Week 1:

Optimization and Constraint Programming

Course Logistics

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Department of Computer Science
Umm Al-Qura University

- About this course

- Who should take this course

- Administrative issues:
  - Assessment scheme
  - Course and class policies
  - Tentative course timeline

About this Course

- This course introduces the basics of optimization theory, numerical algorithms, and applications.
- The course is divided into four main parts:
  - Unconstrained optimization methods (One dimensional and multi-dimensional optimization, first and second order optimization methods).
  - Linear programming (simplex method)
  - Constrained optimization (Lagrange multipliers, Karush-Kuhn-Tucker conditions, interior point methods)
  - Constraint programming.

About this Course - Course Main Objectives

1. Understand the overview of **optimization techniques**, **concepts of design space**, **constraint surfaces** and **objective function**.
2. Understand **unconstrained optimization** methods and algorithms.
3. Understand **constrained optimization** concepts and algorithms.
4. Formulate real-life problems with **Linear Programming** and solve the Linear Programming models using graphical and simplex methods.
5. Understand and apply **constraint programming algorithms**.

About this Course

Topics to be covered

- Introduction to optimization.
- Generic formulation of optimization problems.
- Unconstrained Optimization methods.
- Linear Programming and simplex methods.
- Constrained optimization methods.
- Constraint Programming.

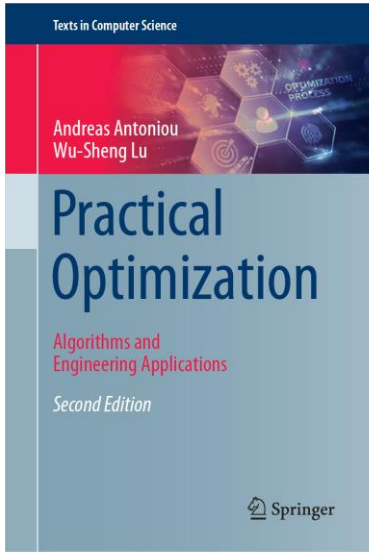
About this Course

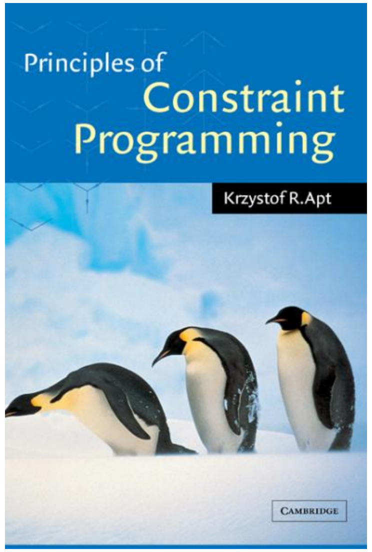
UMM AL-QURA UNIVERSITY

Main Textbooks:

Practical Optimization: Algorithms and Engineering Applications. (2nd Edition)
Andreas Antoniou and Wu-Sheng Lu.

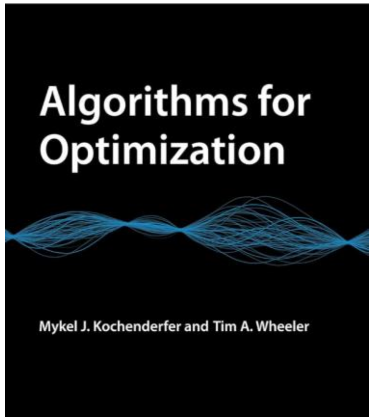
Principles of Constraint Programming.
Apt K.





Alternative Textbook:

Algorithms for Optimization.
Mykel J. Kochenderfer and Tim A. Wheeler.



Who Should Take This Course?

- Knowledge of programming.
  - Topics covered in “Computer Programming (2)”
- Knowledge of linear algebra.
  - Topics covered in “Linear Algebra”
- Good problem-solving skills.
- Good English
  - All references are in English.

Assessment and Workload

- 20% [gr] 5% × **4 Practical assignments**
  - You should form a team of 3 students.
  - Cross-section is NOT allowed.
- 10% [id] 4 **Quizzes.**
  - Only best 3 will be considered.
- 20% [id] **Midterm exam**
- 50% [id] **Final exam**
- +3% [id] **Online DataCamp Courses**

- Workload:
  - For an average grade, 1 credit = 3 hours/week of academic activities: lectures, assignments, reading, discussions, .. etc.
  - Your course has 3 credits (i.e. 3x3=9 hpw).
  - There will be 4 hpw of lectures & labs (i.e. you are expected to work for 9-4=5hpw).

Class Policies

- Please be at the class on time. Latecomers will not be allowed to attend the class.
- Please turn off your mobiles while you are in the class. You would be forced to leave the class if your mobile rang.
- Any absence from any assessment (i.e. quizzes, tutorial assignments, exams) would make you lose their marks unless you got an official excuse.
- If you miss a class, you must demonstrate a valid/official excuse. Otherwise, your absence will be counted. Your valid excuse must be presented no later than 48 hours from your missed class.
- According to the university policies, each student is expected to attend at least 85% of the contact hours. Otherwise, the student will be considered denied (DN-محروم).
- Cheating and plagiarism are considered felonies. Any form of cheating revealed during exams will result in an F grade for the Exam.
- In case plagiarism is discovered in a submitted coursework, all contributors will get a zero mark in that specific coursework.

