Chapter 1

vector Algebra

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena is studied.

قياسيه متجهه
· Scalars and vectors

فقط مقدار
A scalar :- is a quantity that has only magnitude

متجه
A vector :- is a quantity that has both magnitude and direction له مقدار واتجاه

زمن كتله مسافه
example of scalar :- time - mass - distance

example of vector :- force - displacement - acceleration
قوه الازاحه
التسارع

مجال
what is Field ?

is a function that specifies a particular quantity everywhere in a region.
داله تعبر عن كميه محدده في منطقه معينه

Example of field :-

scalar field مجال قياسي
temperature distribution in a building

مجال اتجاهي
vector field
velocity of raindrops in the atmosphere.

Vector :-

$F = 3a_x + 4a_y + 5a_z$

$|F| = \sqrt{3^2+4^2+5^2}$

the vector is written as $\bar{A} = A a_A$

$\bar{A}$ → vector, A → magnitude, a_A → unit vector

$\boxed{A = |\bar{A}|}$

Example :- $\bar{A} = A_x a_x + A_y a_y + A_z a_z = |\bar{A}| a_A$

$$|\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{a}_A = \frac{\bar{A}}{|\bar{A}|} = \frac{A_x a_x + A_y a_y + A_z a_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

متجه الوحدة

unit vector

unity vector

$\frac{\text{المتجه}}{\text{مقداره}}$

vector addition and subtraction :-

$\bar{A} = A_x a_x + A_y a_y$, $\bar{B} = B_x a_x + B_y a_y$

what is $A + B$ and $A - B$?

$$\bar{A} + \bar{B} = (A_x + B_x) a_x + (A_y + B_y) a_y$$

$$\bar{A} - \bar{B} = (A_x - B_x) a_x + (A_y - B_y) a_y$$

Example 1-1 $\boxed{P. 18}$

If $\bar{A} = 10a_x - 4a_y + 6a_z$, $\bar{B} = 2a_x + a_y$

Find a] the component of $\bar{A}$ along a_y $= -4$

b] the magnitude of $3\bar{A} - \bar{B}$

$30a_x - 12a_y + 18a_z - 2a_x - a_y = 28a_x - 13a_y + 18a_z$

c] a unit vector along $\bar{A} + 2\bar{B}$

$10a_x - 4a_y + 6a_z + 4a_x + 2a_y = 14a_x - 2a_y + 6a_z$

Sol:- a] The component of $\bar{A}$ along a_y

is $A_y = -4$

b] $3\bar{A} - \bar{B} = 3[10a_x - 4a_y + 6a_z] - 2a_x + a_y$

$= 30a_x - 12a_y + 18a_z - 2a_x - a_y$

$= (30-2)a_x + (-12-1)a_y + 18a_z$

$3\bar{A} - \bar{B} = 28a_x - 13a_y + 18a_z$

Hence $|3\bar{A} - \bar{B}| = \sqrt{(28)^2 + (-13)^2 + (18)^2}$

$= \sqrt{1277} = 35.74$

c] $\bar{A} + 2\bar{B} = (10a_x - 4a_y + 6a_z) + 2[2a_x + a_y]$

$= 10a_x - 4a_y + 6a_z + 4a_x + 2a_y$

$= (10+4)a_x + (-4+2)a_y + 6a_z$

$\bar{A} + 2\bar{B} = 14a_x - 2a_y + 6a_z$

Lets say that the unit vector of $\bar{A} + 2\bar{B}$

is a_c $\quad \hat{a} = \dfrac{14a_x - 2a_y + 6a_z}{\sqrt{14^2 + 2^2 + 6^2}} = \dfrac{14a_x - 2a_y + 6a_z}{\sqrt{236}}$

$\Rightarrow a_c = \dfrac{\bar{A} + 2\bar{B}}{|\bar{A} + 2\bar{B}|} = \dfrac{14a_x - 2a_y + 6a_z}{\sqrt{(14)^2 + (-2)^2 + (6)^2}}$

$= 0.9113a_x - 0.1302a_y + 0.39a_z$

$a_c = 0.9113\hat{a}_x - 0.1302\hat{a}_y + 0.390\hat{a}_z$

Note that $|a_c| = 1$ as expected.

