Umm Al-Qura University
Computers and Information Systems College

جامعة أم القرى
UMM AL-QURA UNIVERSITY

Computer Science Department
1444 - 2nd Semester
Discrete Structures II – 14011802-3
Discrete Structures II – CS2102

Tutorial Exercises

Week: 1
Main Topic: Counting (Part 1)
Topics Covered: Sum Rule, Product Rule, Subtraction Rule, Division Rule, Tree Diagrams, Pigeonhole Principle.

From Rosen's book - Exercises (Pages 416-420)

1. There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

a) $18 \cdot 325 = 5850$

b) $18 + 325 = 343$

4. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

$12 \cdot 2 \cdot 3 = 72$

6. There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?

$4 \cdot 6 = 24$ routes.

8. How many different three-letter initials with none of the letters repeated can people have?

$26 \cdot 25 \cdot 24 = 15{,}600.$

9. How many different three-letter initials are there that begin with an *A*?

$1 \cdot 26 \cdot 26 = 676$

16. How many strings are there of four lowercase letters that have the letter x in them?

$$\text{Number of strings of length } 4 = 26^4$$
$$\text{Number of strings of length 4 other than } x = 25^4$$
$$26^4 - 25^4 = 66,351 \text{ strings.}$$

26. How many strings of four decimal digits
a) do not contain the same digit twice?
b) end with an even digit?
c) have exactly three digits that are 9s?

a) 10.9.8.7=5040
b) 10.10.10.5= 5000
C) 4.9=36

30. How many license plates can be made using either three uppercase English letter followed by three digits or four uppercase English letters followed by two digits?

$$26^3 10^3 + 26^4 10^2 = 63,273,600$$

34. How many different functions are there from a set with 10 elements to sets with the following numbers of elements?
a) 2 **b)** 3 **c)** 4 **d)** 5

a) $2^{10} = 1024$

b) $3^{10} = 59,049$

c) $4^{10} = 1,048,576$

d) $5^{10} = 9,765,625$

50. How many bit strings of length seven either begin with two 0s or end with three 1s?

first case (beginning with 2 0′s) there are: $1*1*2*2*2*2*2=32$

Second case (end with three 1′s): $2*2*2*2*1*1*1=16$

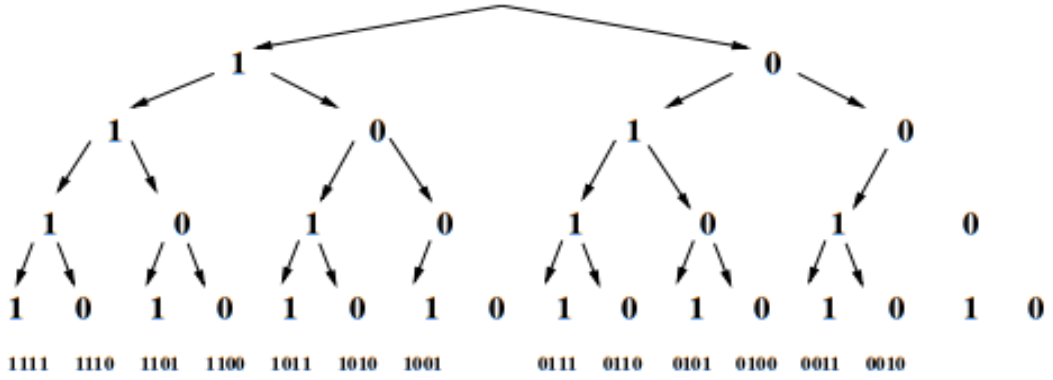
String starting with two 0's end with three 11's : $1*1*2*2*1*1*1=4$

add disjoint cases: $32 + 16 - 4 = 44$

54. Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

$38+23-7 = 54$

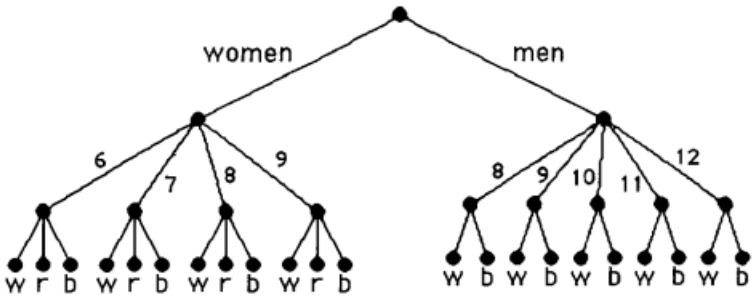
66. Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.



71. **a)** Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes 6, 7, 8, and 9, and the man's shoe comes in sizes 8, 9, 10, 11, and 12. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black. Use a tree diagram to determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women.

b) Answer the question in part (a) using counting rules.

a)



b) possible women's models

$4 \cdot 3 = 12$.

men's models

$5 \cdot 2 = 10$.

the answer is $12 + 10 = 22$.

Exercises (Pages 426-428)

4. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?.

b) How many balls must she select to be sure of having at least three blue balls?

a) $N=k\,(r-1)+1$

$N=2(3-1)+1=5$

b) 13

11. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

$$N = k(r-1) + 1$$

$$50(100-1)+ 1$$

$$50(99) + 1 = 4951$$

17. A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so at least two products must be stored in the same bin?

$$50 \cdot 85 \cdot 5 = 21{,}250$$

$$N = k\,(r-1) + 1$$

$$21{,}250\,(2-1) + 1 = 21{,}251$$

40. Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.

$$4 + (4 \times 4) = 20 \text{ cables.}$$