Tentative Timeline

Week	Theory (100 minutes)		Exercises (50 minutes)	Lab (100 minutes)	Assignment	Exams
	Topic	Reading				
1 26/11	Introduction to optimization	AL-Ch1	Real-world optimization applications	Optimization software tools		
2 3/12	Generic formulation of optimization problems	AL-Ch2	Formulating simple optimization problems	Problem formulation		
3 10/12	Unconstrained optimization methods	AL-Ch3 AL-Ch4	Unconstrained optimization	Unconstrained optimization algorithms implementation	Assignment 1 due	Quiz 1
4 17/12	*Long weekend*		Advanced problems in unconstrained optimization			
5 24/12	Linear programming and simplex methods	AL-Ch11 AL-Ch12	Linear programming problems	Simplex method implementation.	Assignment 2 due	Quiz 2
6 31/12	Constrained optimization methods	AL-Ch10 AL-Ch13	Constrained optimization problems.	Constrained optimization algorithms implementation		Midterm
7 7/1	*Midterm break*					
8 14/1	Constraint programming	AK-Ch1-3	Constraint programming	Simple constraint programming exercises.	Assignment 3 due	Quiz 3
9 21/1		AK-Ch4-5	Advanced constraint programming problems.	Complex exercises in constraint programming.		
10 28/1	*Long weekend*				Assignment 4 due	
11 4/2	Review session		Review session	Review session		Quiz 4
12 11/2	***Final exam***					

Topics and times are subject to modification according to the university regulation.

Week 1:

Optimization and Constraint Programming

Introduction to Optimization

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Today's Objective

After this session, you should be able to:

- **Recall** the basic concepts and definitions of optimization.
- **Explain** the importance and applications of optimization in various fields.
- **Identify** real-world problems where optimization can be applied.

Agenda

- Overview of optimization
- Basic Concepts in Optimization
- The Optimization Problem
- Challenges in Optimization

What is Optimization?

تحسين رياضي

- Optimization process is also known as **mathematical optimization** or **mathematical programming**.

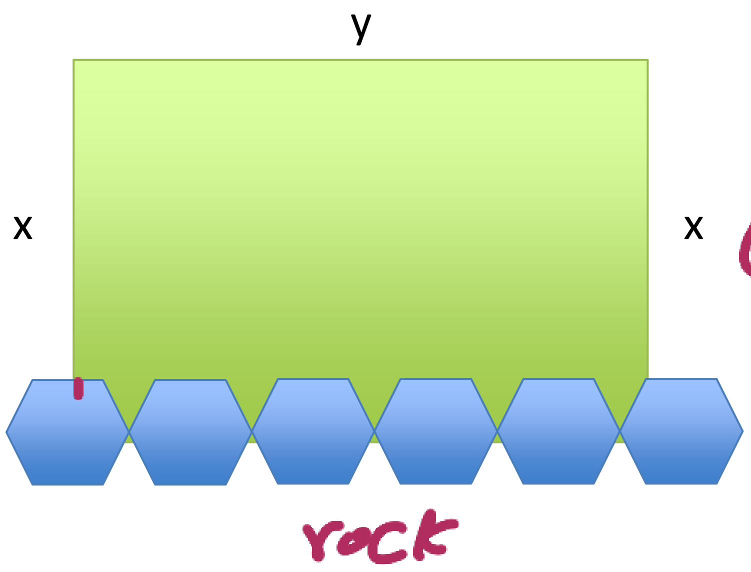
برمجة رياضية

اختيار العنصر الأفضل حسب معايير محددة من خيارات بديلة

- It is the selection of the “best” element, with regard to some criteria, from a set of available alternatives.
  - e.g. Maximize the profits تعظيم الأرباح
  - e.g. Minimize the expenses تقليل المصروفات

Optimization Problem – Example 1

A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 100m of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?



لدي سور (سلك)
طوله ١٠٠ متر واريد
اكون حديقة (١ ، ٢ و ٣)
بحيث تكون المساحة
اكبر ما يمكن.

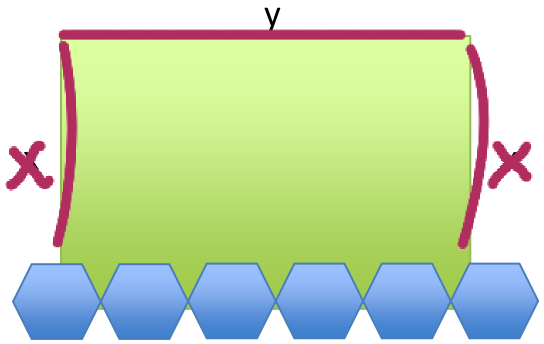
Optimization Problem – Example 1

Let A be the area of the garden that we want to maximize.

$$A = x\,y$$

We want to find the maximum possible area subject to the constraint that the total fencing is 100m. Hence:

$$2x + y = 100 \qquad y = 100 - 2x$$



Substitute the values of y in A:

$$A = x\,(100 - 2x) = \boxed{100x - 2x^2}$$

We cannot have length as negative numbers, so:

$$x > 0$$
$$y > 0$$

الاطوال يجب ان تكون موجبة

$$y = 100 - 2x > 0$$

We can have a maximum value for x. Since $y = 100 - 2x$ and $y > 0$, then:

$$100 > 2x$$

$$x < 50$$

x يجب ان تكون 50

$$50 > x$$

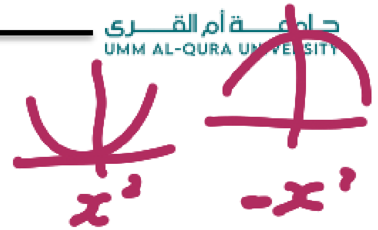
الهدف من السؤال هو تعظيم المساحة

Optimization Problem – Example 1

So we want:

$$\underline{x^*} = \underset{x}{\text{maximize}}\ \boxed{A = 100x - 2x^2},\qquad \underbrace{0 < x < 50}_{\text{Constraint}}$$

