Applied Statistics for Data Science

TUTORIAL 1: REVIEW OF PROBABILITY: AXIOMS, RULES, AND APPROXIMATION

1

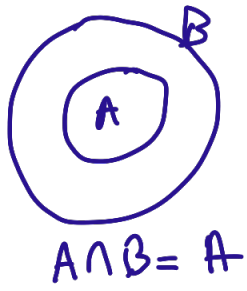
Problem 1

Let A and B be events such that A ⊂ B. Can A and B be independent?

$A \subset B$

$P(A \cap B) = P(A) \times P(B)$

$P(A) = P(A)P(B)$



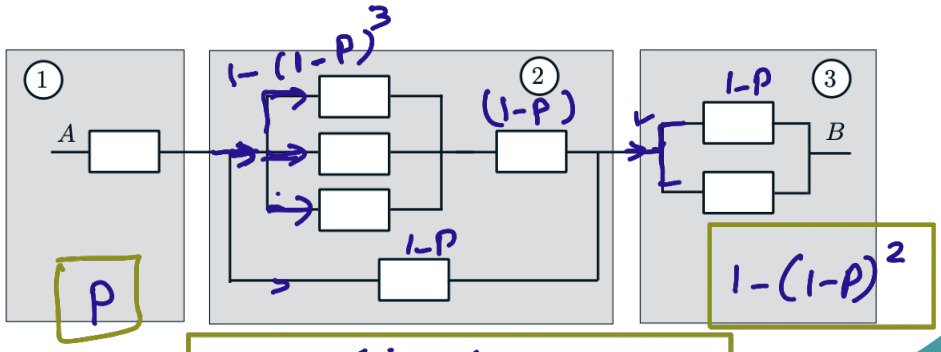
$A \cap B = A$

Thus the event A and B can never be Independent

2

Problem 2

An electrical system consists of identical components that are operational with probability p independently of other components. The components are connected in three subsystems, as shown in the figure. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components. This is the same as requiring that all three subsystems are operational. What are the probabilities that the three subsystems, as well as the entire system, are operational?



$$(1-p)\left(1-p\left(1-(1-p)^3\right)\right)$$

$$= p\left[1-(1-p)^2\right]\left[(1-p)\left(1-p(1-(1-p)^3)\right)\right]$$

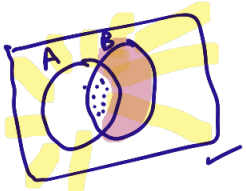
3

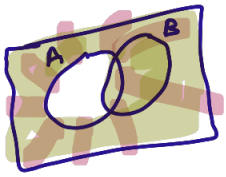
Problem 3

If P(A) = 0.3, P (B) = 0.2, and P(A ∩ B) = 0.1,
determine the following probabilities:

$\bar{A}$ A_c A'

a) P(A') = $1-P(A) = 1-0.3 = 0.7$
b) P(A U B) = $P(A)+P(B)-P(A\cap B) = 0.2+0.3-0.1 = 0.4$
c) P(A' ∩ B) = $P(B)-P(A\cap B) = 0.2-0.1 = 0.1$
d) P(A ∩ B') = $P(A)-P(A\cap B) = 0.3-0.1 = 0.2$
e) P(A U B)' = $1-P(A\cup B) = 1-0.4 = 0.6$
f) P(A' U B) = $P(\bar{A})+P(B)-P(\bar{A}\cap B) = 0.7+0.2-0.1 = 0.8$





$$P(\bar{A}\cap B) = P(B)-P(A\cap B)$$

$$P(\bar{B}\cap A) = P(A)-P(A\cap B)$$

4

$P(G/H) = 0.95$
$P(G/M) = 0.6$
$P(G/P) = 0.1$

$P(H) = 0.4$
$P(M) = 0.35$
$P(P) = 0.25$

Problem 4

Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews.

In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

1) What is the probability that a product attains a good review?
2) If a new design attains a good review, what is the probability that it will be a highly successful product?
3) If a product does not attain a good review, what is the probability that it will be a highly successful product?

0.4	0.35	0.25
H	M	P
G (0.95)	G (0.6)	G (0.1)

① احتمال ان يكون المنتج من فئة جيد

$P(G) = 0.95 \times 0.4 + 0.6 \times 0.35 + 0.1 \times 0.25 = 0.615$

$P(G) = P(G/H)P(H) + P(G/M)P(M) + P(G/P)P(P)$

② $P(H/G) = \dfrac{P(G/H)P(H)}{P(G/H)P(H)+P(G/M)P(M)+P(G/P)P(P)} = \dfrac{0.95 \times 0.4}{0.615} = 0.618$

③ $P(H/\bar{G}) = \dfrac{P(G'/H)P(H)}{P(G'/H)P(H)+P(G'/M)P(M)+P(G/P)P(P)}$

H(0.4)	M(0.35)	P(0.25)
0.05 G'	0.4 G'	0.9 G'

$= \dfrac{0.05 \times 0.4}{0.05 \times 0.4 + 0.35 \times 0.4 + 0.25 \times 0.9} = 0.0536$

5

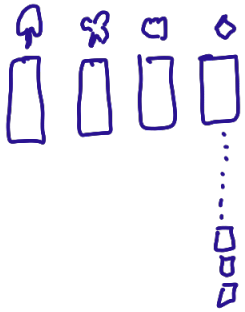
Problem 5

A player is randomly dealt 13 cards from a standard 52-card deck.

(a) What is the probability the 13th card dealt is a king?

(b) What is the probability the 13th card dealt is the first king dealt

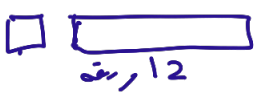
a) $\dfrac{4}{52} = \dfrac{1}{13}$



$13 \times 4 = 52$

4 kings

b) احتمال ان تكون الورقة 13 هي اول king سحوب



12 ورقة

6

$_{52}C_{12}$ العدد الكلي لاحتمال سحب 12 ورقة من المجموعة الكلية

$_{48}C_{12}$ عدد طرق سحب الـ 12 ورقة بدون ورقة king

احتمال سحب 12 ورقة بدون King

$$4 \cdot \frac{{}_{48}C_{12}}{{}_{52}C_{12}}$$

احتمال الورقة 13 ان تكون King

$$\frac{1}{13}$$

الاحتمال الكلي لسحب 12 ورقة تكون King

$$= \frac{1}{13} \cdot 4 \cdot \frac{{}_{48}C_{12}}{{}_{52}C_{12}} = 0.1038$$