\vec{s}$ of the test particle along its path has the same direction as $\vec{E}$.
- Because the path is radial, let us write ds as dr, and substituting the limits R and ∞. So,
- $V_f - V_i = -\int_R^{\infty} E\ dr$
- Next, we set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, so



Potential due to a Charged Particle

$$0 - V = -\int_R^\infty E\,dr = -\frac{q}{4\pi\varepsilon_o}\int_R^\infty \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

- Solving for V and switching R to r, we have the electric potential V due to a particle of charge q at any radial distance r from the particle.

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}}$$

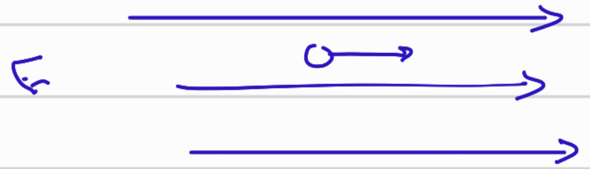
الشحنه موجبه تنتج جهد موجب
والسالبه تنتج جهد سالب

- A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Potential due to a Group of Charged Particles:

- The net electric potential at a point due to a group of charged particles with the help of the superposition principle. We calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. Thus, for n charges, the net potential is

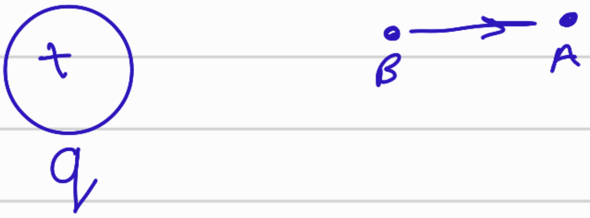
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



$$\Delta V = -E\,\Delta x$$

$$\Delta V = -\int E \cdot ds$$

الجهد الكهربائي الناتج عن شحنة نقطية



$$\Delta V = V_A - V_B$$

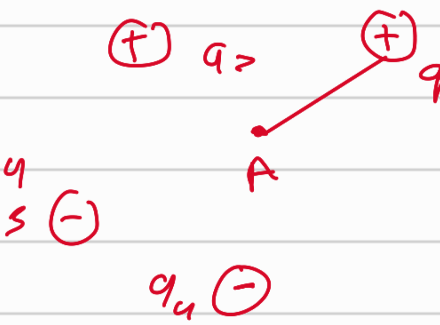
$$\boxed{V_A = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r}}$$

نعوض مع الشحنة سالبة

$$V_A = \frac{1}{4\pi\varepsilon_0}\,\frac{5\times10^{-6}}{2}$$

$$= \frac{9\times10^{9}\times 5\times10^{-6}}{2}$$

$$V = \sum V_i = \frac{1}{4\pi\varepsilon}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r} \cdots\right]$$



Sample Problem 24.03

Net potential of several charged particles

- What is the electric potential at point P, located at the center of the square of charged particles shown in the following Figure?
- The distance d is 1.3 m and the charges are
- $q_1 = +12\ nC,\ \ q_2 = -24\ nC$
- $q_3 = +31\ nC,\ \ q_4 = +17\ nC$
- Solution:
- $V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$
- The sum of the charges is
- $q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\ C = 36 \times 10^{-9}\ C$

القطر =

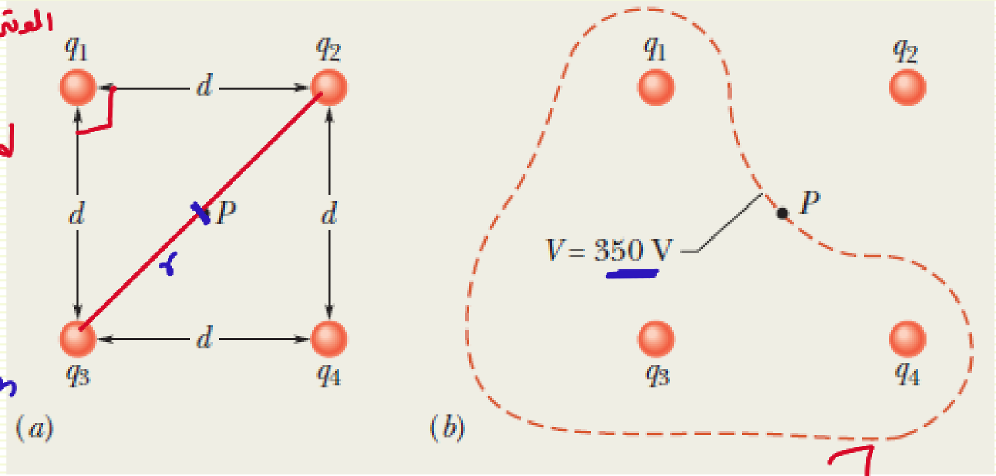
$\sqrt{d^2 + d^2}$

$\sqrt{2d^2} = \sqrt{2}\,d$

$r = \frac{\sqrt{2}}{2} d$

$r = \frac{\sqrt{2}}{2}(1.3)$

$= 0.919\,m$



$V = \frac{1}{4\pi\varepsilon}\left[\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right]$

$V = \frac{1}{4\pi\varepsilon r}\left[12 - 24 + 31 + 17\right] \times 10^{-9}$

$V = \frac{9 \times 10^9}{0.919}\left[12 - 24 + 31 + 17\right] \times 10^{-9} = 350\,V$

Sample Problem 24.03

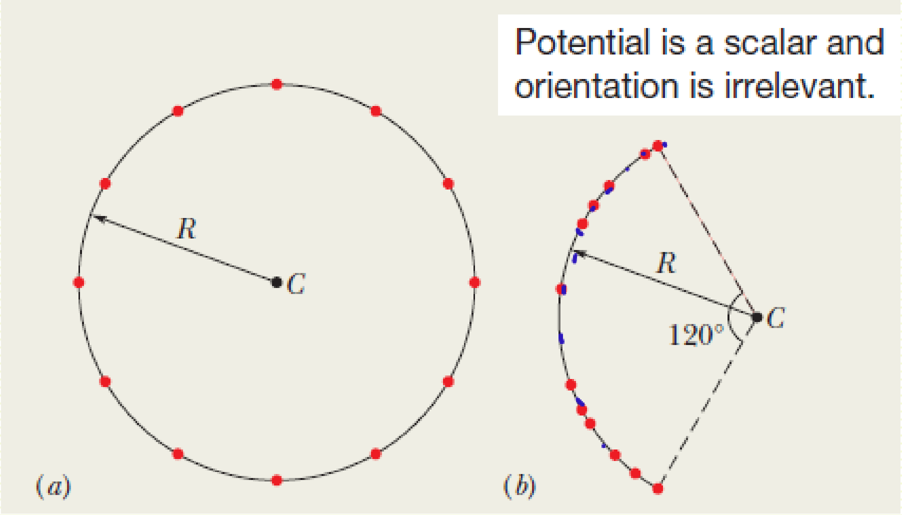
Net potential of several charged particles

- The distance r is
- $(2r)^2 = d^2 + d^2 \quad \Rightarrow \quad 4r^2 = 2d^2$
- $\therefore r = \frac{d}{\sqrt{2}} = \frac{1.3}{\sqrt{2}} = 0.919\ m$
- Thus,
- $V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right) = \frac{(8.99\times10^9\ Nm^2C^{-2})(36\times10^{-9}C)}{0.919\ m} \approx 350\ V$
- Close to any of the three positively charged particles in Figure (a), the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P.
- The curve in Figure (b) shows the intersection of the plane of the figure with the equipotential surface that contains point P

Sample Problem 24.04

Potential is not a vector, orientation is irrelevant

- 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R, as shown in the following Figure. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?
- Solution:
- The electric potential V is the algebraic sum of the electric potentials contributed by all the electrons. Because electric potential is a scalar, the orientations of the electrons do not matter.
- The electric field at C is a vector quantity and thus the orientation of the electrons is important.
- Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, so
- $V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_o}\frac{(-12\,e)}{R}$



المجال الكهربائي في المركز سوف تكون zero لان المجالات متعاكسة تلغي بعض

$V = \frac{1}{4\pi\epsilon}\left[\frac{-12e}{R}\right] = \frac{-12e}{4\pi\epsilon R}$

Sample Problem 24.04

Potential is not a vector, orientation is irrelevant

- Because the symmetry of the arrangement in Figure (a), the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. So, at C
- $\vec{E} = 0$

اذا كانت الالكترونات غير موزعة بشكل منتظم

- If the electrons are moved along the circle until they are nonuniformly spaced over a $120°$ arc (Figure b). At C, find the electric potential and describe the electric field.
- Because the distance between C and each electron is unchanged and orientation is irrelevant, the potential is still given by

لن يتغير الجهد لكن المجال لن يبقى صفر
بعد الان

- $V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_o} \frac{(-12\, e)}{R}$
- The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.
- $\vec{E} \neq 0$

حساب المجال الكهربائي عن الجهد

Calculating the Field From the Potential

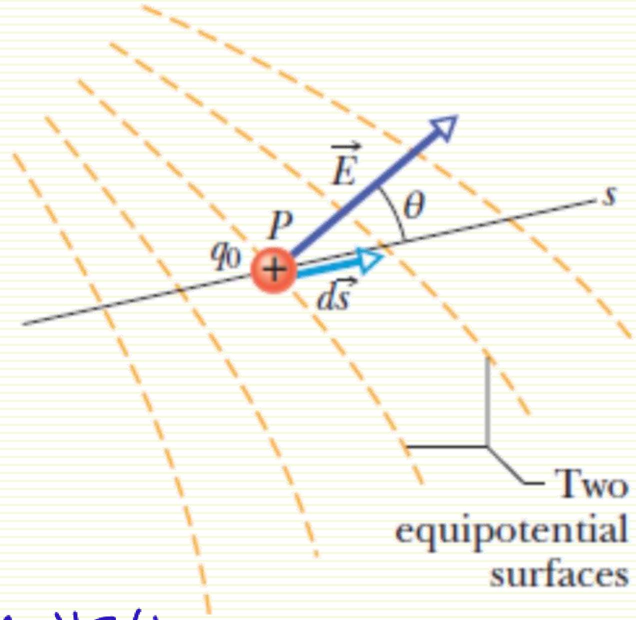
- Suppose that a positive test charge q_o moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface, as shown in the following Figure.
- The work that the electric field does on the test charge during the move is $-q_o dV$. The work done by the electric field may be written as the scalar product

$$\left(q_o \vec{E}\right) . d\vec{s} \quad \text{or} \quad q_o E(\cos\theta) ds$$

- Equating these two expressions for the work yields

$$-q_o dV = q_o E(\cos\theta) ds$$

$$E\cos\theta = -\frac{dV}{ds}$$



- Since $(E\cos\theta)$ is the component of $\vec{E}$ in the direction of $d\vec{s}$, so

$$E_s = -\frac{\partial V}{\partial s}$$

المجال الكهربائي = مشتقة الجهد بالنسبة لـ المسافة

Calculating the Field From the Potential

- The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.
- If we take the s axis to be the x, y, and z axes, we find that the x, y, and z components of $\vec{E}$ at any point are:

$$\boxed{E_x = -\frac{\partial V}{\partial x},} \qquad \boxed{E_y = -\frac{\partial V}{\partial y},} \qquad \boxed{E_z = -\frac{\partial V}{\partial z}}$$

- So, if we know the function $V(x, y, z)$, we can find the components of $\vec{E}$ at any point by taking partial derivatives.

اذا كنا نعرف V يمكننا معرفه مركبات E

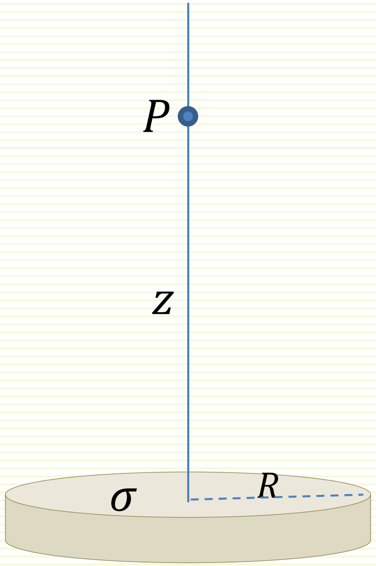
- The component of electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.

المجال الكهربائي تساوي صفر في اي اتجاه سطوح تساوي (لا يوجد
تغير في الجهد

Sample Problem 24.05
Finding the field from the potential

- The electrical potential at any point on the central axis of a uniformly charged disk is given by

$$V = \frac{\sigma}{2\varepsilon_o}\left(\sqrt{z^2 + R^2} - z\right)$$



- Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.
- **Solution:**
- We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z.

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{\sigma}{2\varepsilon_o}\left(\sqrt{z^2 - R^2} - z\right)\right) = -\frac{\sigma}{2\varepsilon_o}\frac{\partial}{\partial z}\left(\left(z^2 - R^2\right)^{1/2} - z\right)$$

$$= -\frac{\sigma}{2\varepsilon_o}\left(\frac{1}{2}\left(z^2 - R^2\right)^{-1/2}.(2z) - 1\right) = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 - R^2}}\right)$$

24.05

$$\sqrt{f(x)} = \frac{f'}{2\sqrt{f(x)}}$$

$$V = \frac{\sigma}{2\varepsilon}\sqrt{z^2+R^2} - z$$

$$E = -\frac{\partial V}{\partial x} \quad E = -\frac{\partial V}{\partial y} \quad E = -\frac{\partial V}{\partial z}$$

zero zero

المجال سوف يثبت فقط باتجاه Z

$$-\frac{\partial V}{\partial z} = \frac{\sigma}{2\varepsilon}$$

$$E = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon}\left[\frac{2z}{2\sqrt{z^2+R^2}} - 1\right]$$

$$E = \frac{\sigma}{2\varepsilon}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

---

q_1

(+) —— r —— (+)

مثال طاقة وضع مخزنة في هذا النظام من شحنات

q_2 • —— r_1 —— • q_1, r_3, • q_3

$$\boxed{U = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r_1}}$$

$$\boxed{U = \frac{1}{4\pi\varepsilon}\left[\frac{q_1 q_2}{r_1} + \frac{q_1 q_3}{r_3} + \frac{q_2 q_3}{r_2}\right]}$$

قياسية وتقاس بوحدة J

طاقة الوضع

Electric Potential Energy of a System of Charged Particles

- We want to calculate the potential energy of a system of two charged particles.
- Our starting point is to examine the work we must do (as an external agent) to bring together two charged particles that are initially infinitely far apart and that end up each other and stationary.

- If the two particles have the same sign of charge, we must fight against their mutual repulsion. Our work is positive and results in a positive potential energy for the final two-particle system.

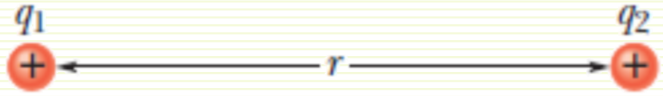
الطاقة تكون موجبة ⊕ ⊕

- If the two particles have opposite signs of charge, our work is negative and results in a negative potential energy for the system.

الطاقة تكون سالب ⊖ ⊕

Electric Potential Energy of a System of Charged Particles

- For a two-particle system as in the Figure, where particle 1 (with positive charge q_1) and particle 2 (with positive charge q_2) have separation r. Although both particles are positively charged, our result will apply also to situations where they are both negatively charged or have different signs.



- We start with particle 2 fixed in place and particle 1 infinitely far away, with an initial potential energy U_i for the two-particle system. Next we bring particle 1 to its final position, and then the system's potential energy is U_f. Our work changes the system's potential energy by

$$\Delta U = U_f - U_i = q_1(V_f - V_i)$$

- The initial potential energy is $U_i = 0$ because the particles are in the reference configuration. The two potentials (V_f and V_i) are due to particle 2 and are given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r}$$

Electric Potential Energy of a System of Charged Particles

- This tells us that when particle 1 is initially at distance $r = \infty$, the potential at its location is $V_i = 0$. when we move it to the final position at distance r, the potential at its location is

$$V_f = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r}$$

- The final configuration has a potential energy of

$$U_f - U_i = U_f - 0 = q_1(V_f - V_i) = q_1\left(\frac{1}{4\pi\varepsilon_o}\frac{q_2}{r} - 0\right)$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}}$$

- The signs of the two charges are included. If the two charges have the same sign, U is positive. If they have opposite signs, U is negative.