Variable/Parameter/Feature



optimal solution or maximizer

الحل الأفضل

objective function

القيود

لحساب القيم القصوى

- حساب المشتقة ونساويها بالصفر

$A = 100x - 2x^2$

$A' = 100 - 4x$

$100 - 4x = 0$

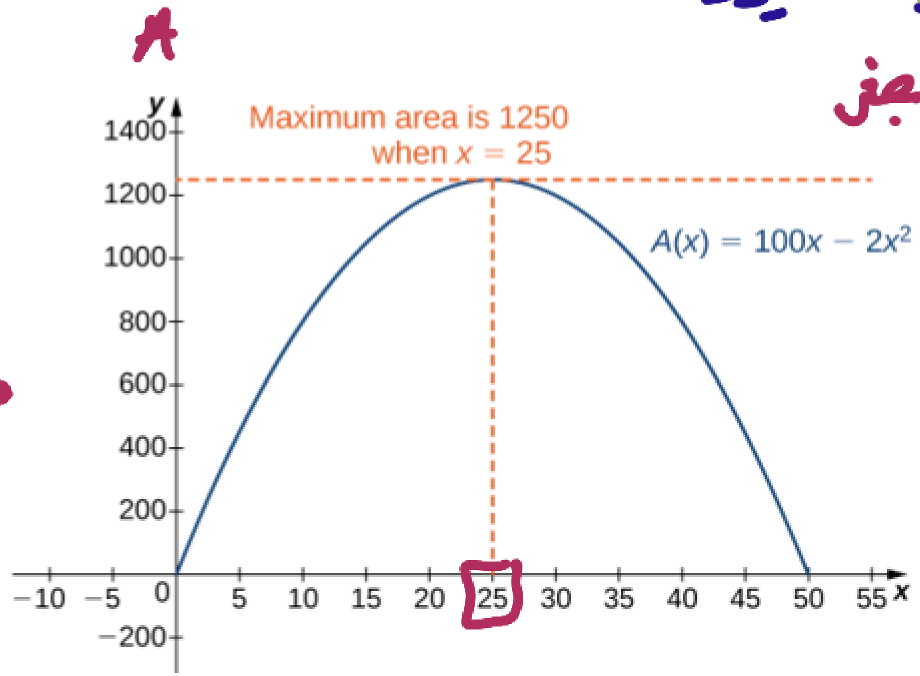
$100 = 4x$

$\boxed{x = 25}$

$A = 100x - 2x^2$

x^* الحل الأفضل

قيمة x التي تعطي أكبر مساحة



Optimization Problem – Example 2

Suppose you have a set of activities you want to engage in during a day, such as **work, exercise, leisure, learning, and rest**. Each activity contributes to your overall productivity and satisfaction in different ways, and **you have a limited number of hours in a day**. The goal is to allocate time to each activity optimally to **maximize your overall well-being and productivity**.

Well-being and Productivity Score for Each Activity:

Work: 5 points per hour
(up to a certain limit, say 8 hours)

Exercise: 10 points per hour
(up to a certain limit, say 2 hours)

Leisure: 8 points per hour
(up to a certain limit, say 4 hours)

Learning: 7 points per hour
(up to a certain limit, say 3 hours)

Rest: 4 points per hour
(at least 7 hours required for health)

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Optimization Problem – Example 2

Well-being and Productivity Score for Each Activity:

x_1 Work: 5 points per hour (up to a certain limit, say 8 hours)

x_2 Exercise: 10 points per hour (up to a certain limit, say 2 hours)

x_3 Leisure: 8 points per hour (up to a certain limit, say 4 hours)

x_4 Learning: 7 points per hour (up to a certain limit, say 3 hours)

x_4 Rest: 4 points per hour (at least 7 hours required for health)

Let x_1, x_2, x_3, x_4, x_5 are number of hours allocated to Work, Exercise, Leisure, Learning, and Rest, respectively.

ناتج كل النشاطات التي نقوم

Let Z be the Well-being and Productivity Score.

$$Z = 5x_1 + 10x_2 + 8x_3 + 7x_4 + 4x_5$$

How will you compute Z?

Is there any condition or constraint on the number of hours to be allocated?

Optimization Problem – Example 2

Well-being and Productivity Score to be maximize:

$$Z = 5x_1 + 10x_2 + 8x_3 + 7x_4 + 4x_5$$

<u>**Constraints:**</u> قيود

1. **Time Constraint**: Total allocated time should not exceed 24 hours.

   $x_1 + x_2 + x_3 + x_4 + x_5 \leq 24$

   القيد ١ :- مجموع عدد الساعات يجب ان تكون اقل من 24

2. **Activity-specific Constraints**: Limits on hours for each activity for optimal benefit.
   1. Work: $x_1 \leq 8$
   2. Exercise: $x_2 \leq 2$
   3. Leisure: $x_3 \leq 4$
   4. Learning: $x_4 \leq 3$
   5. Rest: $x_5 \leq 7$

   القيد 2 :- عدد الساعات لكل نشاط يجب انه لا تتعدى القيمه المسموحة.