متجه الوحده دائماً طوله = ١

Vector multiplication

$$\bar{A} = A_x a_x + A_y a_y + A_z a_z \quad , \quad \bar{B} = B_x a_x + B_y a_y + B_z a_z$$

1] Scalar product (dot product) :-

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

where $a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

and $a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

or $\bar{A} \cdot \bar{B} = |A||B| \cos\theta$ where θ is the angle between $\bar{A}$ and $\bar{B}$

It is called scalar because the result of this operation is a scalar quantity

2] Cross product :-

$$\bar{A} \times \bar{B} = |A||B| \sin\theta \, a_n$$

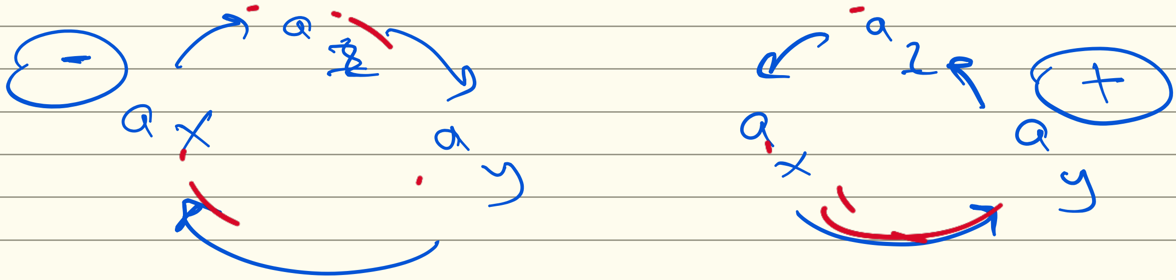
where a_n is the unit vector normal to the plane containing $\bar{A}$ and $\bar{B}$

$$\bar{A} \times \bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = a_x(A_y B_z - A_z B_y) - a_y(A_x B_z - A_z B_x) + a_z(A_x B_y - A_y B_x)$$

Note that $\quad a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$

and $\quad a_x \times a_y = a_z \;,\; a_y \times a_z = a_x \;,\; a_z \times a_x = a_y$

$\quad a_y \times a_x = -a_z \;,\; a_z \times a_y = -a_x \;,\; a_x \times a_z = -a_y$



Scalar triple product :-

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \bar{B} \cdot (\bar{C} \times \bar{A}) = \bar{C} \cdot (\bar{A} \times \bar{B})$$

vector triple product :-

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$$

problem 1.22 (p.37)

$\bar{E}$ and $\bar{F}$ are vector fields given by $\bar{E}$

$$\bar{E} = 2x\, a_x + a_y + yz\, a_z$$

$2a_x + a_y + 6a_z$ $\qquad |E| = \sqrt{4+1+6^2}$

$$\bar{F} = xy\, a_x - y^2 a_y + xyz\, a_z$$

Determine :-

a] $|\bar{E}|$ at (1,2,3)

b] the component of $\bar{E}$ along $\bar{F}$ at (1,2,3) $\qquad E_F = (E \cdot F) \dfrac{\vec{F}}{|F|^2}$

c] A vector perpendecular to both $\bar{E}$ and $\bar{F}$ at (0,1,-3) whose magnitude is unity

، الذي بعوضه

a] $|\bar{E}| = \sqrt{(2x)^2 + (1)^2 + (yz)^2} = \sqrt{4x^2 + 1 + y^2 z^2}$

$|\bar{E}|$ at $(1,2,3) \Rightarrow |\bar{E}| = \sqrt{4 + 1 + 4(9)}$

$\Rightarrow |\bar{E}| = \sqrt{41} = 6.403$

b] at $(1,2,3) \rightarrow \bar{F} = 2a_x - 4a_y + 6a_z$

$$E_F = (\bar{E} \cdot a_F)\, a_F = \left(\bar{E} \cdot \frac{\bar{F}}{|F|}\right) \frac{\bar{F}}{|F|}$$

$$= (\bar{E} \cdot \bar{F}) \frac{\bar{F}}{|F|^2}$$

$\Rightarrow \bar{E} \cdot \bar{F} = 2x^2 y - y^2 + x y^2 z^2$

at $(1,2,3) \Rightarrow \bar{E} \cdot \bar{F} = \cancel{4} - \cancel{4} + 36 = 36$