Electric Potential Energy of a System of Charged Particles

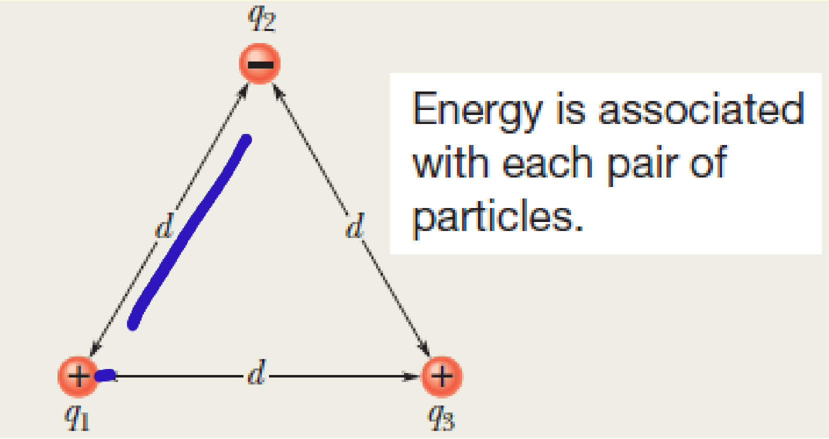
- If we next bring in a third particle, with charge q_3, we repeat our calculation, starting with particle 3 at an infinite distance and then bringing it to a final position at distance r_{31} from particle 1 and distance r_{32} from particle 2.
- At the final position, the potential V_f at the location of particle 3 is the algebraic sum of the potential V_1 due to particle 1 and the potential V_2 of particle 2. so,
- The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

الطاقة الكلية تساوي مجموع الطاقات المختزنة بين
كل زوج من الشحنات

Sample Problem 24.06

Potential energy of a system of three charged particles

- Three charged particles held in fixed positions by forces that are not shown in the following Figure. What is the electric potential energy U of this system of charges? Assume that $d = 12\ cm$ and that $q_1 = +q$, $q_2 = -4q$, and $q_3 = +2q$, in which $q = 150\ nC$. $d = 0.12$



- Solution:
- Starting with one of charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. So, the potential energy U_{12} associated with the pair of charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{d}$$

$$U = \frac{1}{4\pi\varepsilon}\left[\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_3 q_2}{d}\right]$$

Sample Problem 24.06

Potential energy of a system of three charged particles

- Then, we bring the last charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2.
- $W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_3}{d}$
- The total potential energy U of the three-charge system is the sum of the potential energies associated with the three pairs of charges.

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right)$$

$$= \frac{1}{4\pi\varepsilon_o\, d}\left((+q)(-4q) + (+q)(+2q) + (-4q)(+2q)\right) = \frac{1}{4\pi\varepsilon_o d}\left(-4q^2 + 2q^2 - 8q^2\right) = -\frac{10q^2}{4\pi\varepsilon_o d}$$

$$= -\frac{\left(8.99 \times 10^9\ Nm^2C^{-2}\right)(10)\left(150 \times 10^{-9}\ C\right)^2}{0.12\ m} = -1.7 \times 10^{-2}\ J = -17\ mJ$$

$$U = \frac{1}{4\pi\varepsilon}\frac{1}{d}\left[q_1 q_2 + q_2 q_3 + q_1 q_3\right] = 8.99\times10^9 \frac{1}{0.12}\left[q(-4q) + (-4q)(2q) + q(2q)\right]$$

$$= 8.99\times10^9 \frac{1}{0.12}\left[-4 - 8 + 2\right](150\times10^{-9})^2 = 0.017\ J$$

جامعة أم القرى
UMM AL-QURA UNIVERSITY

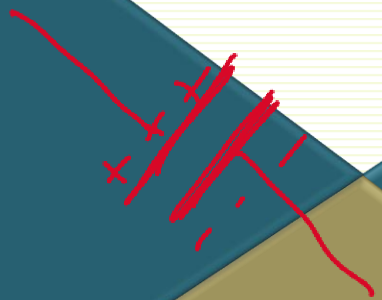
General Physics for Engineering

Code: PHY1119-4

Ch 25

Capacitors and Dielectrics

مواسعات   المواد العازلة



Physics Department

College of Science

Contents

Capacitance

- The basic elements of any capacitor are shown in Figure 1 (two isolated conductors of any shape, no matter what their geometry, flat or not, we call these conductors plates).

الواح ذو اللوحين المتوازيين

- In Figure (a), a $\boxed{\text{parallel –plate capacitor}}$, made up of two plates of area A separated by a distance d. The charges on the facing plate surfaces have the same magnitude q but opposite signs.

منتظم

- In Figure (b), as the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

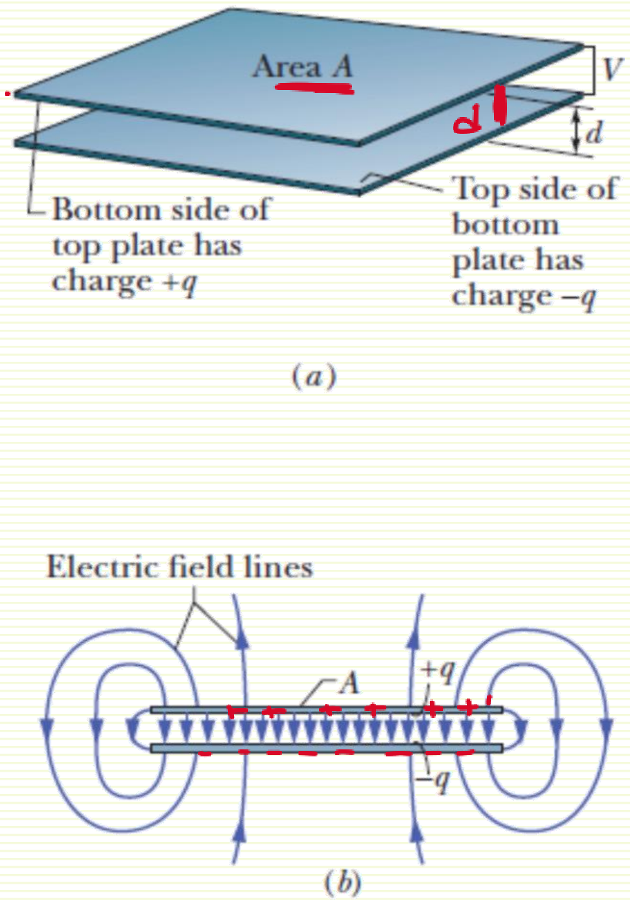


Figure 1

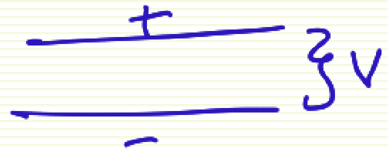
السعه كهربائيه

Capacitance C (فاراد F)

- When a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$. We refer to the *charge of a capacitor* as being q, the absolute value of these charges on the plates. (Note that q is not the net charge on the capacitor, which is zero).
- Because the plates are conductors, they are equipotential surfaces; all points on a plate are at the same electric potential. Moreover, there is a potential difference between the two plates.

  سطح متساوي جهد

- The charge q and the potential difference V for a capacitor are proportional to each other; that is,



$$q = CV$$

- The proportionality constant C is called the **capacitance** of the capacitor.
- The value of capacitance depends only on the geometry of the plates and not on their charge or potential difference.

السعه: نسبه بين الشحنه و فرق الجهد

$$C = \frac{q}{V}$$

قيمه (C) ثابتة بالنسبه للمواضع وتعتمد على الابعاد الهندسيه

$$C = \frac{\text{كولوم}}{\text{فولت}} = \text{فاراد}$$

Capacitance

مقياس لمقدار الشحنة على الصفائح لتنتج فرق جهد معين

- The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: *the greater the capacitance, the more charge is required.*

كلما كانت السعة اكبر احتاجت شحنات اكبر لنفس الجهد

- The SI unit of capacitance is coulomb per volt (C/V), this unit occurs so often that it is given a special name, the *farad* (F):

$$1\text{ farad} = 1\ F = 1\text{ Coulomb per Volt} = 1\ C/V$$

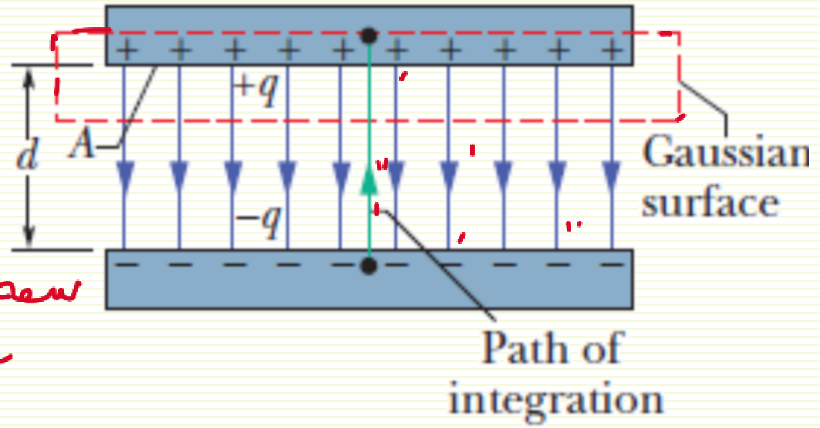
1×10^{-6}

- The farad is a very large unit, submultiples of the farad, such as the microfarad ($1\ \mu F = 10^{-6} F$) and the picofarad ($1\ pF = 10^{-12} F$), are more convenient units in practice.

The Capacitance of A parallel-Plate Capacitor

For a parallel-plate capacitor, we assume, that the plates are so large and so close together . For this case the capacitance is given by

$$\boxed{C = \frac{q}{V} = \frac{\varepsilon_o E A}{E d} = \frac{\varepsilon_o A}{d}}$$



$\frac{A\ \text{المساحه} \times \varepsilon_0}{d\ \text{المسافه}} = \frac{\text{الشحنه}}{\text{الجهد}} =$ سعة المواسع C

- Where:
- A is the plate area , and
  مساحه
- d is the plate separation.
  المسافه

تعتمد C فقط على الابعاد الهندسيه A و d

- That is the capacitance depend only on geometrical factors- namely, the plate area A and the plate separation d.

$C = \frac{\varepsilon A}{d}$ طردي عكسي

C تزداد بازدياد A تقل بزيادة d

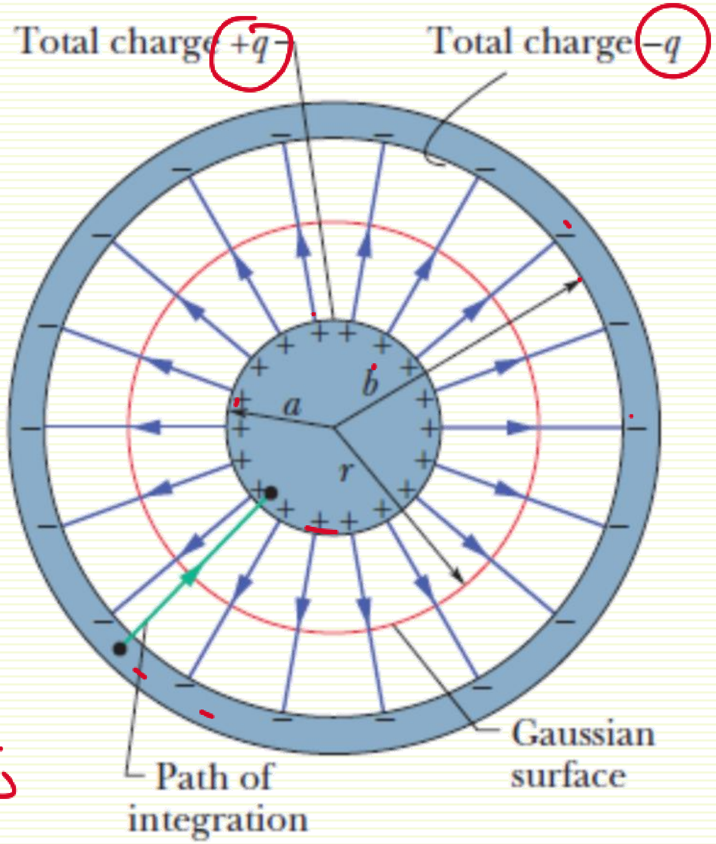
- Note that C increases as we increase area A or decrease separation d.

مواسع اسطواني

The Capacitance of A Cylindrical Capacitor

متحدتين في المحور

- A cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume that $L \gg b$, and each plate contains a charge of magnitude q.



- The capacitance is

$$C = \frac{q}{V} = 2\pi \varepsilon \frac{L}{\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{q}{V} = 2\pi\varepsilon_o \frac{L}{\ln(b/a)}}$$

- That is the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case the length L and the two radii b and a.

تعتمد السعة على الابعاد الهندسية

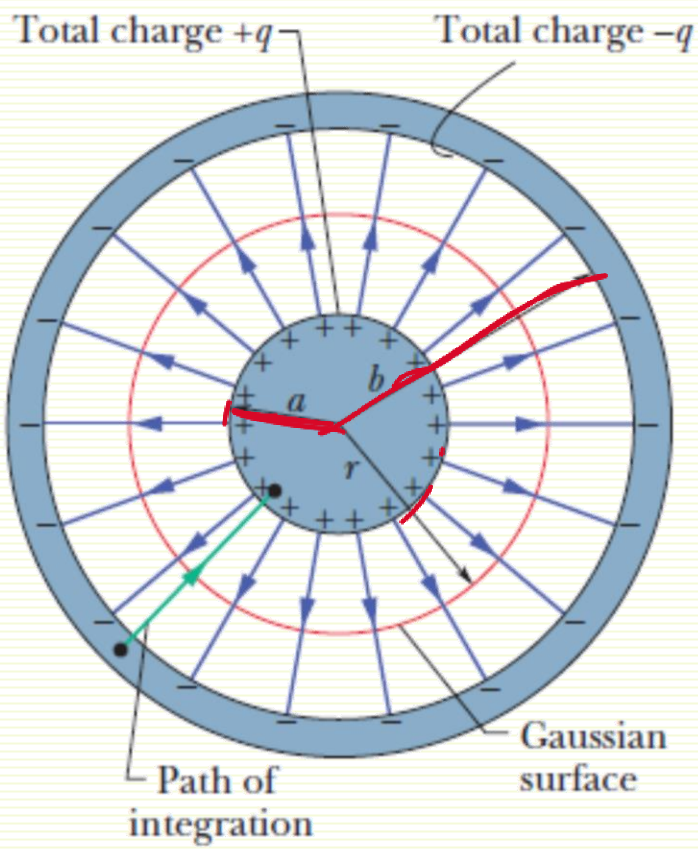
تزداد C بزيادة طول الاسطوانة وتقل بنقصان الوف

بين a و b

The Capacitance of A Spherical Capacitor

قشرتين كرويتين متحدين بالمركز:

- If we consider a capacitor that consists of two concentric spherical shells, of radii a and b. the capacitance of a spherical capacitor is



$$\boxed{C = \frac{q}{V} = 4\pi\varepsilon_o \left(\frac{ab}{b-a}\right)}$$

تعتمد C على الابعاد الهندسية فقط على نصف القطر a و b

- That is The capacitance of a spherical capacitor depends only on geometrical factors, in this case the two radii of spherical shells b and a.

$$C = \frac{q}{V} = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)$$

Sample Problem 25.01

Charging the plates in a parallel-plate capacitor

- In the following Figure, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12\ V$. The lower capacitor plate has thickness $L = 0.50\ cm$ and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}\ electrons/m^3$. From what depth d within the plate (Figure b) must electrons move to the plate face as the capacitor becomes charged?

عدد الشحنات في وحدة الحجوم

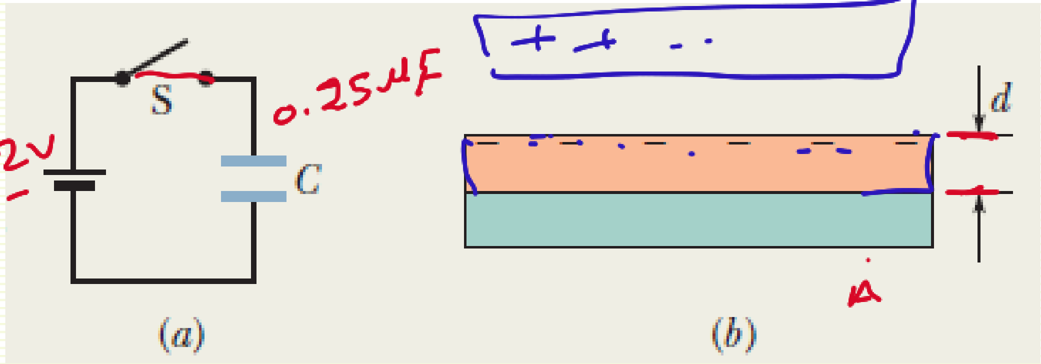
1.