3. **Non-negativity Constraints**:

   $x_1, x_2, x_3, x_4, x_5 \geq 0$

   القيد 3 :- لا يمكن ان تكون احدى القيم سالبة

Optimization Problem – Example 2

Can be written as follows: تلخيص المطلوب من السؤال والقيود

المطلوب

$$\text{maximize} \quad Z = 5x_1 + 10x_2 + 8x_3 + 7x_4 + 4x_5$$

القيود

<u>subject to:</u>

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 24$$
$$x_1 \leq 8$$
$$x_2 \leq 2$$
$$x_3 \leq 4$$
$$x_4 \leq 3$$
$$x_5 \leq 7$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Optimization Problem – Example 2

Variables/Parameters/Features

$$\underset{x_1, x_2, x_3, x_4, x_5}{\text{maximize}} \quad \boxed{Z = 5x_1 + 10x_2 + 8x_3 + 7x_4 + 4x_5}$$

objective function

الدالة/الهدف
التي نريد حساب
تحسين لها

subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 24$$
$$x_1 \leq 8$$
$$x_2 \leq 2$$
$$x_3 \leq 4$$
$$x_4 \leq 3$$
$$x_5 \leq 7$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Constraints

القيود

Optimization Problem – Example 2

We can simplify the notation of the objective function using the **matrix notation**. صيغة، رموز

$$\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5]$$

معاملات

Let's call the factors for each parameter α:

$$\boldsymbol{\alpha} = [5 \quad 10 \quad 8 \quad 7 \quad 4]$$

So the objective function will be:

$$Z = \boldsymbol{\alpha}^T \mathbf{x}$$

يمكن كتابة المعادلات بشكل مختصر على صوره
مصفوفات

$$\alpha = \begin{bmatrix} 5 \\ 10 \\ 8 \\ 7 \\ 4 \end{bmatrix}$$

$$X = [x_1 \; x_2 \; x_3 \; x_4 \; x_5]$$

$$Z = \alpha^T X$$

Basic Concepts of Optimization

متغيرات القرار

- **Variables**, aka **decision variables**, are variables adjust to reach the optimal solution. They can be written as a single column-vector $\mathbf{x}$.

  عليه كتابه المتغيرات على شكل عمود متجه

  – e.g. height, width in Example 1 x y

  – e.g. number of hours for each activity in Example 2 x_1 x_2 x_3 ..

تعبير رياضي يصف هدف المعادلة التي نريد تعظيمه او تصغيره

- **Objective function** is a mathematical expression that describes the problem's goal to be maximize or minimize.

  – e.g. Area, A, in Example 1

  – e.g. Well-being and Productivity Score, Z, in Example 2

المحددات :. حدود او قيود للمتغيرات تمثل المسموح به (حدود مادية ، حدود معا

Basic Concepts of Optimization

القيود

- **Constraints**, are restrictions or limits over the variables to define the feasible region. They could be physical limitations, resource limitations, or other kinds of limitations specific to the problem.

  منطقة السماح

  - Equality constraints in the form: $x = 0$
  - Inequality constraints in the form: $x \geq 0$

تحسين مقيد

التحسين التي معادلة تعتمد على قيود

- An optimization problem may entail a set of equality and/or inequality constraints is said to be a **constrained** optimization problem. Otherwise, it is **unconstrained** optimization problem.

تحسين غير مقيد (بدون قيود)

Feasible Region

منطقة السماح

- **Feasible region** is the set of all points that satisfy the constraints.

  كل النقاط التي تحقق القيود

  $x+y \le 5$

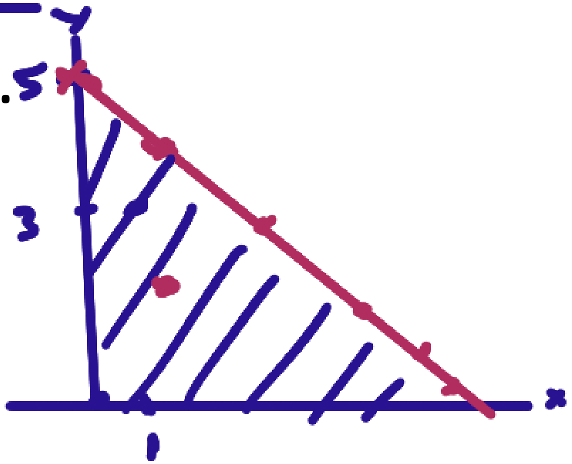
  - Any point not in the feasible region is a **nonfeasible point**

    نقطة غير مسموح

  - The optimum point, $\mathbf{x}^*$, must be in the feasible region.

يجب ان تكون افضل قيمه (اكبر، اصغر) ممكن

منطقة السماح



$$
\begin{array}{ll}
\underset{\mathbf{x}}{\text{minimize}} & f(\mathbf{x}) \\
\text{subject to} & \mathbf{x} \in \mathcal{X}
\end{array}
$$