- Solution:

$q = C \times V$

$= 0.25 \times 10^{-6} \times 12$

$q = 3 \times 10^{-6}\ C$



0.25μF

12V

2. عدد الالكترونات =

$N = \frac{q}{e} = \frac{3 \times 10^{-6}}{1.6 \times 10^{-19}}$

$N = 1.87 \times 10^{13}$ electron

- Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. The total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ F)(12\ V) = 3.0 \times 10^{-6}\ C$$

$n = \frac{N}{\text{حجم}}$

Sample Problem 25.01

Charging the plates in a parallel-plate capacitor

- The number N of conduction electrons that come up to the face is

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.6 \times 10^{-19}\ C} = 1.873 \times 10^{13} \text{ electrons}$$

- These electrons come from a volume that is the product of the face area A and the depth d. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad}$$

3 كثافة = عدد الالكترونات / حجم

$$n = \frac{N}{Ad}$$

$$d = \frac{N}{An}$$

- The depth d is

$$d = \frac{N}{An} = \frac{(1.873 \times 10^{13}\ \text{electrons})}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)} = 1.1 \times 10^{-12}\ m = 1.1\ pm$$

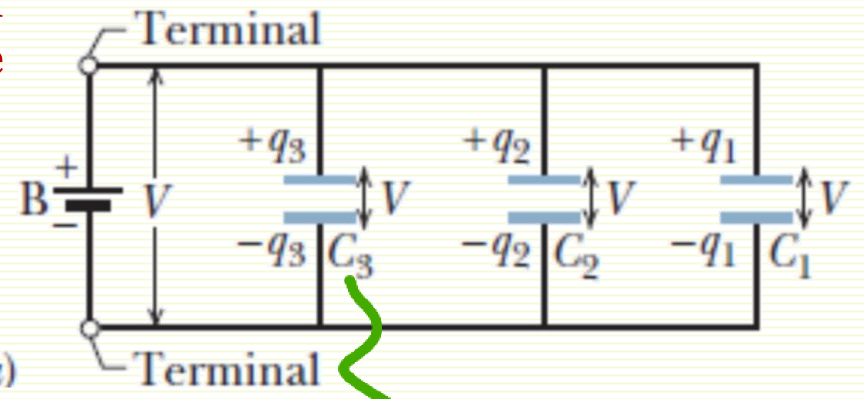
$$d = \frac{1.87 \times 10^{3}}{2 \times 10^{-4} \times 8.49 \times 10^{28}}$$

$$= 1.1 \times 10^{-12}\ m = 1.1\ pm$$

توازي توالي

Capacitors in Parallel and in Series

- Capacitors in Parallel
- Figure (a) shows an electric circuit in which three capacitors are connected in parallel to battery B. “in parallel” means that the capacitors are directly wired together at one plate and directly wired together at the other plate, and the same potential difference V is applied across the two groups of wired-together plates. Thus, each capacitor has the same potential difference V, which produces charge on the capacitor.
- The charge on each actual capacitor is
- $q_1 = C_1 V$, $q_2 = C_2 V$, and $q_3 = C_3 V$
- The total charge on the parallel combination is
- $q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$
- The equivalent capacitance, with the same total charge q and applied potential difference V is
- $C_{eq} = \frac{q}{V} = \frac{(C_1+C_2+C_3)V}{V} = \boxed{C_1 + C_2 + C_3}$ (in parallel)



(a)

$q_1 = C_1 V$

Capacitors in Parallel and in Series

- Capacitors in Series
- Figure a shows three capacitors connected in series to battery B. "in series" means that the capacitors are wired serially, one after the other, and that a potential difference V is applied across the two ends of the series. The potential differences that exist across the capacitors in series produce identical charges q on them.
- The potential difference of each actual capacitor is

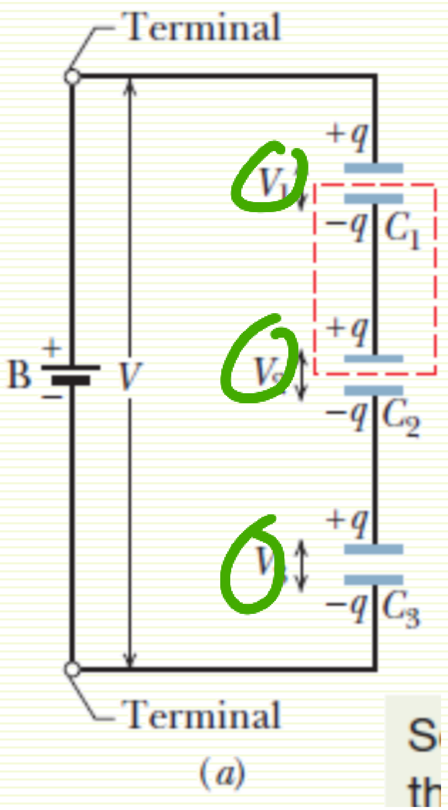
$$V_1 = \frac{q}{C_1}, \qquad V_2 = \frac{q}{C_2}, \qquad \text{and} \qquad V_3 = \frac{q}{C_3}$$

- The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

- The equivalent capacitance is

$$C_{eq} = \frac{q}{V} = \frac{q}{q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)} = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}$$



Parallel التوازي

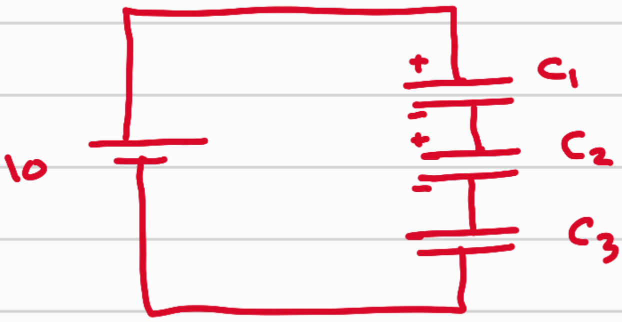
$$q = q_1 + q_2 + q_3$$

$$C_{eq} = \frac{q}{V}$$

$$V = V_1 = V_2 = V_3$$

$$C_{eq} = C_1 + C_2 + C_3 \ldots$$

التوالي Series



$$q = q_1 = q_2 = q_3$$

$$V = V_1 + V_2 + V_3$$

$$C_{eq} = \frac{q}{V}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

في حال مواسعين توالي

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Parallel and in Series

- or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{(in series)}$$

For any number n of capacitors in parallel and in series,

$$C_{eq} = \sum_{j=1}^{n} C_j \qquad \text{(n capacitors in parallel)} \qquad C_1 + C_2 + C_3 \ldots$$

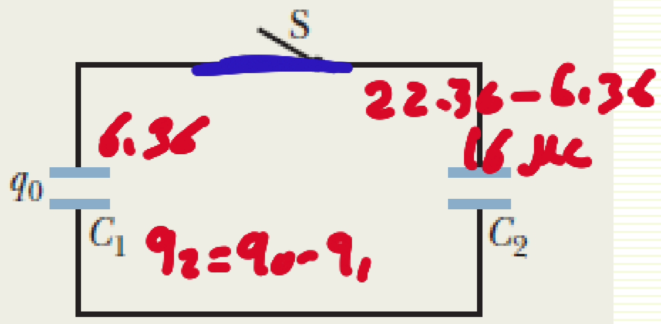
$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad \text{(n capacitors in series)} \qquad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

Sample Problem 25.02

Capacitors in parallel and in series

- Capacitor 1, with $C_1 = 3.55\,\mu F$, is charged to a potential difference $V_o = 6.30\,V$, using a $6.30\,V$ battery. The battery is then removed, and the capacitor is connected as in the following Figure to an uncharged capacitor 2, with $C_2 = 8.95\,\mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

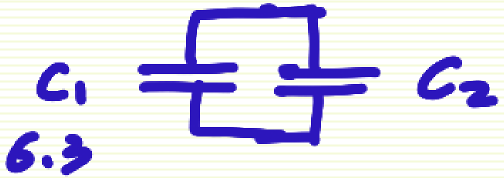
After the switch is closed, charge is transferred until the potential differences match.



- Solution:
- When capacitor 1 is connected to the battery, the charge it acquires is

$$q_o = C_1 V_o = (3.55 \times 10^{-6} F)(6.30\,V) = 22.365 \times 10^{-6} C$$

- When switch S is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until
- $V_1 = V_2$ (equilibrium)



الشحنه الكليه هي نفسه

الشحنه على جوانب الاول قبل التوصيل

$$q_0 = C_1 V = 3.55 \times 10^{-6} \times 6.3 = 22.36 \times 10^{-6} C$$

Sample Problem 25.02

Capacitors in parallel and in series

$$V_1 = V_2 \quad \text{(equilibrium)}$$

- or

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \quad \text{(equilibrium)}$$

Because the total charge cannot change, the total after the transfer must be

$$q_1 + q_2 = q_o \quad \Rightarrow \quad q_2 = q_o - q_1$$

The equilibrium equation is

$$\frac{q_1}{C_1} = \frac{q_o - q_1}{C_2} \quad \Rightarrow \quad \frac{q_1}{q_o - q_1} = \frac{C_1}{C_2} = \frac{3.55\ \mu F}{8.95\ \mu F} = 0.397$$

$$\therefore q_1 = 0.397(q_o - q_1) = 0.397\, q_o - 0.397\, q_1 \quad \Rightarrow \quad 1.397\, q_1 = 0.397\, q_o$$

$$\therefore q_1 = \frac{0.397}{1.397} q_o = \frac{0.397}{1.397}(22.365 \times 10^{-6}\ C) = 6.36 \times 10^{-6}\ C = 6.36\ \mu C$$

The rest of the initial charge must be on capacitor 2:

$$q_2 = q_o - q_1 = 22.365 \times 10^{-6}\ C = 6.36 \times 10^{-6} C = 16.0 \times 10^{-6}\ C = 16.0\ \mu C$$

$q_1 + q_2 = q_0$

$q_2 = q_0 - q_1$

$q_1 + q_2 = q_0$

$V_1 = V_2 \Rightarrow \frac{q_1}{C_1} = \frac{q_2}{C_2}$

$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}$

$\frac{C_2}{C_1} = \frac{q_0 - q_1}{q_1}$

$\frac{C_2}{C_1} = \frac{q_0}{q_1} - 1$

$\frac{q_0}{q_1} = 1 + \frac{C_2}{C_1}$

$q_1 = \frac{q_0}{1 + \frac{C_2}{C_1}}$

Energy Stored in an Electric Field

$$q_1 = \frac{22.365 \times 10^{-6}}{1 + \frac{8.95}{3.55}}$$

$$q_1 = 6.3 \times 10^{-6}$$

- Work must be done by an external agent to charge a capacitor. We can imagine doing the work ourselves by transferring electrons from one plate to the other, one by one.
- As the charges build, so does the electric field between the plates, which opposes the continued transfer. So, greater amounts of work are required.
- Actually, a battery does all this for us, at the expense of its stored chemical energy. We visualize the work as being stored as electric potential energy in the electric field between the plates.
- Suppose that, at a given instant, a charge q has been transferred from one plate of a capacitor to the other. The potential difference V between the plates at that instant will be q/C. If an extra increment of charge dq is then transferred, the increment of work required will be

$$dW = V\,dq = \frac{q}{C}\,dq$$

$$W = \Delta V q$$

$$dW = V\,dq$$

$$\int dW = \int \frac{q}{c}\,dq$$

$$W = \frac{q^2}{2C}$$

Energy Stored in an Electric Field

- The work required to bring the total capacitor charge up to a final value q is

$$W = \frac{q^2}{2C} = U$$

الشغل المبذول = الطاقة

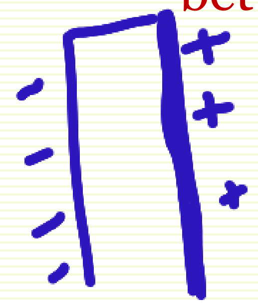
$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

- The work is stored as potential energy U in the capacitor, so that

$$U = \frac{q^2}{2C} = \frac{1}{2C}(CV)^2 = \frac{1}{2C}(C^2V^2) = \frac{1}{2}CV^2 \quad \text{(potential energy)}$$

Energy Density

- In a parallel-plate capacitor, neglecting fringing, the electrical field has the same value at all points between the plates. Thus, the energy density u (The potential energy per unit volume between the plates) should be uniform.



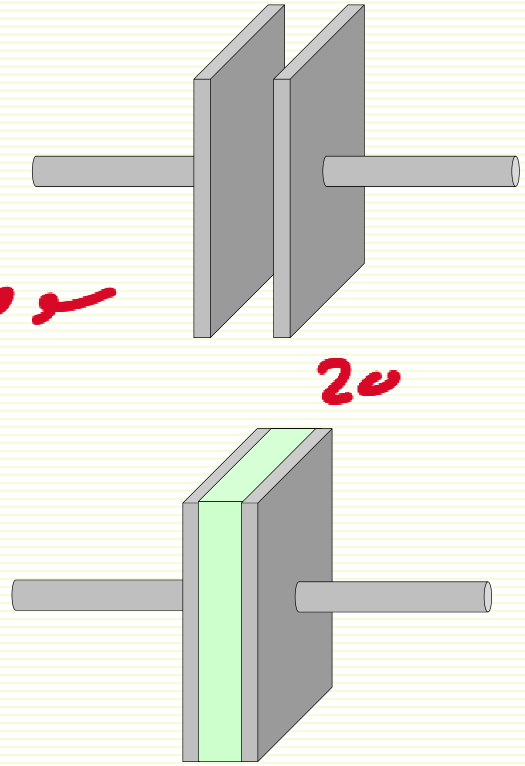
$$u = \frac{U}{volume} = \frac{U}{Ad} = \frac{(CV^2/2)}{Ad} = \frac{CV^2}{2Ad} = \left(\frac{\varepsilon_o A}{d}\right)\left(\frac{V^2}{2Ad}\right) = \frac{1}{2}\varepsilon_o\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_o E^2 \quad J/m^3$$

$V = Ed$

$$u = \frac{\text{الطاقة}}{\text{الحجم}} = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \frac{\varepsilon \cancel{A}}{d\,\cancel{A}\,2d}V^2 = \frac{1}{2}\varepsilon\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon E^2$$

Capacitor with a Dielectric

- If you fill the space between the plates of a capacitor with a dielectric, which is an insulating material such as mineral oil or plastic, what happens to the capacitance? عازلة
- Michael Faraday – to whom the whole concepts of capacitance is largely due and for whom the SI unit of capacitance is named – first looked into this matter in 1837. (معامل العزل) K بمعامل C سوف تزداد
- Using simple equipment much like that shown in the Figure, he found that the capacitance increased by a numerical factor κ which he called the *dielectric constant* of the insulating material.
- The dielectric constant of a vacuum is unity by definition. Because air is mostly empty space, its measured dielectric constant is only slightly greater than unity. Even common paper can significantly increase the capacitance of a capacitor.



20

للهواء $K = 1$

$$C = \frac{\varepsilon A}{d}$$

Capacitor with a Dielectric

يصبح ضمن حد أعلى لفرق جهد

- Another effect of the introduction of a dielectric is to limit the potential difference that can be applied between the plates to a certain value V_{max}, called the *breakdown potential*.
- If this value is substantially exceeded, the dielectric material will break down and form a conducing path between the plates. Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown.

قوة عازلة

اعلى قيمة تتحملها المادة العازلة دون انهيار

- The capacitance of any capacitor in air is

$$\boxed{C_{air} = \varepsilon_o \frac{A}{d}}$$

تقريب

- Faraday's discovery was that, with a dielectric completely filling the space between the plates, so

$$\boxed{C = \kappa \varepsilon_o \frac{A}{d} = \kappa\, C_{air}}$$

لأي مواسع مكونه مادة عازلة

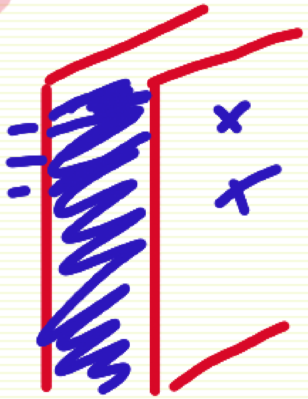
- Where κ (*Kapa*) is constant called dielectric constant of the materials between the plates.

k هي تعتبر ثابت العزل و هو يعتمد على نوع المادة

Sample Problem 25.03

One capacitor charging up another capacitor

- A parallel-plate capacitor whose capacitance C_{air} is $13.5\,pF$ is charged by a battery to a potential difference $V = 12.5\,V$ between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.
- (a) what is the potential energy of the capacitor before the slab is inserted?
- (b) what is the potential energy of the capacitor-slab device after the slab is inserted?