x تنتمي لمنطقة السماح

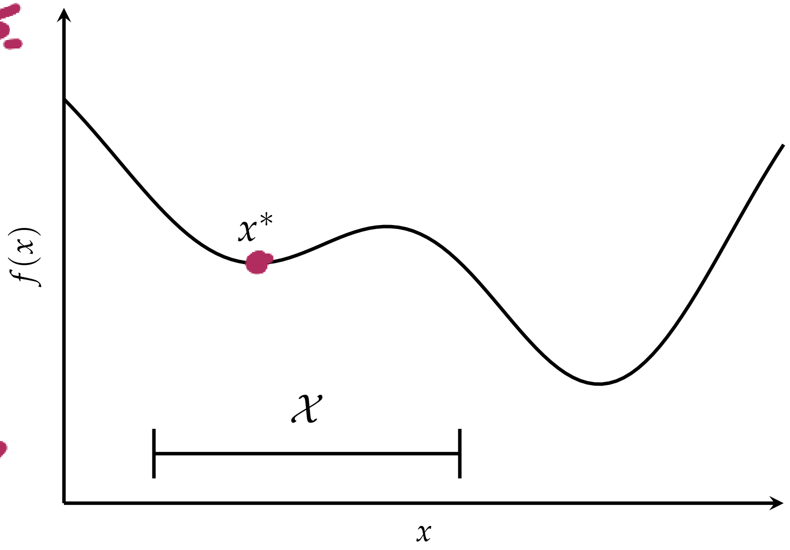


Image source: Kochenderfer & Wheeler (2019). *Algorithms for optimization*. Mit Press.

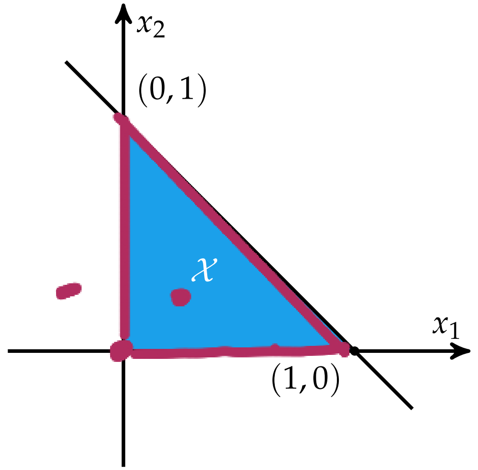
Feasible Region

Inequality constraints, e.g. $x_1 \geq 0$, divide the points in the domain space into three types:

- نقاط داخلية
- Interior points are feasible points. e.g. when $x_1 > 0$
- نقاط الحدود
- Boundary points may or may not be feasible. e.g. when $x_1 = 0$
- نقاط خارجية
- Exterior points are nonfeasible points. e.g. when $x_1 < 0$

Example:

$$
\begin{aligned}
\underset{x_1, x_2}{\text{minimize}} \quad & f(x_1, x_2) \\
\text{subject to} \quad & x_1 \geq 0 \\
& x_2 \geq 0 \\
& x_1 + x_2 \leq 1
\end{aligned}
$$



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النقاط الحرجة

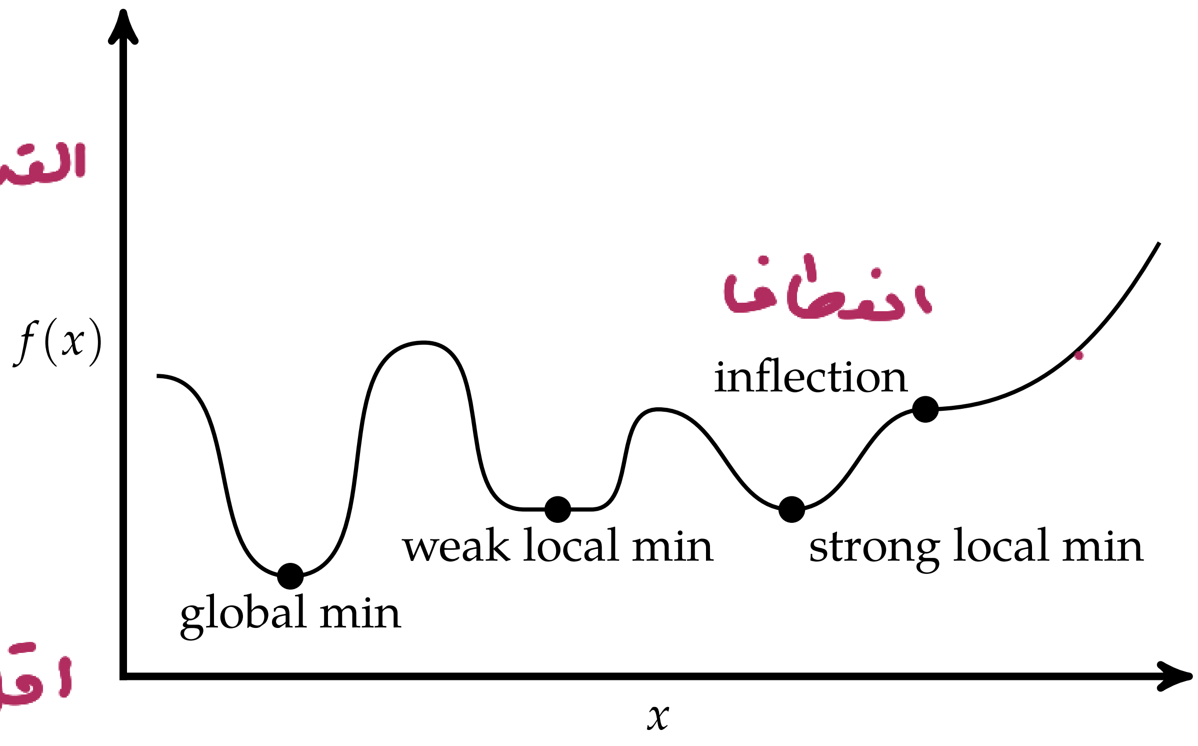
Critical Points

قيمة صغرى محلية

- A **local minimum** of a function is a point where the function value is smaller than at nearby points. القيمة أصغر من القيم المحيطة بها

قيمة صغرى مطلقة

- A **global minimum** is a point where the function value is smaller than at all other feasible points. أقل قيمة للجميع، لقيم الأخرى منها



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Critical Points

- A **local minimum** of a function is a point where the function value is smaller than at nearby points.