$C = \kappa C_{air}$

- Solution:
- The potential energy before the slab is inserted is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12} F)(12.5\,V)^2 = 1.055 \times 10^{-9}\,J = 1055\,pJ$$

- (b) Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential does change.

$$U_{Air} = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12})(12.5)^2 = 1.055 \times 10^{-9}\,J$$

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_{Air}}{\kappa} = \frac{1.055 \times 10^{-9}}{6.5} = 162\,pJ$$

لم نستخدم قانون الجهد لان الجهد تغير (قانون C,q)

Sample Problem 25.03

One capacitor charging up another capacitor

- So, the potential energy of the capacitor-slab is

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ pJ}{6.50} = 162\ pJ$$

- When the slab is introduced, the potential energy decreases by a factor of κ.
- The missing energy would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)pJ = 893\ pJ$$

- If the slab were allowed to slide, between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a constant mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch 26

Current and Resistance

Physics Department

College of Science

Contents

Electric Current

- The current can be defined as the rate at which charge moves through a point. The current is normally due to the motion of conduction electrons that are driven by electric fields (such as those set up in a wire by a battery).
- An electric current i in a conductor is defined by

$$i = \frac{dq}{dt}$$

- where dq is the amount of positive charge that passes in time dt.
- The direction of electric current is taken as the direction in which positive charge carriers would move.
- The SI unit of electric current is the ampere (A), where

$$1\ A = 1\ C/s$$

Electric Density

- Sometimes we are interested in the current i in a particular conductor. At other times we take a localized view and study the flow of charge through a cross section of the conductor at a particular point.
- To describe this flow, we can use the **current density** $\vec{J}$, which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.
- The magnitude J is equal to the current per unit area through that element.
- The total current through the surface is

$$i = \int \vec{J}.d\vec{A}$$

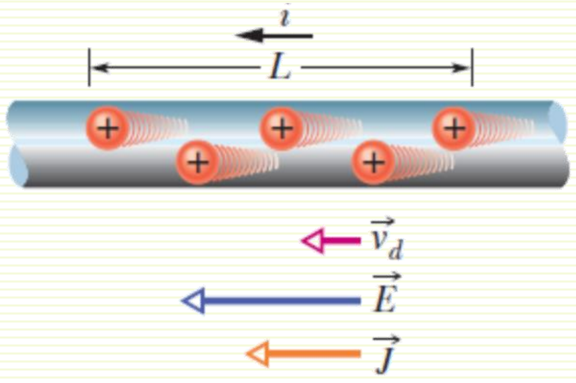
- where $d\vec{A}$ is the area vector of the element, perpendicular to the element. If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$. So,

$$i = \int J\,dA = J \int dA = JA \qquad \Longrightarrow \qquad J = \frac{i}{A}$$

Electric Density

- Where A is the total area of the surface. The SI unit for current density is the ampere per square meter (A/m^2).
- Drift Speed
- When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction.
- When the conductor does have a current through it, its conduction electrons still move randomly and they tend to drift with a drift speed v_d in the direction opposite that of the applied electric field that causes the current.
- Let us assume that the charge carriers all move with the same drift speed and that the current density J is uniform across the wire cross-sectional area A.
- The number of charge carriers in a length L is (nAL)
- Where n is the number of carriers per unit volume.
- The total charge of carries in the length L is

$$q = (nAL)e$$



Electric Density

- Because the carriers all move along the wire with speed v_d, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}$$

- The current is

$$i = \frac{q}{t} = \frac{(nALe)}{\left(\frac{L}{v_d}\right)} = nAev_d$$

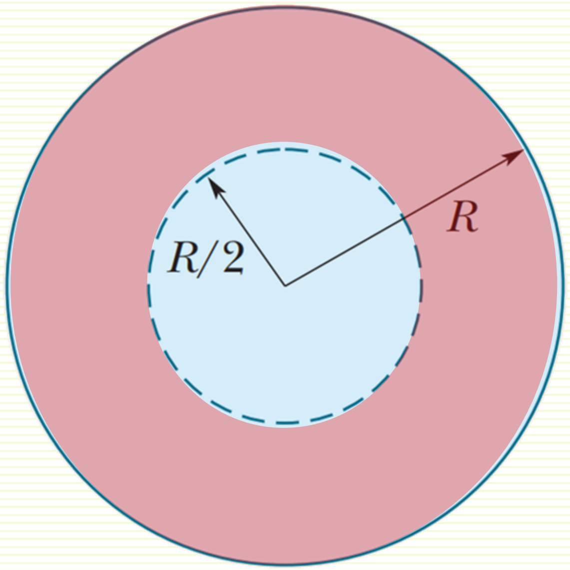
- The drift speed v_d is

$$v_d = \frac{i}{nAe} = \frac{J}{ne} \qquad \text{or} \qquad \vec{J} = (ne)\,\vec{v}_d$$

- The product (ne), whose SI unit is the coulomb per cubic meter (C/m^3), is the carrier charge density.
- For positive carriers, (ne) is positive, and $\vec{J}$ and $\vec{v}_d$ have the same direction.
- For negative carriers, (ne) is negative and , and $\vec{J}$ and $\vec{v}_d$ have opposite direction.

Sample Problem 26.02
Current density

- The current density in a cylindrical wire of radius R = 2.0 mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5\ A/m^2$. What is the current through the outer portion of the wire between radial distances $R/2$ and R
- Solution
- Because the current density is uniform across the cross section, the current density J, the current i, and the cross-sectional area A are related by $J = i/A$.
- The read area, $(A' = \pi R^2 - \pi(R/2)^2 = \frac{3}{4}\pi R^2 = 9.42 \times 10^{-6}\ m^2)$,
- Then, the current through the read area is given by
- $i = J\ A' = \left(2.0 \times 10^5\ A/m^2\right) \times \left(9.42 \times 10^{-6}\ m^2\right) = 1.9\ A.$



Sample Problem 26.03

In a current, the conduction electrons move very slowly

- What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\ \mu m$ when it has a uniform current $i = 17\ mA$? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.
- Solution
- $v_d = \frac{i}{nAe} = \frac{J}{ne}$
- $n = N_A \left(\frac{1}{M}\right) \rho_{mass} = (6.02 \times 10^{23}\ mol^{-1}) \left(\frac{1}{63.54\times10^{-3} kg/mole}\right) (8.96 \times 10^3 kg/m^3)$
- $= 8.49 \times 10^{28}\ \ electrons/m^3$
- $A = \pi r^2 = 3.14 \times (900\mu m)^2 = 2.54 \times 10^{-6}\ m^2.$
- $v_d = \frac{i}{nAe} = \frac{17\times10^{-3}\ A}{(8.49\times10^{28}\ electrons/m^3)(2.54\times10^{-6}\ m^2)(1.6\times10^{-19} C)} = 4.9 \times 10^{-7}\ m/s = 1.8\ mm/h$

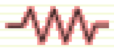
Resistance and Resistivity

- If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical resistance.
- We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is

$$R = \frac{V}{i}$$

- The SI unit for resistance is volt per ampere and ohm (symbol Ω), that is

$$1\ \text{ohm} = 1\ \Omega = 1\ \text{volt per ampere} = 1\ V/A$$

- A conductor whose function in a circuit is to provide a specified resistance is called a resistor. In a circuit diagram, we represent a resistor and a resistance with the symbol  .
- The current is

$$i = \frac{V}{R}$$

- For a given V, the greater the resistance, the smaller the current.

Resistance and Resistivity

- Now we focusing not on the potential difference V across a particular resistor but on the electric field $\vec{E}$ at a point in a resistive material. Instead of dealing with the current i through the resistor, we deal with the current density $\vec{J}$ at the point in question. Instead of the resistance R of an object, we deal with the resistivity ρ of the material, so

$$\rho = \frac{E}{J}$$

- The unit of resistivity is $\Omega.m$, where

$$\frac{unit\ (E)}{unit\ (J)} = \frac{(V/m)}{(A/m^2)} = \frac{V}{A}m = \Omega.m$$

- In vector form

$$\vec{E} = \rho\,\vec{J}$$

- The conductivity σ of a material is the reciprocal of its resistivity, so

$$\sigma = \frac{1}{\rho}$$

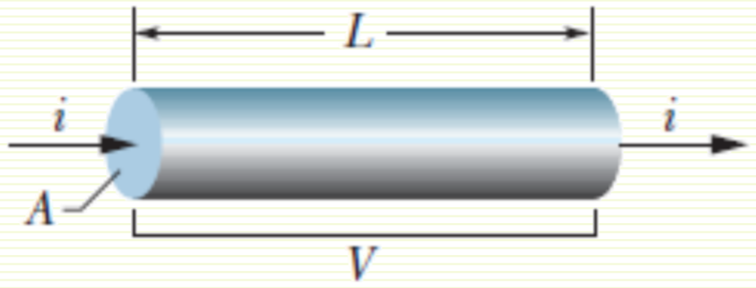
- The SI unit of conductivity is $(\Omega.m)^{-1}$. The unit name (mhos per meter) is sometimes used (mho is ohm backwards).

Resistance and Resistivity

- From the definition of σ, so
- $\because \vec{E} = \rho\,\vec{J} \qquad \Rightarrow \qquad \vec{J} = \frac{1}{\rho}\,\vec{E}$
- $\therefore \vec{J} = \sigma\,\vec{E}$

Calculating Resistance from resistivity:

- Resistance is a property of an object. Resistivity is a property of a material.
- If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let A is the cross-sectional area of the wire, let L is its length, and let a potential difference V exist between its ends.
- If the electric field and the current density are constant for all points within the wire, so
- $E = \frac{V}{L}$ and $J = \frac{i}{A}$
- The resistivity is
- $\rho = \frac{E}{J} = \frac{(V/L)}{(i/A)} = \left(\frac{V}{i}\right)\left(\frac{A}{L}\right) = R\,\left(\frac{A}{L}\right)$



Resistance and Resistivity

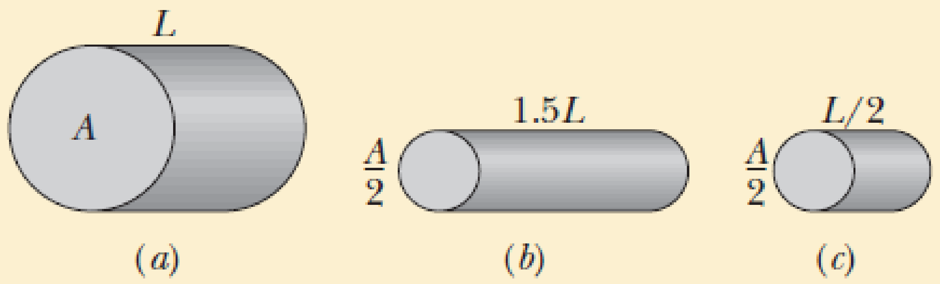
- The resistance R is

$$R = \rho \frac{L}{A}$$

- This equation can be applied only to a homogenous isotropic conductor of uniform cross section, with the potential difference applied as in the Figure.
- (Note: Isotropic materials are the materials whose electrical properties are the same in all directions)
- The macroscopic quantities V, i, and R are of greatest interest when we are making electrical measurements on specific conductors.
- The microscopic quantities E, J, and ρ are of greatest interest in the fundamental electrical properties of materials.

Checkpoint

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference *V* is placed across their lengths.



- In Figure (a):

$$R = \rho \frac{L}{A} = \rho \frac{L}{A}$$

- In Figure (b):

$$R = \rho \frac{1.5\,L}{(A/2)} = \rho \frac{(2 \times 1.5)\,L}{A} = \rho \frac{3L}{A}$$

- In Figure (c):

$$R = \rho \frac{(L/2)}{(A/2)} = \rho \frac{L}{A}$$

- The greatest current occurs in Figures (a) and (c), and then in Figure (b).

Sample Problem 26.04

A material has resistivity, a block of the material has resistance

- A rectangular block of iron has dimensions $1.2\ cm \times 1.2\ cm \times 15\ cm$. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces. What is the resistance of the block if the two parallel sides are (1) the square ends with dimensions $1.2\ cm \times 1.2\ cm$, and (2) two rectangular sides with dimensions $1.2\ cm \times 15\ cm$, if the resistivity of iron is $9.68 \times 10^{-8}\ \Omega.m$?
- Solution:
- For arrangement 1, we have

$$L = 15\ cm = 0.15\ m$$

$$A = 1.2\ cm \times 1.2\ cm = 1.44\ cm^2 = 1.44 \times 10^{-4}\ m^2$$

- The resistance R is

$$R = \rho \frac{L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega.m)(0.15\ m)}{(1.44 \times 10^{-4} m^2)} = 1.0 \times 10^{-4} \Omega = 100\ \mu\Omega$$

- For arrangement 2, we have

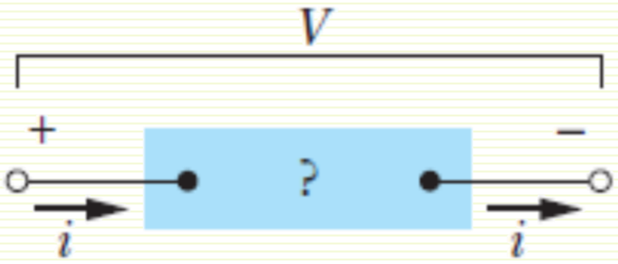
$$L = 1.2\ cm = 1.2 \times 10^{-2}\ m$$

$$A = 1.2\ cm \times 15\ cm = 18\ cm^2 = 18 \times 10^{-4} m^2$$

$$R = \rho \frac{L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega.m)(1.2 \times 10^{-2}\ m)}{(18.0 \times 10^{-4} m^2)} = 6.5 \times 10^{-7} \Omega = 0.65\ \mu\Omega$$

Ohm's Law

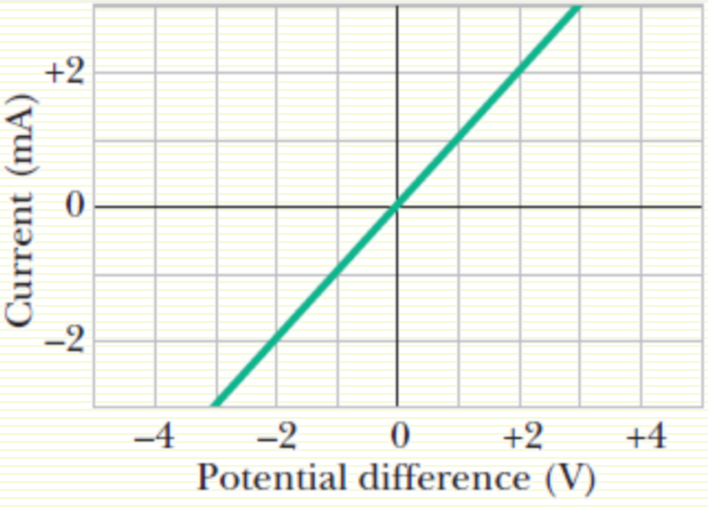
- As we know, a resistor is a conductor with a specified resistance. It has that same resistance no matter what the magnitude and direction (polarity) of the applied potential difference. Other conducting devices, however, might have resistances that changes with the applied potential difference.
- Figure (a) shows how to distinguish such devices. A potential difference V is applied across the device being tested, and the resulting current i through the device is measured as V is varied in both magnitude and polarity. The polarity of V is arbitrary taken to be positive when the left terminal of the device is at a higher potential than the right terminal. The direction of the resulting current (from left to right) is arbitrarily assigned a plus sign.



(a)

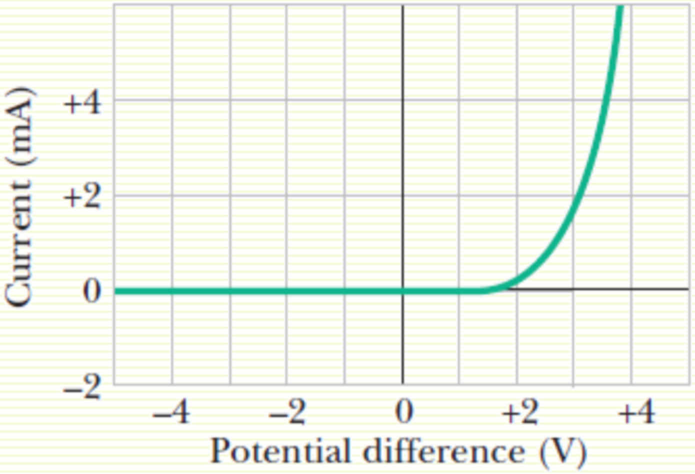
Ohm's Law

- Figure (b) is a plot of i versus V for one device. This plot is a straight line passing through the origin, so the ratio i/V (which is the slope of the straight line) is the same for all values of V. This means that the resistance $R = V/I$ of the device is independent of the magnitude and polarity of the applied potential difference V.



(b)

- Figure (c) is a plot for another conducting device. Current can exist in this device only when the polarity of V is positive and the applied potential difference is more than about 1.5 V. When current does exist, the relation between i and V is not linear; it depends on the value of the applied potential difference V.



(c)

Ohm' Law

- We distinguish between the two types of device by saying that one obeys Ohm's law and the other does not.

- A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

- A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

- It is often contended that $V = iR$ is a statement of Ohm's law. That is not true. This equation is the defining equation for resistance, and it applies to all conducting devices, whether they obey Ohm's law or not. If we measure the potential difference V across, and the current i through, any device, even a pn junction diode, we can find its resistance at that value of V as $R = V/i$.

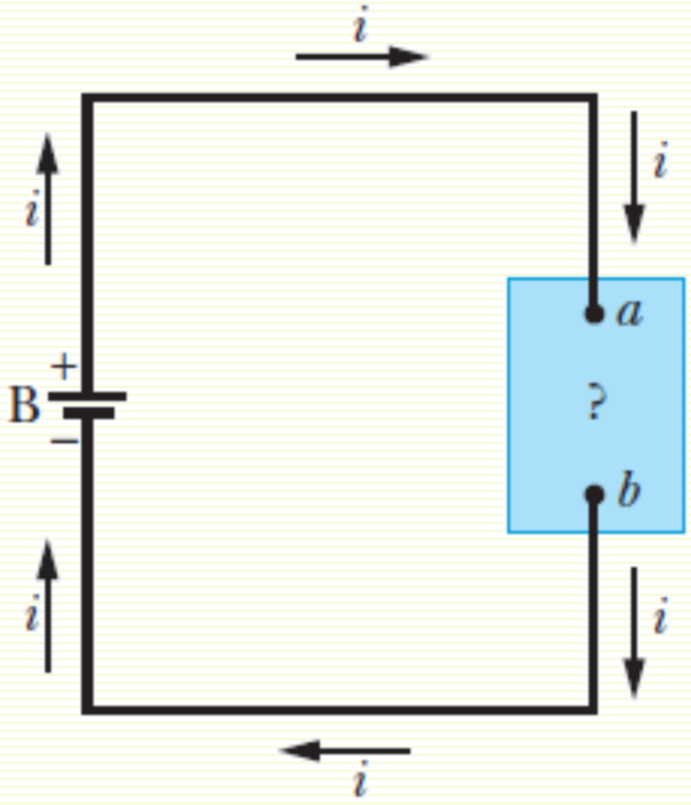
- The essence of Ohm's law, however, is that a plot of i versus V is linear; that is, R is independent of V.

- All homogenous materials, whether they are conductors like copper or semiconductor like pure silicon or silicon containing special impurities, obey Ohm's law within some range of values of the electric field. If the field is too strong, however, there are departures from Ohm's law in all cases.

Power in Electric Circuits

- The Figure shows a circuit consisting of a battery B that is connected by wires, which we assume have a negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude V across its own terminals.