  نقطة صغرى محلية قوية

  - It can be a **strong local minimum** if it is unique within its neighborhood. Otherwise, it is a **weak local minimum**.

    مميزه

    لاتكون مميزه

    نقطه صغرى محليه ضعيفه

- A **global minimum** is a point where the function value is smaller than at all other feasible points.

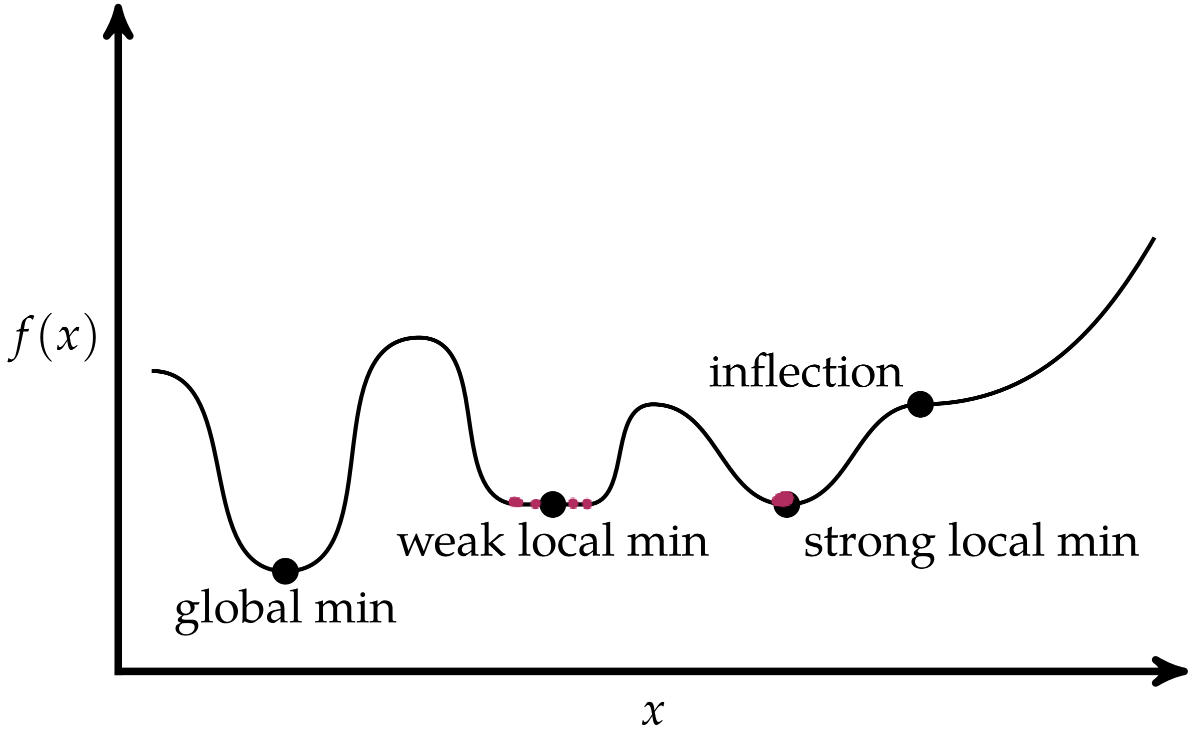


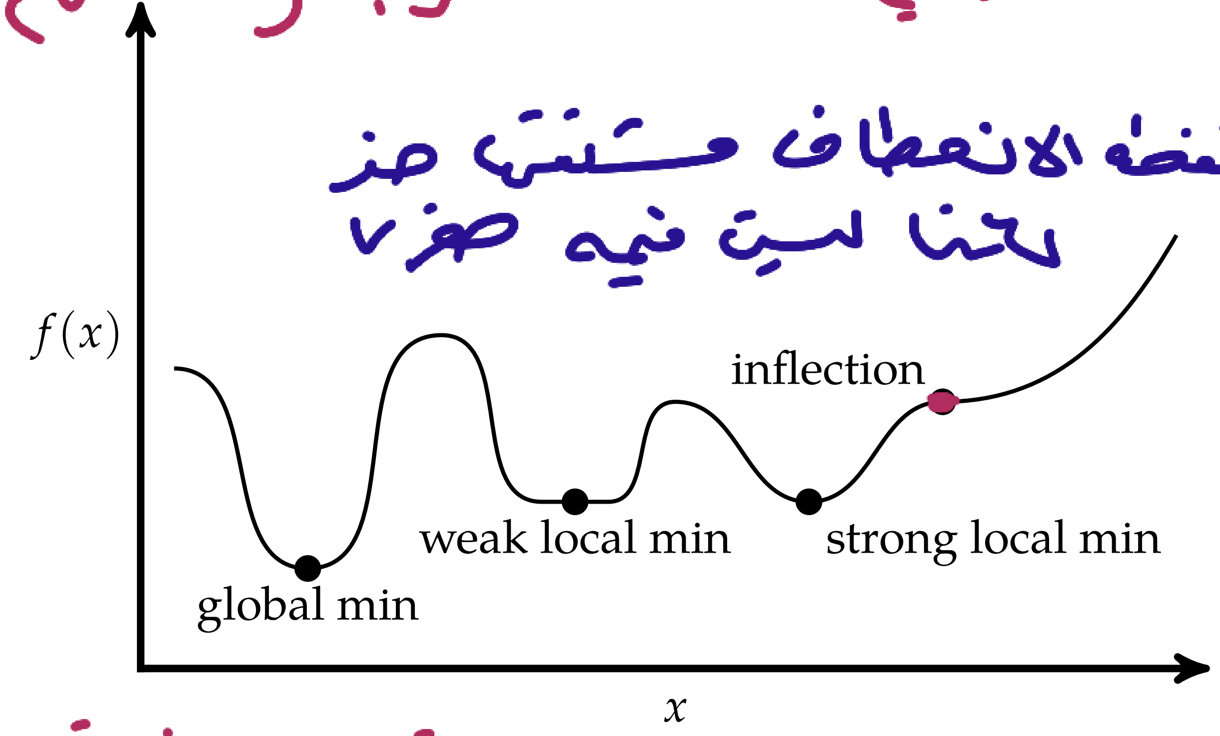
Image source: Kochenderfer & Wheeler (2019). *Algorithms for optimization*. Mit Press.