- A steady current i is produced in the circuit, directed from terminal a to terminal b. The amount of charge dq that moves between those terminals in time interval dt is equal to (idt). This charge dq moves through a decrease in potential of magnitude V and thus its electric potential energy decrease in magnitude by the amount



$$dU = dq\,V = (i\,dt)V$$

Power in Electric Circuits

- The principle of conservation of energy tells us that the decrease in electric potential energy from a to b is accompanied by a transfer of energy to some other form.
- The power P associated with that transfer is the rate of transfer

$$P = \frac{dU}{dt} = \frac{(i\ dt\ V)}{dt} = i\,V \quad \text{(rate of electrical energy transfer)}$$

- The unit of power is volt. Ampere $(V.A)$, where

$$1\ V.A = \left(1\frac{J}{C}\right)\left(1\frac{C}{s}\right) = 1\frac{J}{s} = 1\ W$$

- As an electron moves through a resistor at constant drift speed, its average kinetic energy remains constant and its lost electric potential energy appears as thermal energy in the resistor and the surroundings. On a microscopic scale this energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice.

Power in Electric Circuits

- The mechanical energy that transferred to thermal energy is dissipated (lost) because the transfer cannot be reversed.
- For a resistor with resistance R, the rate of electrical energy dissipation due to a resistance is

$$P = iV = i(iR) = i^2R$$

- Or

$$P = iV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

Checkpoint

A potential difference V is connected across a device with resistance R, causing current i through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) V is doubled with R unchanged, (b) i is doubled with R unchanged, (c) R is doubled with V unchanged, (d) R is doubled with i unchanged.

- The rate at which electrical energy is converted to thermal energy due to R is

$$P_a = \frac{(2V)^2}{R} = 4\frac{V^2}{R}, \qquad P_b = (2i)^2R = 4\,i^2R$$

$$P_c = \frac{V^2}{(2R)} = \frac{1}{2}\frac{V^2}{R}, \qquad P_d = i^2(2R) = 2i^2R$$

- The greatest in case (a) and (b), then in case (d), then in case (c).

Sample Problem 26.06

Rate of energy dissipation in a wire carrying current

- You are given a length of uniform heating wire made of a nickel-chromium-iron alloy called Nichrome; it has a resistance R of $72\ \Omega$. At what rate is energy dissipated if a potential difference of $120\ V$ is applied across the full length of the wire.
- Solution:

$$P = \frac{V^2}{R} = \frac{\left((120\ V)^2\right)}{72\ \Omega} = 200\ W$$

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch 27

DC Circuits

Physics Department
College of Science

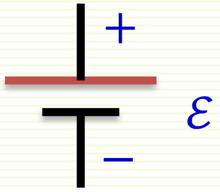
Contents

Electromotive Force

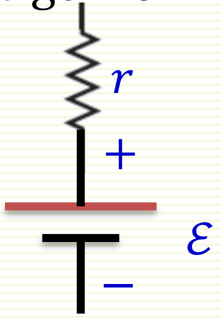
- An electromotive force device is a battery with two terminals, positive terminal and negative terminal.
- The emf of an emf device is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal, so

$$\mathcal{E} = \frac{dW}{dq}$$

- The SI unit for emf is the joule per coulomb (J/C) or volt.
- A real electromotive force device include an internal resistance r
- For Ideal electromotive force device, the internal resistance $r = 0$.



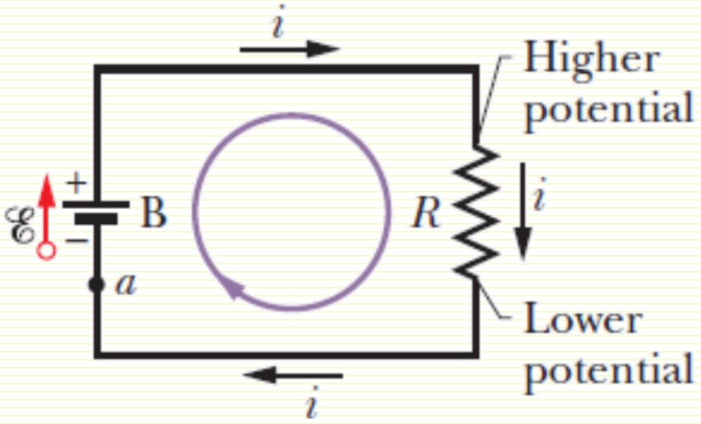
Ideal emf.



Real emf.

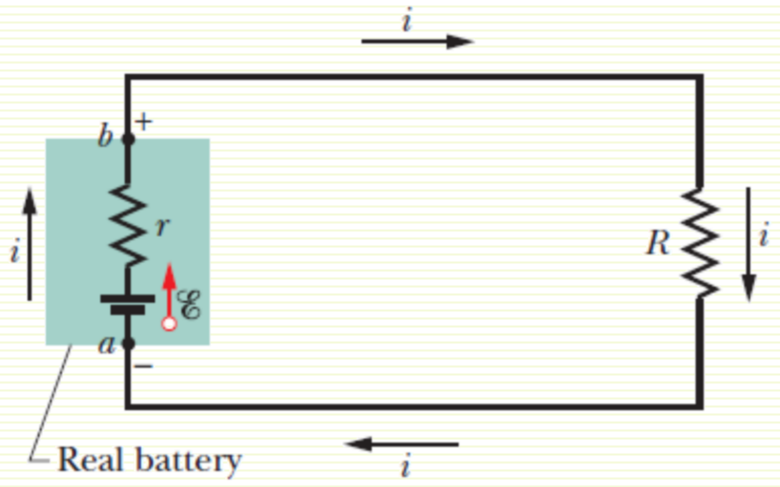
- In a simple single loop circuit, which consists of an ideal battery B with emf $\mathcal{E}$, a resistor of resistance R, and two connecting wires have negligible resistance. Then the relation between the emf and the current is

$$\mathcal{E} = iR \quad \Rightarrow \quad i = \frac{\mathcal{E}}{R}$$



Other Single-Loop Circuits

- For a simple loop connected to a real emf with internal resistance r, the figure shows a real battery, with internal resistance r, wires to an external resistor of resistance R.
- The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an unremovable feature of the battery.
- The battery is drawn as if it could be separated into an ideal battery with emf $\mathcal{E}$ and a resistor of resistance r.
- In this case, the emf can be written as

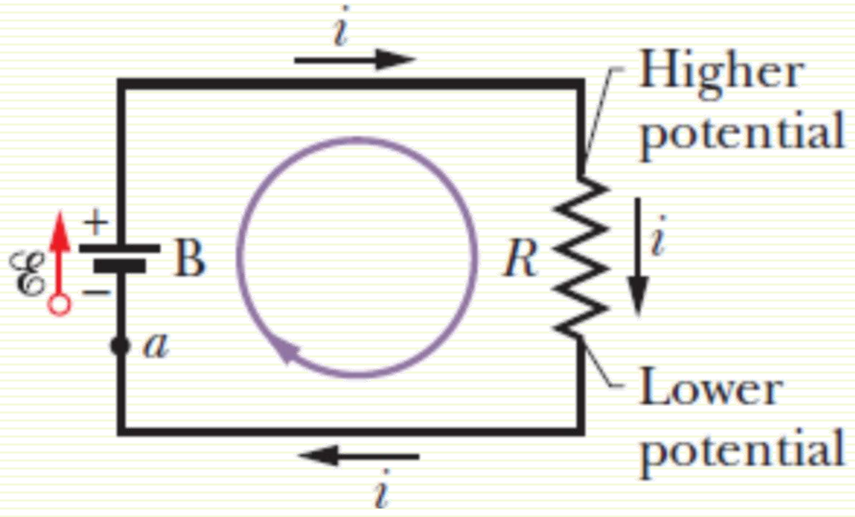


$$\mathcal{E} - ir - iR = 0 \quad \Rightarrow \quad \mathcal{E} = ir + iR = i(r + R)$$

$$\therefore i = \frac{\mathcal{E}}{r + R}$$

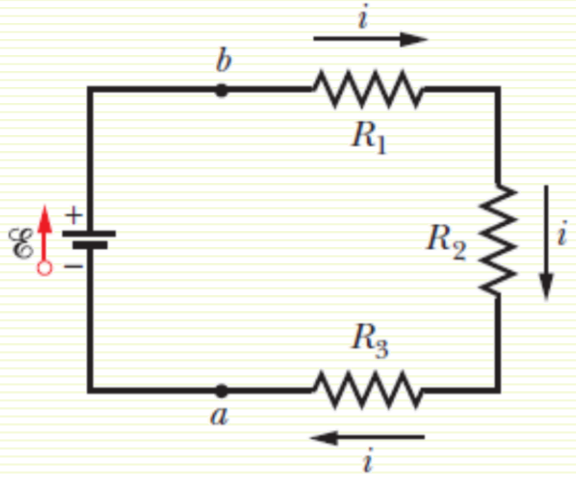
Calculating the Current in a Single Loop

- **Kirchhoff's loop rule or Kirchhoff's voltage law**
- The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.
- $\mathcal{E} - iR = 0$
- To apply this rule, we choose a direction suppose that is clockwise direction, then we apply the following rules.
- Resistance Rule: For a move through a resistance in the direction of the current, the change in potential is $-iR$; and in the opposite direction is $+iR$.
- EMF Rule: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; and in the opposite direction is $-\mathcal{E}$.



Calculating the Current in a Single Loop

- Resistance in Series: The following Figure shows three resistance connected in series to an ideal battery with emf $\mathcal{E}$. The resistances are connected one after another between a and b, and a potential difference is maintained across a and b by the battery.

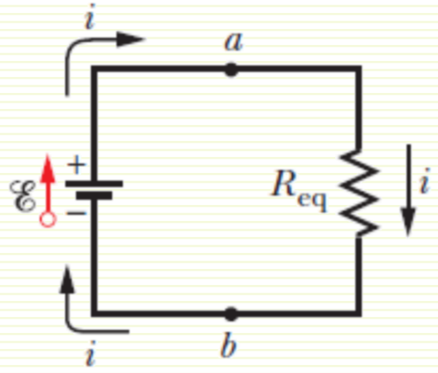


- The potential difference that then exist across the resistance in the series produce identical current i in them. In general,

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0 \quad \Rightarrow \quad \mathcal{E} = i(R_1 + R_2 + R_3)$$

$$\therefore i = \frac{\mathcal{E}}{(R_1 + R_2 + R_3)}$$

- If we replaced the three resistances with a single equivalent resistance R_{eq}, we find



$$\mathcal{E} - iR_{eq} = 0 \quad \Rightarrow \quad i = \frac{\mathcal{E}}{R_{eq}}$$

- So,

$$R_{eq} = R_1 + R_2 + R_3$$

Potential Difference Between Two Points

- We want to find the potential difference between two points in a circuit. For example, what is the potential difference $V_b - V_a$ between points a and b?
- Note: $V_b - V_a = iR$, is the potential difference across the resistance R.
- Then we have

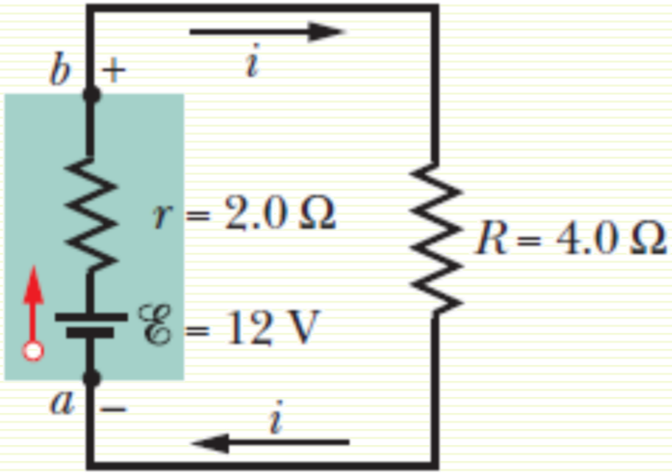
$$\mathcal{E} - ir - iR = 0 = \mathcal{E} - ir - (V_b - V_a) = 0 \quad \Rightarrow \quad V_b - V_a = \mathcal{E} - ir$$

But $i = \frac{\mathcal{E}}{R+r}$, So,

$$\therefore V_b - V_a = \mathcal{E} - \left(\frac{\mathcal{E}}{R+r}\right) r = \mathcal{E}\left(\frac{R+r}{R+r}\right) - \mathcal{E}\left(\frac{r}{R+r}\right) = \mathcal{E}\frac{R}{R+r}$$

$$\therefore V_b - V_a = \mathcal{E}\frac{R}{R+r} = (12\ V)\frac{4.0\ \Omega}{4.0\ \Omega + 2.0\ \Omega} = 8.0\ V$$

The internal resistance reduces the potential difference between the terminals.



Potential Difference Across a Real Battery

- Points a and b are located at the terminals of the battery. Thus, the potential difference $V_b - V_a$ is the terminal-to-terminal potential difference V across the battery. So, $V = \mathcal{E} - ir$
- If the internal resistance r of the battery were zero, so the potential difference V would be equal to the emf $\mathcal{E}$ of the battery. If the internal resistance r of the battery is greater than zero, so the potential difference V is less than the emf $\mathcal{E}$ of the battery.

Power, Potential, and emf

- When a battery or some other type of emf device does work on the charge carriers to establish a current i, the device transfers energy from its source of energy to the charge carriers.
- Because a real emf device has an internal resistance r, it also transfers energy to internal thermal energy via resistive dissipation.

1. The net rate P of energy transfer from the emf device to the charge carriers is

$$P = iV = i(\mathcal{E} - ir) = i\mathcal{E} - i^2 r$$

2. The term $i^2 r$ is the rate P_r of energy transfer to thermal energy within the emf device.

$$P_r = i^2 r \quad \text{(internal dissipation rate)}$$

3. The term $i\mathcal{E}$ is the rate P_{emf} at which the emf device transfers energy both to the charge carriers and to internal thermal energy

$$P_{emf} = i\mathcal{E} \quad \text{(power of emf device)}$$

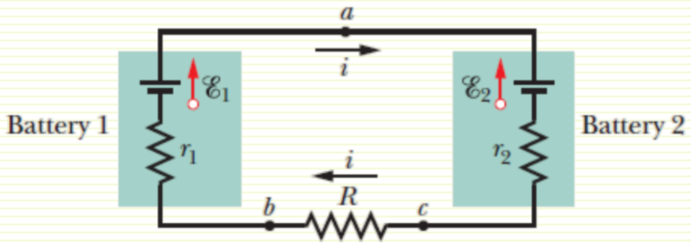
Sample Problem 27.01

Single-loop circuit with two real batteries

- The emfs and resistances in the circuit have the following values:

$$\mathcal{E}_1 = 4.4\,V, \qquad \mathcal{E}_2 = 2.1\,V$$

$$r_1 = 2.3\,\Omega, \qquad r_2 = 1.8\,\Omega, \qquad R = 5.5\,\Omega$$



(a) What is the current i in the circuit?

Solution:

- Because $\mathcal{E}_1$ is greater than $\mathcal{E}_2$, battery 1 controls the direction of i, so the direction is clockwise. Let us apply the loop rule by going clockwise, against the current, and starting at point a. We find

$$\mathcal{E}_1 - ir_1 - iR - ir_2 - \mathcal{E}_2 = 0$$

$$\mathcal{E}_1 - \mathcal{E}_2 = i(r_1 + r_2 + R)$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R} = \frac{4.4\,V - 2.1\,V}{5.5\,\Omega + 2.3\,\Omega + 1.8\,\Omega} = 0.2396\,A \cong 240 \times 10^{-3}\,A = 240\,mA$$

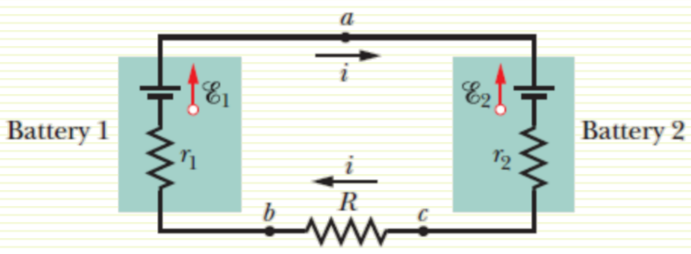
Sample Problem 27.01
Single-loop circuit with two real batteries

- The emfs and resistances in the circuit have the following values:

$$\mathcal{E}_1 = 4.4\ V, \qquad \mathcal{E}_2 = 2.1\ V$$

$$r_1 = 2.3\ \Omega, \qquad r_2 = 1.8\ \Omega, \qquad R = 5.5\ \Omega$$

(b) What is potential difference between the terminals of battery 1?