Critical Points

المشتقه تساوي صفر عند
جميع النقاط (الحرجة) (عظمى، صغرى)

- The derivative is zero at all local and global minima.

- A zero derivative is a ***necessary*** condition for a local minimum, it is ***not a sufficient*** condition.
  - e.g. inflection has a zero derivative but it is not a local minimum.



عند حساب المشتقة ومساواتها = صفر اذا طلبت
ان نقول ان هذه النقطة قيمة صغرى لكن ليس شرط
لانه ممكن ان تكون نقطة انعطاف

Image source: Kochenderfer & Wheeler (2019). *Algorithms for optimization*. Mit Press.

Types of Optimization

- **Linear Programming (LP):**
  - Linear objective functions and linear constraints.
  - No powers or products of variables.

البرمجة الخطية
- المعادلات والقيود كلها خطية
لا أس ولا ضرب متغيرات

- **Integer Programming (IP):**
  - Some or all of the decision variables integers.

- برمجة الصحيحة
بعض المتغيرات (اعداد صحيحة)

- **Quadratic Programming (QP):**
  - Quadratic objective functions and linear constraints.

- برمجة تربيعية
المعادلة الهدف تكون تربيعية
القيود تكون خطية

- **Nonlinear Programming (NLP):**
  - Nonlinear objective functions or nonlinear constraints.
  - ~~No~~ powers or products of variables.

- برمجة غير خطية
معادلة الهدف والقيود غير خطية

- **Dynamic Programming (DP):**
  - Problems to be broken down into simpler subproblems, which are solved independently..
  - Effective for optimization in stages or over time.

- برمجة ديناميكية

يتم تقسيم المشكلة الى مسائل جزئية يتم حل كل واحدة على حدى
فعالة في حالة التحسين في مراحل أو مدد زمنية

تطبيقات

Real-World Applications of Optimization

جامعة أم القرى
UMM AL-QURA UNIVERSITY

- **Application in Logistics:** تطبيقات اللوجستية
  - Minimize delivery times and transportation costs. تقليل كلفة المواصلات و زمن التوصيل
  - Considering factors like traffic, distance, and package sizes.

- **Finance Sector:** مجال مالي
  اختيار افضل تجميع للبضائع يؤدي الى اكبر عائد
  - Choosing the best combination of stocks and bonds to maximize returns while minimizing risk.

الرعاية الصحية
- **Healthcare Industry:** التحديد الافضل للمصادر و جداول الموظفين وغرف العمليات
  - Resource allocation and scheduling, such as schedule staff and operating rooms efficiently.

الانتاج والتصنيع
- **Manufacturing and Production:** الحفاظ على جودة مع بقاء السعر ثابت
  - 'Lean Manufacturing,' helps in maintaining high-quality standards while keeping costs low.

Real-World Applications of Optimization

جامعة أم القرى
UMM AL-QURA UNIVERSITY

- **Energy Sector:** مجال الطاقة

  امثلا، الخلطة الافضل من مصادر الطاقة، تحقق متطلبات الطاقة وتقلل الاثار البيئية

  – Determining the optimal mix of energy sources like solar, wind, and fossil fuels to meet energy demands while reducing environmental impact.

  الاتصالات

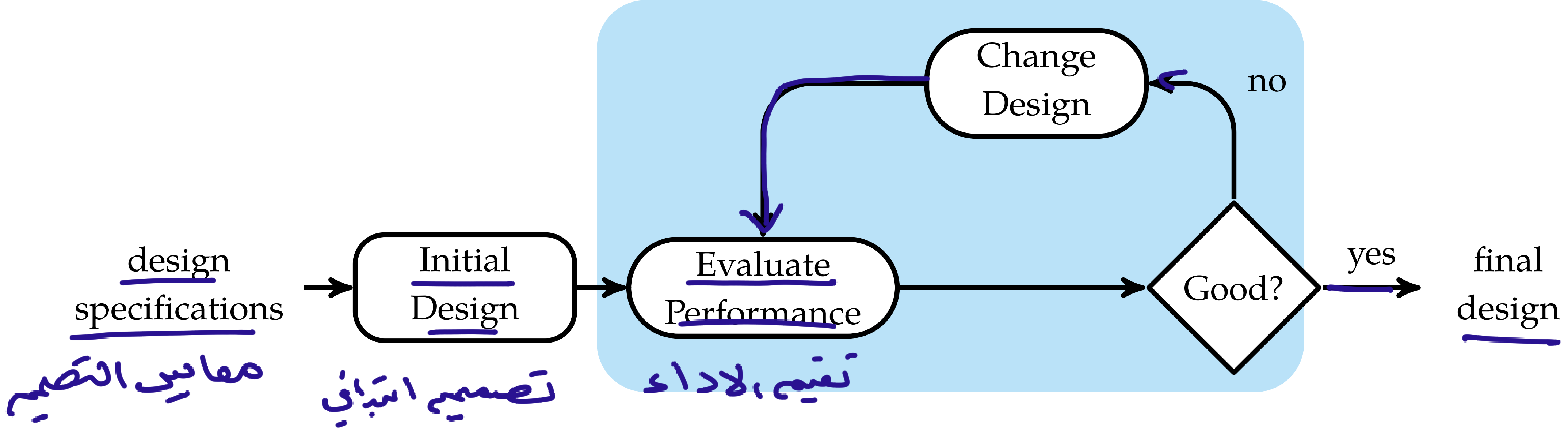
- **Telecommunications:**
  – Placement of cell towers and routing of data to ensure maximum coverage and data speeds with minimal infrastructure costs.