Solution:

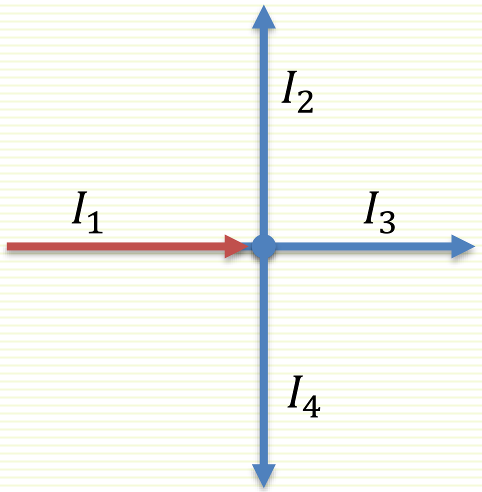
- We need to sum the potential differences between points a and b. Let us start at point b (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point a (effectively the positive terminal). We find that

$$(V_b - V_a) - ir_1 + \mathcal{E}_1 = 0$$

$$V_a - V_b = \mathcal{E}_1 - ir_1 = 4.4\ V - (0.2396\ A)(2.3\ \Omega) = +3.84\ V \approx 3.8\ V$$

Multiloop Circuits

- **Kirchhoff's Junction Rule:**
- The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.
- This rule is often called Kirchhoff's junction rule (or Kirchhoff's current law).
- Our basic tools for solving complex circuits are the loop rule (based on the conservation of energy) and the junction rule (based on the conservation of charge).
- $I_1 = I_2 + I_3 + I_4$



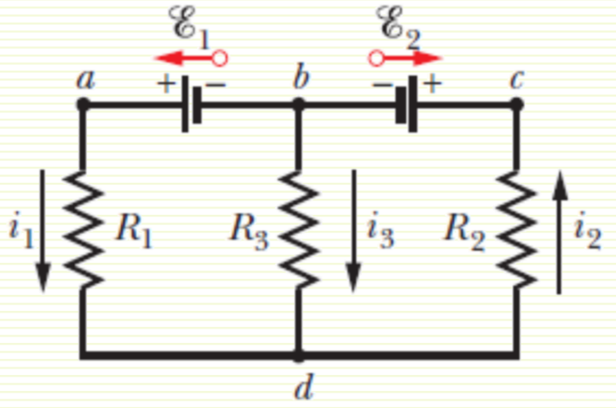
Multiloop Circuits

- The following figure shows a circuit containing more than one loop. There are two junctions in this circuit, at b and d, and there are three branches connecting these junctions.
- We have two junctions at b and d, and two loops.
- For junction d : incoming currents i_1 and i_3, and it leaves via outgoing current i_2.
- $i_1 + i_3 = i_2$
- For the left-hand loop ($badb$), If we traverse in a counterclockwise direction from point b, so
- $\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$
- For the right-hand loop ($bcdb$), If we traverse in a counterclockwise direction from point b, so

$$-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$$

- We now have three equations in the three unknown currents, and they can be solved.

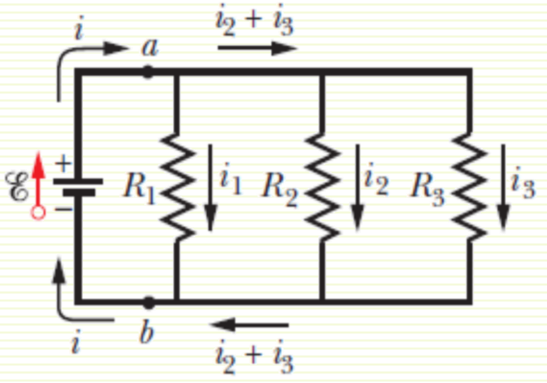
The current into the junction must equal the current out (charge is conserved).



Multiloop Circuits

- **Resistance in Parallel:**
- Figure (a) shows three resistances connected in parallel to an ideal battery of emf $\mathcal{E}$. The term in parallel means that the resistance are directly wired together on one side and directly wired together on the other side, and that a potential difference V is applied across the pair of connected sides.
- All three resistances have the same potential difference V across them, producing a current through each.

$$i_1 = \frac{V}{R_1}, \qquad i_2 = \frac{V}{R_2}, \qquad i_3 = \frac{V}{R_3}$$



(a)

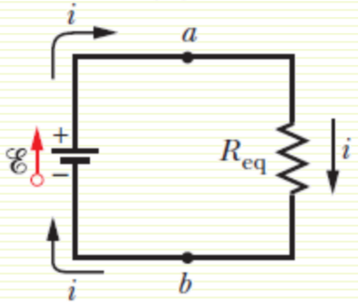
- If we apply the junction rule at point a, so

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

- If we replaced the parallel combination with the equivalent resistance

$$\because i = \frac{V}{R_{eq}}$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



(b)

Multiloop Circuits

- For the case of two resistance, the equivalent resistance is their product divided by their sum; that is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- **Note:** When two or more resistances are connected in parallel, the equivalent resistance is smaller than any of the combining resistance.

Multiloop Circuits

- Now we summarizes the equivalence relations for resistors and capacitors in series and in parallel.
- For resistors:

$$R_{eq} = \sum_{j=1}^{n} R_j$$ (in series: same current through all resistors)

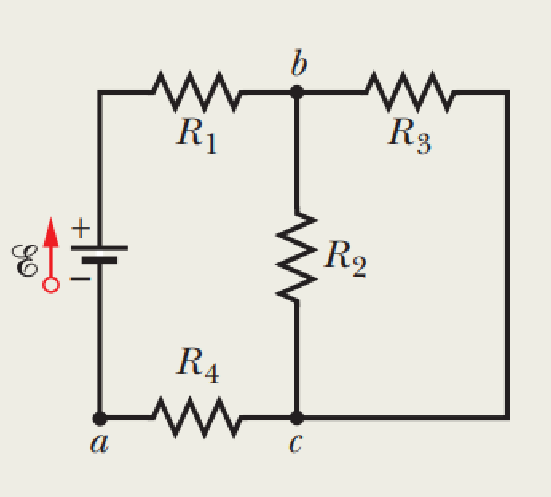
$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$ (in parallel: same potential difference across all resistors)

- For Capacitors:

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$ (in series: same charge on all capacitors)

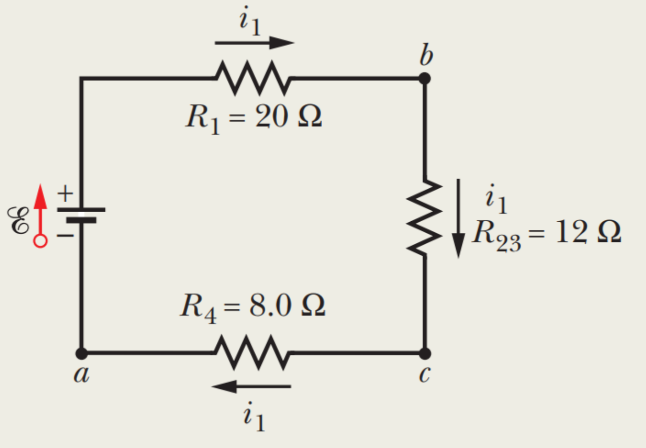
$$C_{eq} = \sum_{j=1}^{n} C_j$$ (in parallel: same potential difference across all capacitors)

Sample Problem 27.02
Resistors in parallel and in series



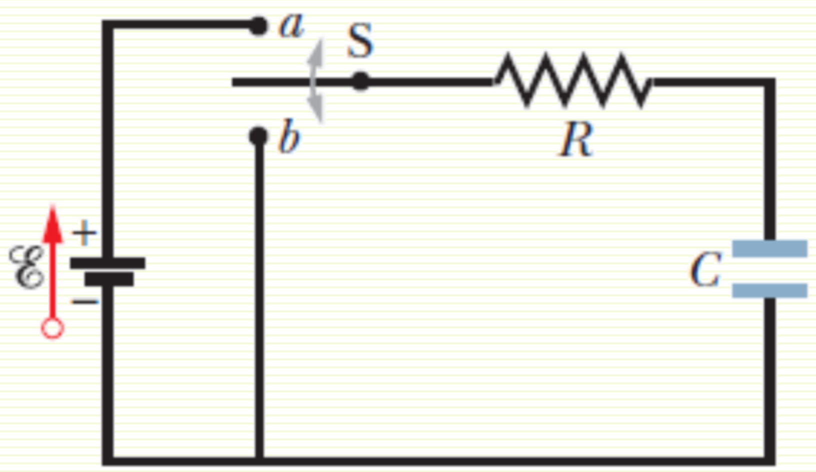
- The figure shows a multiloop circuit containing one ideal battery and four resistances with the following values: $R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$, $\mathcal{E} = 12$ V, $R_3 = 30\ \Omega$, $R_4 = 8.0\ \Omega$. What is the current through the battery?
- Solution:
- First, we calculate the equivalent resistance R_{23} for R_2, and R_3 in parallel.
- $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20} + \frac{1}{30} \Rightarrow \quad R_{23} = 12\ \Omega$
- Note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule yields
- $\mathcal{E} - i_1R_1 - i_1R_{23} - i_1R_4 = 0$
- $12\ V - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0$
- $i_1 = \frac{12\ V}{40\ \Omega} = 0.3\ A$

The equivalent of parallel resistors is smaller.



RC Circuits

- **Charging Capacitor:**
- Let a capacitor of capacitance C is initially uncharged. To charge it, we close switch S on point a. This completes an RC series circuit consisting of the capacitor, an ideal battery of emf $\mathcal{E}$, and a resistance R.
- During charging capacitor, a current increases the charge q on the plates and the potential difference $V_C = q/C$ across the capacitor.
- When V_C, the potential difference across the capacitor equals the potential difference across the battery (which is equal to the emf $\mathcal{E}$), the current is zero.
- From the equation $q = CV$, the equilibrium (final) charge on the then fully charged capacitor is equal to $C\mathcal{E}$.



RC Circuits

- But how the charge $q(t)$ on the capacitor plates, the potential difference $V_C(t)$ across the capacitor, and the current $i(t)$ in the circuit vary with time during the charging process?
- We begin by applying the loop rule to the circuit, traversing it clockwise from the negative terminal of the battery. We find

$$\mathcal{E} - iR - V_C = 0 \quad \text{or} \quad \mathcal{E} - iR - \frac{q}{C} = 0$$

$$\because i = \frac{dq}{dt} \quad \Rightarrow \quad \therefore R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \quad \text{(charging equation)}$$

- The solution of this equation is

$$q = C\,\mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{(charging a capacitor)}$$

The derivative of $q(t)$ is the current $i(t)$ charging the capacitor:

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\,e^{-t/RC} = I_o\;e^{-t/RC} \text{(charging a capacitor)}$$

Where $\frac{\mathcal{E}}{R} = I_o$ is the maximum current at time $t = 0$. This tells us that $i = \mathrm{I_o}$ at $t = 0$ and that $i = 0$, when the capacitor becomes fully charged as $t \to \infty$.

RC Circuits

- The potential difference $V_C(t)$ across the capacitor during the charging process is

$$V_C = \frac{q}{C} = \frac{C\,\mathcal{E}\left(1 - e^{-t/RC}\right)}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{(charging a capacitor)}$$

- This tells us that $V_C = 0$ at $t = 0$ and that $V_C = \mathcal{E}$ when the capacitor becomes fully charged as $t \to \infty$.

The time constant

- The product RC that appears in the equations of charging a capacitor has the dimensions of time (because the argument of an exponential must be dimensionless and because, in fact, $1.0\ \Omega \times 1.0\ F = 1.0\ s$)
- The product RC is called the capacitive time constant of the circuit and is represented with the symbol τ:

$$\tau = RC \quad \text{(time constant)}$$

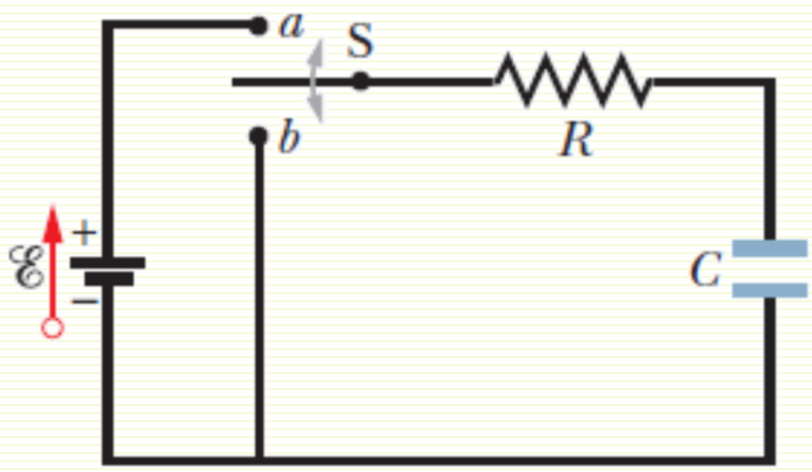
RC Circuits

- Discharging a Capacitor:
- Assume now that the capacitor is fully charged to a potential V_o equal to the emf $\mathcal{E}$ of the battery. At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R.
- How do the charge $q(t)$ on the capacitor and current $i(t)$ through the discharge loop of capacitor and resistance now vary with time?
- The differential equation describing $q(t)$ is like the equation in charging of a capacitor except that now, with no battery in the discharge loop, $\mathcal{E} = 0$. thus,

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{(discharging equation)}$$

- The solution to this differential equation is

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor)}$$



RC Circuits

- This equation tells us that q decreases exponentially with time, at a rate that is set by the capacitive time constant $\tau = RC$.
- At time $t = \tau$, the capacitor's charge has been reduced to $q_0 e^{-1}$, or about 37% of the initial value. Note that a greater τ means a greater discharge time.
- The current is

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(q_0 e^{-t/RC}\right) = q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{q_0}{RC}e^{-\frac{t}{RC}} \quad \text{(discharging a capacitor)}$$

- This tells us that the current also decreases exponentially with time, at a rate set by τ.
- The minus sign means that the capacitor's charge q is decreasing.

جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch. 28

Magnetic Fields

Physics Department

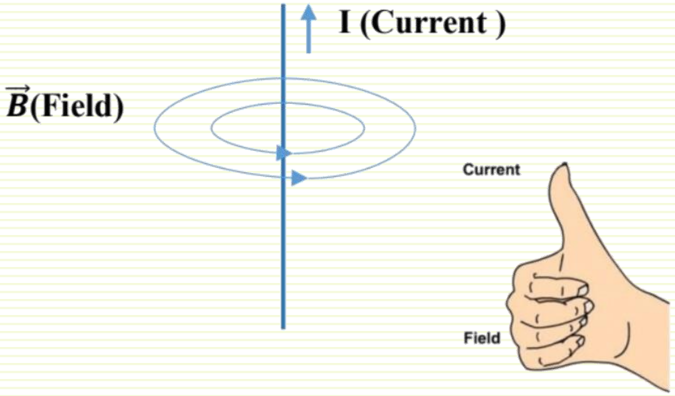
College of Science

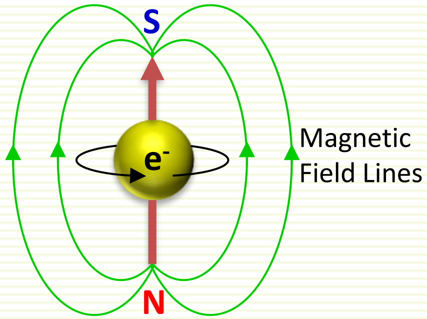
Contents

The Magnetic Field $\vec{B}$

- **What produces a Magnetic Field $\vec{B}$?**
- There are two ways:

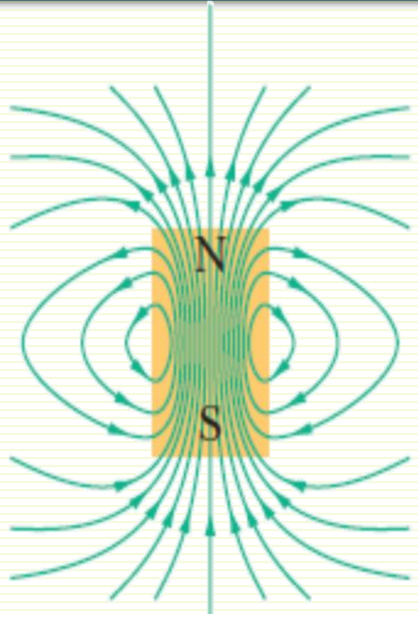
1. One way is to use moving electrically charged particles, such as a current in a wire, to make an electromagnet. The electric current produces a magnetic field
2. The other way to produce a magnetic field is by means of elementary particles such as electrons because these particles have an intrinsic magnetic field around them. That is, the magnetic field is a basic characteristic of each particle just as mass and electric charge are basic characteristics.



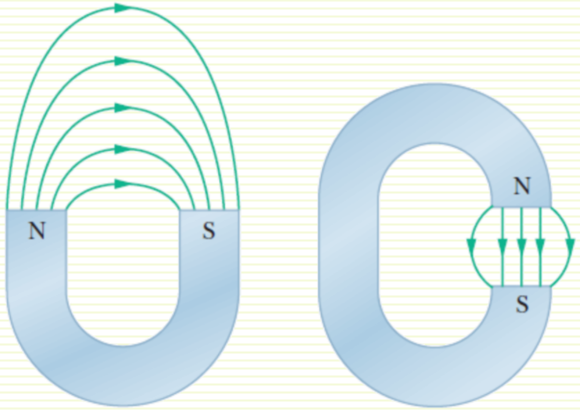


The Magnetic Field B

- **Two magnetic Poles (Magnetic Dipole):**
- The closed field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where the field lines enter the magnet, is called the *south pole*. Because a magnet has two poles, it is said to be a *magnetic dipole*.



- **Magnetic Field Lines**
- We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply:
  1) The direction of the tangent to a magnetic field line at any point gives the direction of $\vec{B}$ at that point.
  2) The density of the lines (number of normal lines per unit area) represents the magnitude of $\vec{B}$.