  مواقع الابراج والمسارات التي تضمن اكبر تغطية وسرعة بيانات و اقل كلفة بنية تحتية

- **E-Commerce and Digital Marketing:** تجارة الالكترونية
  – Personalized advertising and product recommendations.

  توصيات المنتجات التسويق الشخصي

General Structure of Optimization Algorithms



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General Structure of Optimization Algorithms

Step 1
(a) Set $k = 0$ and initialize $\mathbf{x}_0$.
(b) Compute $F_0 = f(\mathbf{x}_0)$.
Step 2
(a) Set $k = k + 1$.
(b) Compute the changes in $\mathbf{x}_k$ given by column vector $\mathbf{\Delta x}_k$ where

$$\mathbf{\Delta x}_k^T = [\Delta x_1 \;\; \Delta x_2 \;\; \cdots \;\; \Delta x_n]$$

by using an appropriate procedure.
(c) Set $\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{\Delta x}_k$
(d) Compute $F_k = f(\mathbf{x}_k)$ and $\Delta F_k = F_{k-1} - F_k$.
Step 3
Check if convergence has been achieved by using an appropriate criterion, e.g., by checking ΔF_k and/or $\mathbf{\Delta x}_k$. If this is the case, continue to Step 4; otherwise, go to Step 2.
Step 4
(a) Output $\mathbf{x}^* = \mathbf{x}_k$ and $F^* = f(\mathbf{x}^*)$.
(b) Stop.

$k=1$ — 1] 2
$k=2$ — 3]
$k=3$ — 5

Source: Antoniou, A., & Lu, W. S. (2007). Practical optimization: algorithms and engineering applications. New York: Springer.

General Structure of Optimization Algorithms

تقارب النقاط الموجوده في سلسلة الحلول عند ازمنه التكرار

- Convergence the stable point found at the end of a sequence of solutions via an iterative optimization algorithm.

تقارب

- Checking for convergence:

  عندما يصبح الفرق بين F(k) غير مهم

  – When the reduction in Fk between any two iterations has become insignificant:

    $$|\Delta F_k| = |F_{k-1} - F_k| < \varepsilon_F$$

    العتبة المسموح بها في دالة الهدف

    where ε_F is optimization tolerance for the objective function.

  – When the changes in all variables have become insignificant:

    $$|\Delta x_i| < \varepsilon_x \quad for\ i = 1,2,\dots,n$$

    عندما تكون الفروقات في كل القيم صغيره

    where ε_F is optimization tolerance for the variables.

  – When both criteria are satisfied.

وتحقق في الحالتين

تحديات التحسين

Challenges in Optimization

- Complexity

التعقيد

  - The size of the problem, the non-linearity of the objective function, or the number of constraints.

تكون المسألة معقدة إذا كانت حجم المسألة
او تكون غير خطية او تكون القيود كثيرة

التوسع

- Scalability
  - Ensuring that an increase in problem size does not lead to an exponential increase in computational time.

التأكد أن زيادة حجم المشكلة
لا يؤدي إلى التضخم خلال فترة الحساب

عدم الدقة

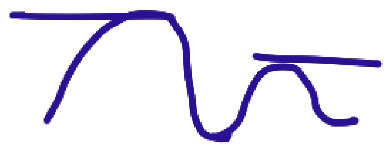
- Dealing with Uncertainty
  - In real-world scenarios, we often have to make decisions with incomplete or uncertain information.

في العالم الواقعي نضطر إلى وضع قرارات
بالاعتماد على معلومات غير كاملة

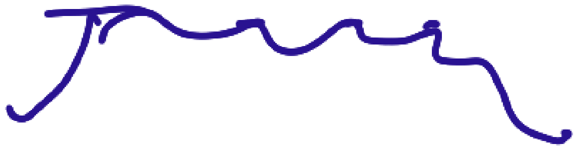
Challenges in Optimization

عدم التحدب

- Non-Convexity
  وجود عدد كبير من القيم الصغرى
  - This non-convexity can lead to numerous local minima



البيئة الديناميكية



- Dynamic Environments
  - the underlying conditions change over time.
  الظروف تتغير مع الوقت
- Algorithm Selection and Customization
  - no one-size-fits-all solution.
  تصميم واختيار الخوارزمية على حسبه
- Multi-Objective Optimization
  مشاكل ذات اهداف متعددة
  - can be competing with each other.

What’s Next?

- Reading:
  - AL:1
  - KW:1