The magnetic Force on a moving charge

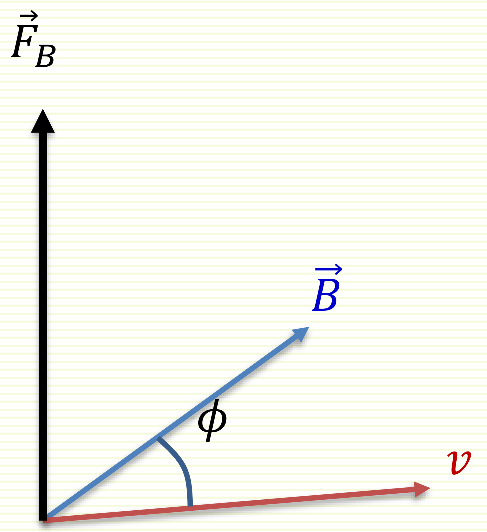
- When a charged particle moves through a magnetic field $\vec{B}$, a magnetic force acts on the particle as given by

$$\vec{F}_B = q\left(\vec{v} \times \vec{B}\right) \qquad (1)$$

- where q is the particle's charge (sign included) and $\vec{v}$ is the particle's velocity.
- The right-hand rule for cross products gives the direction of $\vec{v} \times \vec{B}$. The sign of q then determines whether $\vec{F}_B$ in the same direction as $\vec{v} \times \vec{B}$ or in the opposite direction.
- The magnitude of $\vec{F}_B$ is given by

$$F_B = |q|\, v\, B \sin\phi \qquad (2)$$

- Where ϕ is the angle between $\vec{v}$ and $\vec{B}$.
- The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is always perpendicular to $\vec{v}$ and $\vec{B}$.



The magnetic Force on a moving charge

- From Eq.(2), the magnetic field can be written as:

$$B = \frac{F_B}{|q| v \sin \phi} \qquad (3)$$

- So, the SI unit for $\vec{B}$ is the newton per coulomb-meter per second or tesla (T)

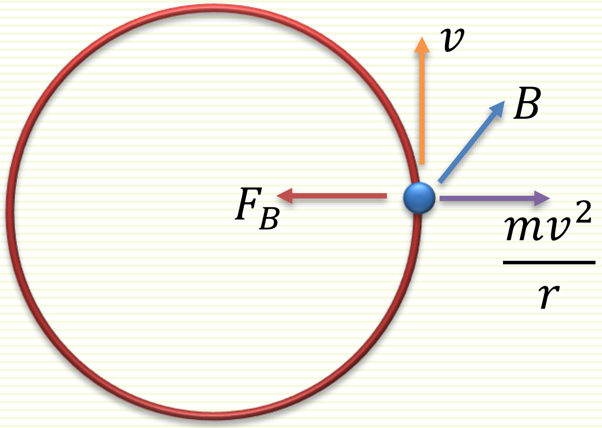
$$1 \text{ tesla} = 1\, T = 1 \frac{\text{newton}}{(\text{coulomb})\left(\frac{\text{meter}}{\text{second}}\right)} = 1 \frac{\text{N}}{\left(\frac{\text{coulomb}}{\text{second}}\right)(\text{meter})} = 1 \frac{\text{N}}{\text{A. m}}$$

- An earlier (non-SI) unit for $\vec{B}$, still in common use, is the gauss (G), where,

$$1 \text{ tesla} = 10^4 \text{ gauss}$$

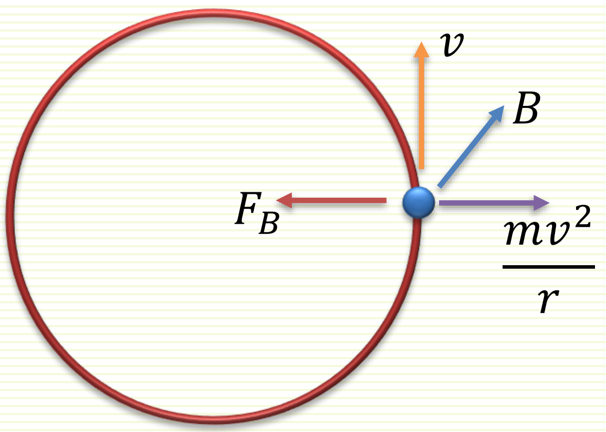
A Circulating Charged Particle

- If a charged particle with charge q and mass m moves with constant velocity v, then it enters the space where a magnetic field B is applied perpendicular to v, then there will be a magnetic force F_B act on the normal direction of both v and B.
- The particle will move under the influence of F_B, which act to bend the path of the moving particle. Since this force acts always normally to v, then the particle will move in a curve.
- In fact, there will be another force, that is the centrifugal force which equals $\frac{mv^2}{r}$ act to draw the particle outwards against F_B.
- At balance, the particle will move in a circular path with radius r, and
- $F_B = qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad (7)$



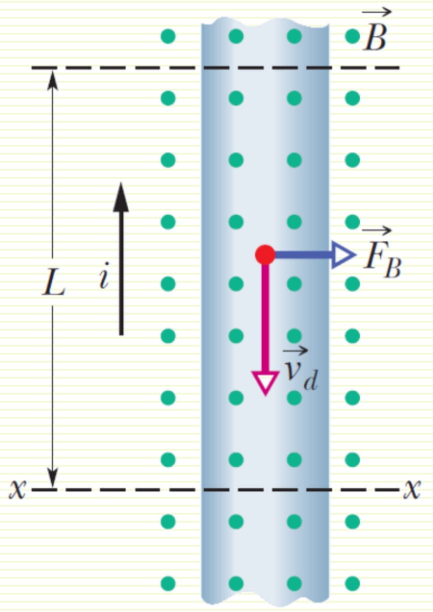
A Circulating Charged Particle

- The periodic time T of this cycle (the time for one full revolution) is given by
- $T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$ (8)
- The frequency f of the motion (the number of revolutions per unit time) is given by
- $f = \frac{1}{T} = \frac{qB}{2\pi m}$ (9)
- The angular frequency ω of the motion is given by
- $\omega = 2\pi f = \frac{qB}{m}$ (10)



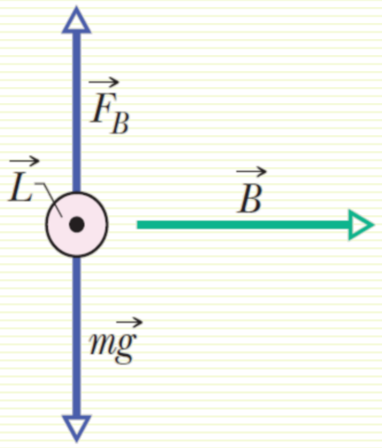
Magnetic Force on a Current-carrying Wire

- Consider a length L of the wire is placed in a magnetic field B. All the electrons in this section of wire will drift past plane xx in a time $t = \frac{L}{v_d}$. Thus, in that time a charge given by $q = it = i\frac{L}{v_d}$ will pass through that plane, where v_d is the drift velocity of the charge.
- From eq.(2), the magnetic force acting on this charge is given by
- $F_B = |q|\, v_d\, B \sin\phi = i\frac{L}{v_d}\, v_d\, B \sin\phi = iLB \sin\phi \qquad (11)$
- If the angle ϕ (ϕ is the angle between the L and B) equals 90°, then the force on the wire is
- $F_B = iLB \qquad (12)$
- Equation (11) can be written as
- $\overrightarrow{F_B} = i\,\vec{L} \times \vec{B} \qquad (13)$



Example 1

- A straight, horizontal length of copper wire has a current $i = 28\ A$ through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire – that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is $46.6\ g/m$.
- Solution:
- Let L is the length of the wire , and m its mass
- $m = 46.6 \times 10^{-3} \times L\ kg$
- Since the wire is horizontal, then to get the force upward against the gravity, we have to apply a magnetic field in the horizontal plane normal to the wire, as shown in the figure.
- So, at the balance, we get
- Then $F_B = mg \Rightarrow\ ILB = 46.6 \times 10^{-3} \times L$
- Then $B = \frac{46.6\times10^{-3}}{i} = \frac{46.6\times10^{-3}}{28} = 1.6 \times 10^{-2}\ T$



جامعة أم القرى
UMM AL-QURA UNIVERSITY

General Physics for Engineering

Code: PHY1119-4

Ch. 29

Magnetic Fields due to Current

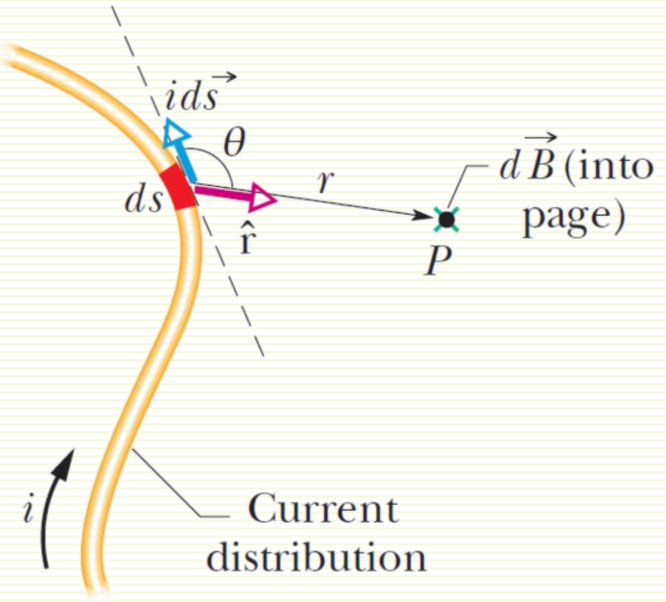
Physics Department

College of Science

Contents

Magnetic Field due to a Current

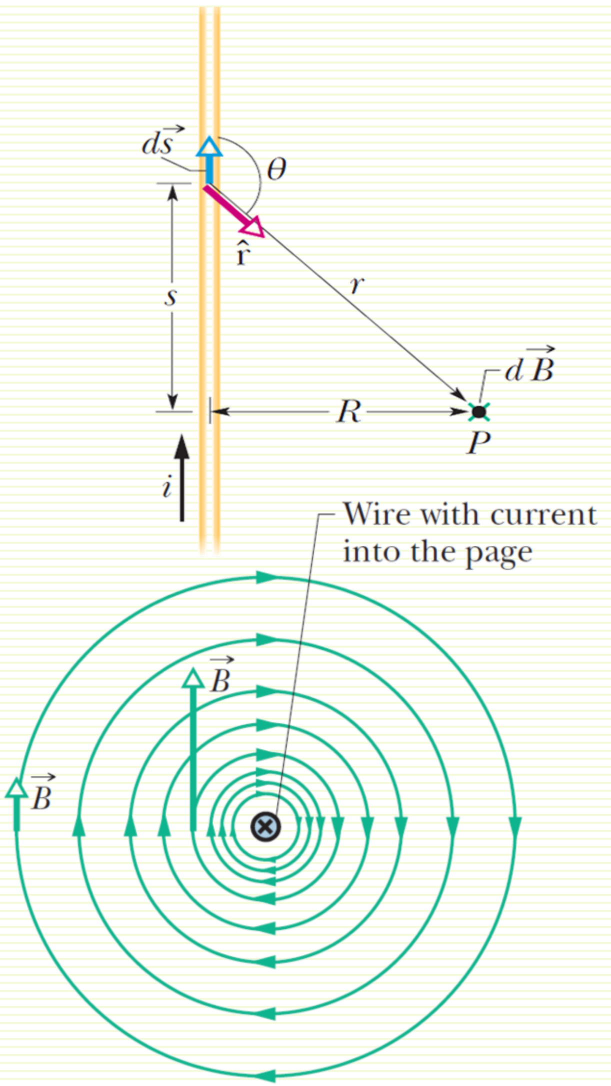
- **Biot and Savart Law:**
- **The magnitude** of the field dB produced at point P at distance r by a current-length element $i\ ds$ can be expressed as
- $dB = \frac{\mu_o}{4\pi}\frac{i\ ds \sin\theta}{r^2}$ (1)
- where θ is the angle between the directions of ds and r (vector that points from ds towards P). Symbol μ_o is a constant, called the permeability constant, whose value is defined to be exactly
- $\mu_o = 4\pi \times 10^{-7}\ T \cdot m/A$
- **The direction** of dB, is shown in the figure as being pointed into the page and is given as the cross product $d\vec{s} \times \vec{r}$. We can therefore write Eq. (1) in vector form as
- $d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\ d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_o}{4\pi}\frac{i\ d\vec{s}\times\vec{r}}{r^3}$ (2)



Magnetic Field due to a Current

Magnetic Field Due to a Current in a Long Straight Wire

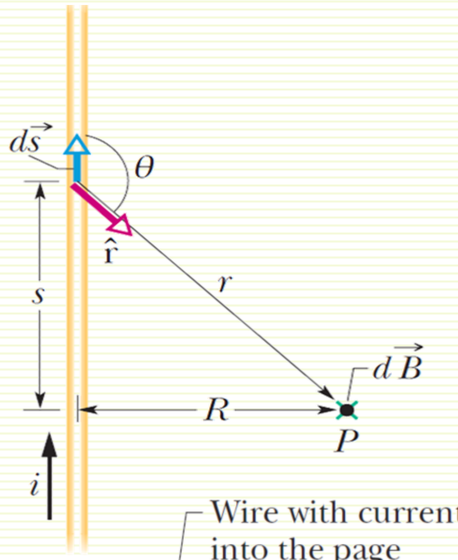
- Using the law of Biot and Savart, we get
- $dB = \frac{\mu_o}{4\pi}\frac{i\, ds \sin\theta}{r^2}$ (1)
- The direction of dB in the figure is that directed into the page.
- The magnitude of dB at point P has this same direction for all the current-length elements into which the wire can be divided. Thus, we can find the magnitude of the magnetic field produced at P by the current-length elements in the upper half of the infinitely long wire by integrating dB in Eq. (1) from 0 to ∞.
- $B = 2\int_o^\infty dB = \frac{\mu_o\, i}{2\pi}\int_0^\infty \frac{\sin\theta\, ds}{r^2}$ (3)
- Form the figure we get
- $r = \sqrt{s^2 + R^2}$ & $\sin\theta = \frac{R}{\sqrt{s^2+R^2}}$

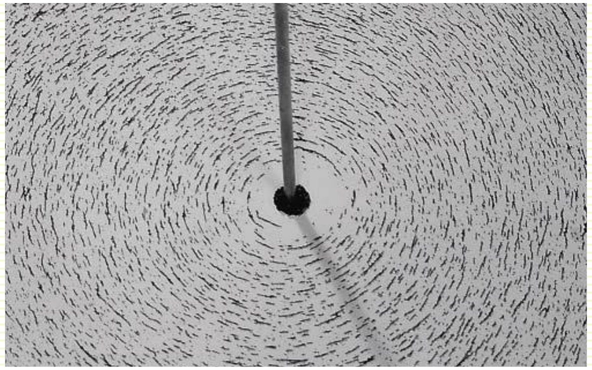


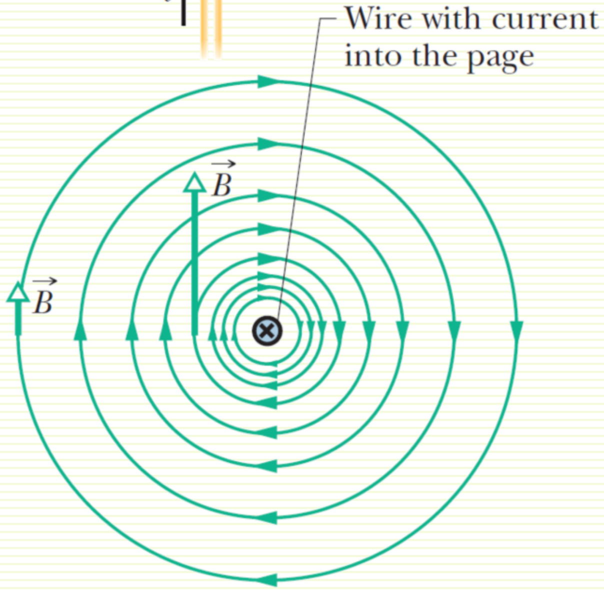
Magnetic Field due to a Current

Magnetic Field Due to a Current in a Long Straight Wire

- Substituting in eq.(3), we get
- $B = \frac{\mu_o\, i}{2\pi} \int_0^{\infty} \frac{R\, ds}{(s^2+R^2)^{3/2}} = \frac{\mu_o\, i}{2\pi\, R} \left[\frac{s}{(s^2+R^2)^{1/2}}\right]_o^{\infty}$
- $\therefore \quad B = \frac{\mu_o i}{2\pi R} \qquad (4)$



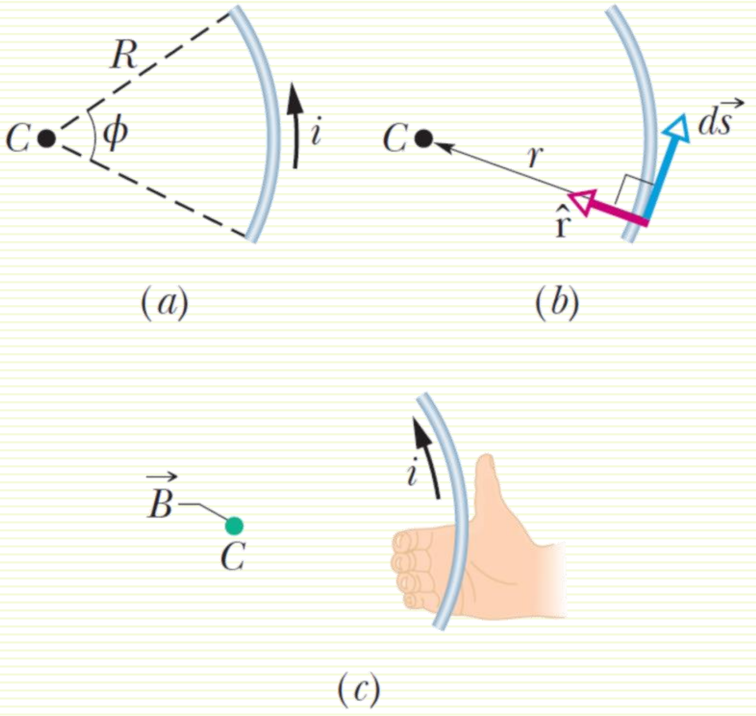




Magnetic Field due to a Current

Magnetic Field Due to a Current in a Circular ARC of Wire

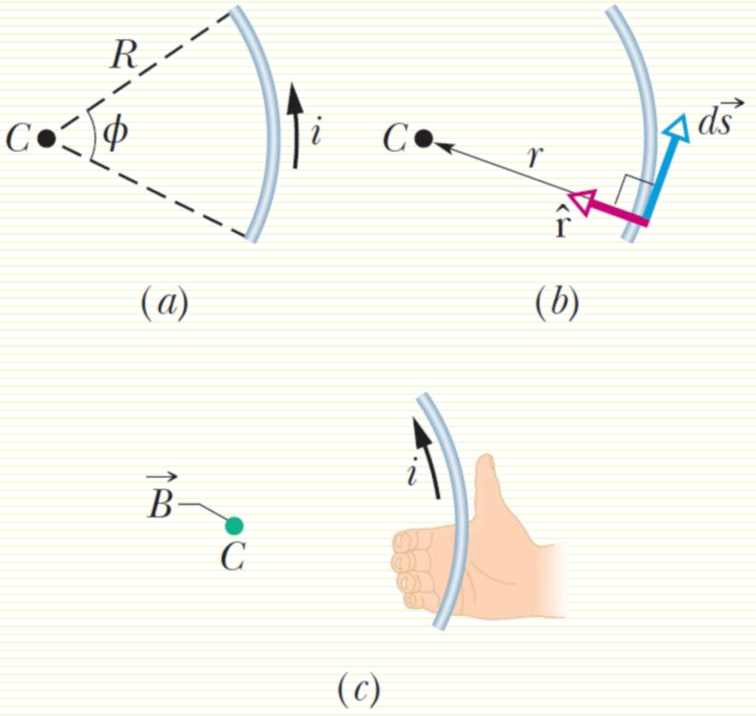
- Figure (a) shows an arc-shaped wire with central angle ϕ, radius R, and center C, carrying current i.
- At C, each current-length element $i\ ds$ of the wire produces a magnetic field of magnitude dB given by Eq. (1). Moreover, as Fig. (b) shows, no matter where the element is located on the wire, the angle θ between the vectors ds and r is 90°; also, $r = R$.
- Therefore, Eq. (1) becomes;
- $dB = \dfrac{\mu_o}{4\pi}\dfrac{i\ ds \sin 90°}{R^2} = \dfrac{\mu_o}{4\pi}\dfrac{i\ ds}{R^2}$ (1)
- **The direction** of the field dB is perpendicular to a radial line extending through point C from the element, either into the plane of Fig. (a) or out of it. To tell which direction is correct, we use the right-hand rule for any of the elements, as shown in Fig. (c). So, dB is out of the plane of the figure,



Magnetic Field due to a Current

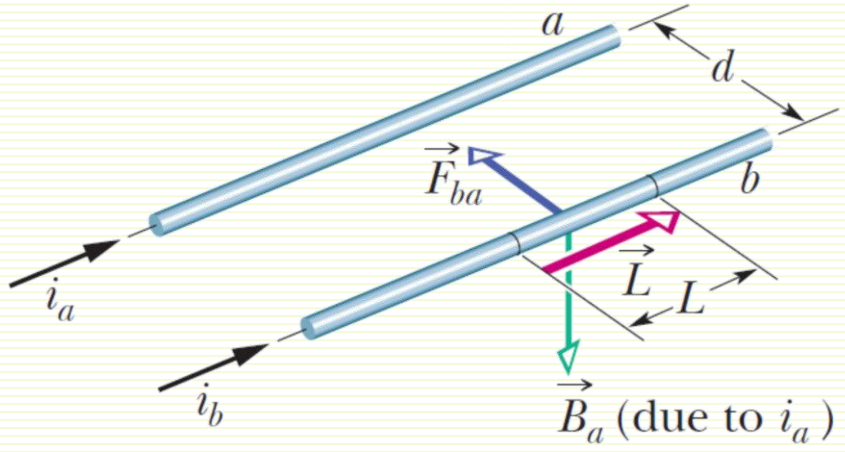
Magnetic Field Due to a Current in a Circular ARC of Wire

- **The total field** can be determined by the integration of eq. (1), so
- $B = \int dB = \frac{\mu_o\, i}{4\pi} \int_0^{\phi} \frac{R\, d\phi}{R^2} = \frac{\mu_o\, i}{4\pi\, R} \int_0^{\phi} d\phi =$
- $\therefore \quad B = \frac{\mu_o\, i\, \phi}{4\pi\, R} \qquad (5)$
- For a wire of a shape of closed circle, we find $\phi = 2\pi$, so the magnetic field at the center of the circle is given by
- $B = \frac{\mu_o\, i}{2\, R} \qquad (6)$



Force Between two Parallel Currents

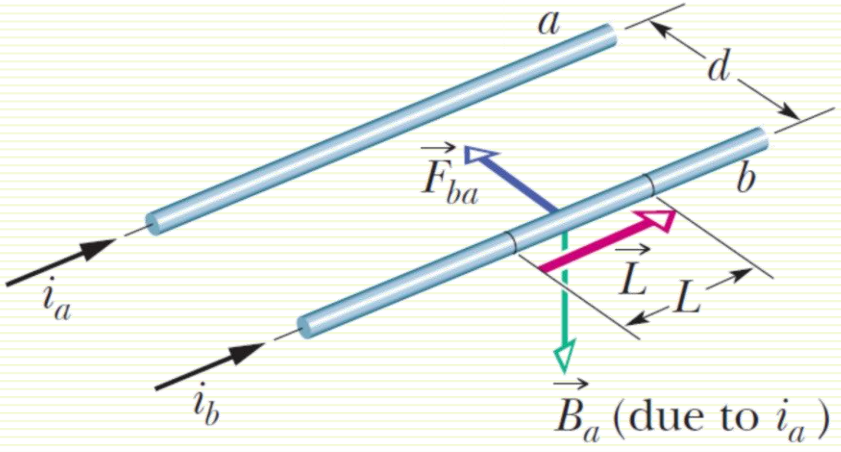
- Two long parallel wires carrying currents exert forces on each other.
- The figure shows two such wires, separated by a distance d and carrying currents i_a and i_b. Let us analyze the forces on these wires due to each other.
- The force F_{ba} on wire b due to the current in wire a can be written as



- $F_{ba} = i_b\, L \times B_a$ (7)
- Where B_a is the filed at b due to the current i_a, and given by
- $B_a = \frac{\mu_o i_a}{2\pi\, d}$
- And L is the length vector of the wire. Since both L and B_a are perpendicular to each other, equation (7) can be written as

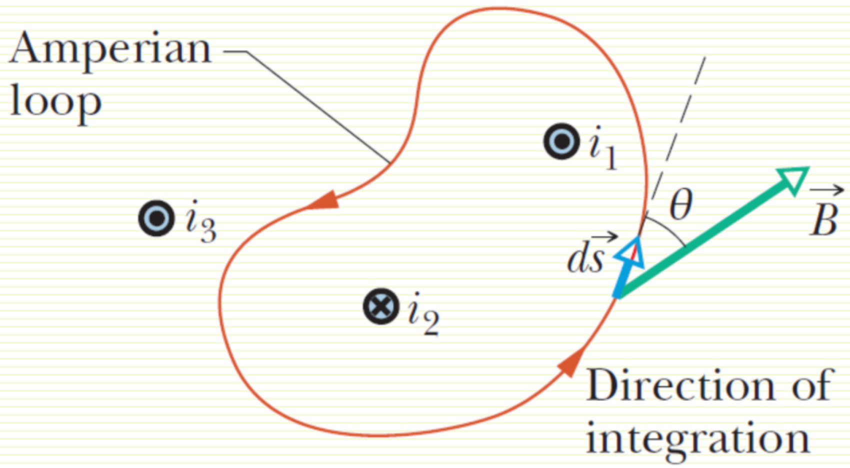
Force Between two Parallel Currents

- $F_{ba} = i_b \, L B_a \sin\theta = \frac{\mu_o \, L \, i_a \, i_b}{2\pi \, d}$ (8)
- The direction of F_{ba} is the direction of the cross product $L \times B_a$.
- Applying the right-hand rule for cross products to $L \times B_a$ in the figure, we see that F_{ba} is directly toward wire a, as shown.
- That is Parallel currents attract each other, and antiparallel currents repel each other.



Ampere's Law

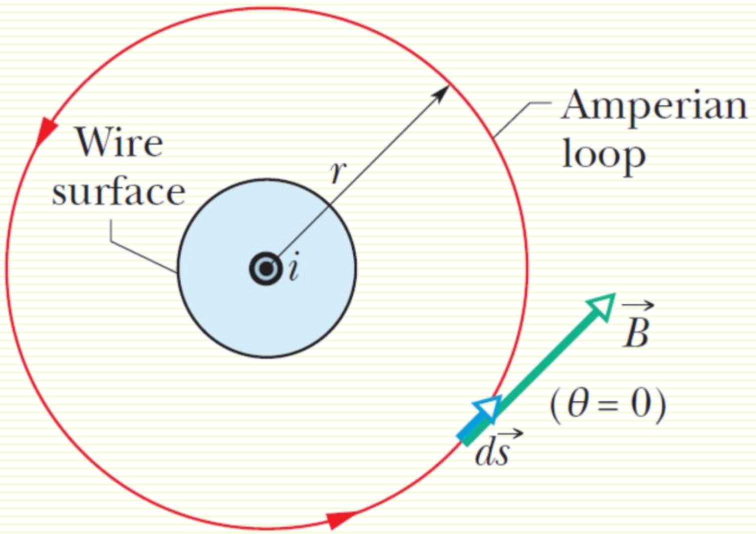
- **Ampere's law is**
- $\oint \vec{B}.d\vec{s} = \mu_o\, i_{enc}$ (9)
- That is the loop integral of the magnetic field around a closed loop ($\oint B.ds$) is equal to the net current i_{enc} enclosed within the loop multiplied by μ_o.
- To apply Ampere's law, we mentally divide the loop into differential vector elements $d\vec{s}$ that are everywhere directed along the tangent to the loop in the direction of integration.



- Thus, Ampere's law can be written as
- $\oint \vec{B}.d\vec{s} = \oint B\cos\theta\ ds = \mu_o\, i_{enc}$ (10)
- **Notes:**
  1. The closed loop through which the linear integration is carried out is called Amperian loop which should be chosen in such a way to facilitate the solution.
  2. The net current within the Amperian loop is $i_{enc} = \boldsymbol{i_1} + \boldsymbol{i_2}$

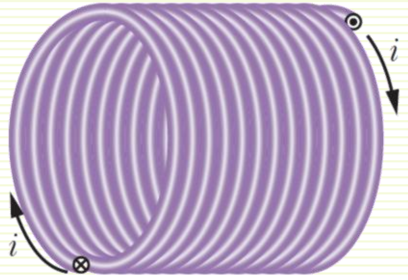
Magnetic Field Outside a Long Straight Wire with Current

- The figure shows a long straight wire that carries current i directly out of the page.
- Equation (9) tells us that the magnetic field $\vec{B}$ produced by the current has the same magnitude at all points that are the same distance r from the wire; that is, the field $\vec{B}$ has cylindrical symmetry about the wire.
- We can take advantage of that symmetry to simplify the integral in Ampere's law (Eqs. (9) and (10)) if we encircle the wire with a concentric circular Amperian loop of radius r, as in the figure.
- The magnetic field then has the same magnitude B at every point on the loop. We shall integrate counterclockwise, so that $d\vec{s}$ has the direction shown in the figure.
- $\oint \vec{B}.d\vec{s} = \oint B\cos\theta \; ds = B\oint \; ds = B(2\pi r) = \; \mu_o i$
- $\Rightarrow \quad B = \dfrac{\mu_o i}{2\pi r} \qquad\qquad (11)$

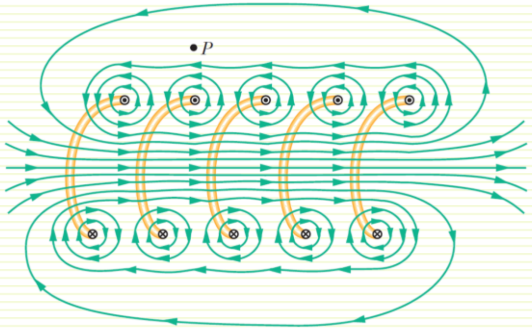


Magnetic Field (inside) of a Solenoid

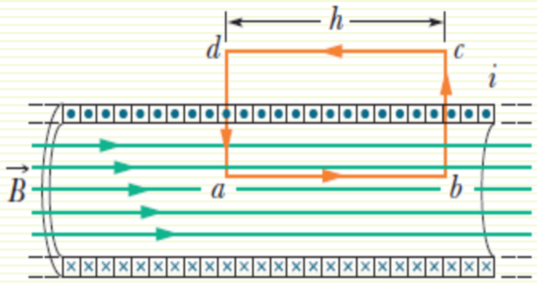
- The solenoid is a helical coil of wire such as in the figure (a).
- The magnetic field due to the current passing in the solenoid is shown in figure (b). It is shown that the magnetic field is uniform inside the solenoid.
- Now let us chose the Amperian loop in shape of rectangular ($abcd$) as shown in figure (c). And let the magnetic field outside the solenoid is zero.
- Now apply Ampere law (Eq. (9))
- $\oint \vec{B}.d\vec{s} = \mu_o \, i_{enc}$
- $\oint \vec{B}.d\vec{s} = \int_a^b \vec{B}.d\vec{s} + \int_b^c \vec{B}.d\vec{s} + \int_c^d \vec{B}.d\vec{s} + \int_d^a \vec{B}.d\vec{s}$
- $\therefore \; \oint \vec{B}.d\vec{s} = \int_a^b \vec{B}.d\vec{s} = Bh = \mu_o \, i_{enc}$



(a)



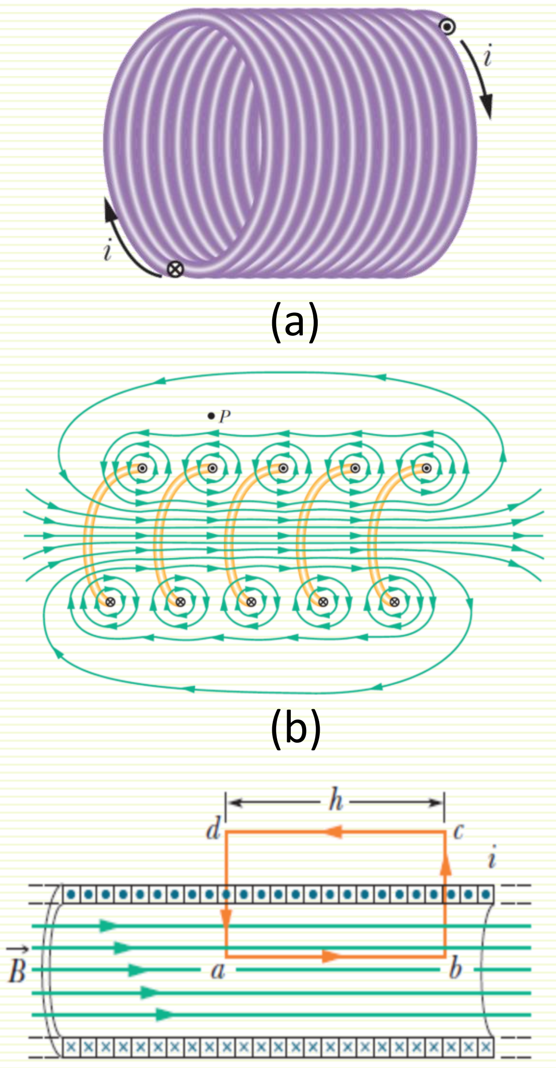
(b)



(c)

Magnetic Field (inside) of a Solenoid

- The second and the fourth term has been vanished because $d\vec{s}$ is perpendicular to $\vec{B}$. The third term has been vanished because the magnetic field outside the solenoid is zero.
- The net current within the loop is given by
- $i_{enc} = i\, n\, h$
- Where n be the number of turns per unit length of the solenoid, and i is the current through one turn, h is the length of the rectangular.
- Now Ampere's law become
- $Bh = \mu_o\, i_{enc} = \mu_o\, i\, n\, h$
- Then
- $B = \mu_o\, i\, n \qquad (12)$



(a) (b) (c)

Solenoids and Toroids

- The toroid, may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet.
- From the symmetry, we see that the lines of B form concentric circles inside the toroid, directed as shown in figure (b).
- The magnetic field B inside the toroid (inside the hollow of the bracelet) can be determined using Ampere's law.
- The Amperian loop is taken as a closed circle of radius r which located inside the toroids as shown in figure (b). Applying Ampere's law yields
- $(B)(2\pi r) = \mu_o iN$
- Where i is the current through the toroid, and N is the total number of turns. Then
- $B = \frac{\mu_o iN}{2\pi}\frac{1}{r}$ (13)