Now $$\frac{\bar{F}}{|F|^2} = \frac{xy\, a_x - y^2 a_y + xyz\, a_z}{\left(\sqrt{(xy)^2 + (-y^2)^2 + (xyz)^2}\right)^2}$$

$$= \frac{2a_x - 4a_y + 6a_z}{4 + 16 + 36} = \frac{2a_x - 4a_y + 6a_z}{56}$$

then $$\bar{E}_F = (\bar{E} \cdot \bar{F}) \frac{\bar{F}}{|F|^2} = \frac{36}{56}(2a_x - 4a_y + 6a_z)$$

$$= 1.286\, a_x - 2.57\, a_y + 3.857\, a_z$$

c] at $(0,1,-3)$

$\bar{E} = a_y - 3a_z \quad , \quad \bar{F} = -a_y$

$$\bar{E} \times \bar{F} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = a_x(0-3) - a_y(0-0) + a_z(0-0)$$

$$= -3a_x$$

$$\boxed{\hat{a}_{\bar{E}\times\bar{F}} = \frac{\bar{E}\times\bar{F}}{|\bar{E}\times\bar{F}|}} = \frac{-3a_x}{|-3|} = \pm a_x$$

Preview problems

1.7, 1.9, 1.10, 1.11 p (34

1.7 Let $\mathbf{F} = 2\mathbf{a}_x - 6\mathbf{a}_y + 10\mathbf{a}_z$ and $\mathbf{G} = \mathbf{a}_x + G_y\mathbf{a}_y + 5\mathbf{a}_z$. If $\mathbf{F}$ and $\mathbf{G}$ have the same unit vector, G_y is

(a) 6
(b) -3
(d) 0
(e) 6

1.8 Given that $\mathbf{A} = \mathbf{a}_x + \alpha\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = \alpha\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, if $\mathbf{A}$ and $\mathbf{B}$ are normal to each other, α is

(a) -2
(b) $-1/2$
(c) 0
(d) 1
(e) 2

1.9 The component of $6\mathbf{a}_x + 2\mathbf{a}_y - 3\mathbf{a}_z$ along $3\mathbf{a}_x - 4\mathbf{a}_y$ is

(a) $-12\mathbf{a}_x - 9\mathbf{a}_y - 3\mathbf{a}_z$
(b) $30\mathbf{a}_x - 40\mathbf{a}_y$
(c) 10/7
(d) 2
(e) 10

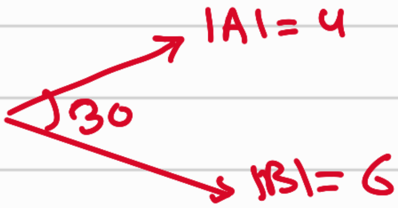
1.10 Given $\mathbf{A} = -6\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$, the projection of $\mathbf{A}$ along $\mathbf{a}_y$ is

(a) -12
(b) -4
(c) 3
(d) 7
(e) 12

Answers: 1.1d, 1.2a, 1.3b,e, 1.4b, 1.5a, 1.6b,c, 1.7b, 1.8b, 1.9d, 1.10c.

ضرب المتجهات

1. الضرب القياسي $(A \bullet B)$



$$A = 3a_x + 4a_y + 5a_z$$
$$B = 2a_x + a_y - 3a_z$$

$$\vec{A} \bullet \vec{B} = |A||B| \cos\theta$$
$$= 4 \times 6 \times \cos 30$$
$$= 20.8$$

$$\vec{A} \cdot \vec{B} = (3\times 2) + (4\times 1) + (5\times -3)$$
$$= -5$$

اذا كان المتجهين متعامدين

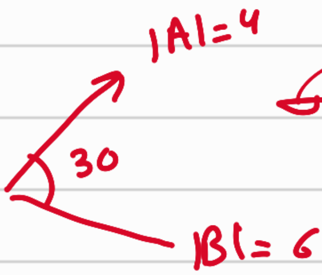
$$A \bullet B = 0$$
$$a_x \cdot a_y = a_x \cdot a_z = a_y \cdot a_z = 0$$

اذا كان متجهين نفس الاتجاه الضرب $a_x \cdot a_x = 1$

$$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$$

$$\boxed{\theta = \cos^{-1}\left[\frac{\vec{A} \cdot \vec{B}}{|A||B|}\right]}$$

2 الضرب الاتجاهي $\vec{A} \times \vec{B}$



$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{A} \times \vec{B}| = |A||B| \sin\theta$$
$$4 \times 6 \times \sin 30 = 12$$

$\vec{A} = 2a_x - 3a_y + a_z$

$\vec{B} = -2a_x + a_y + a_z$

$$\vec{A} \times \vec{B} = \begin{array}{ccc} a_x & a_y & a_z \end{array} \begin{vmatrix} 2 & -3 & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

$(-3 - 1)\,a_x - (2 - -2)\,a_y + (2 - 6)\,a_z$

$\vec{A} \times \vec{B} = -4a_x - 4a_y - 4a_z$

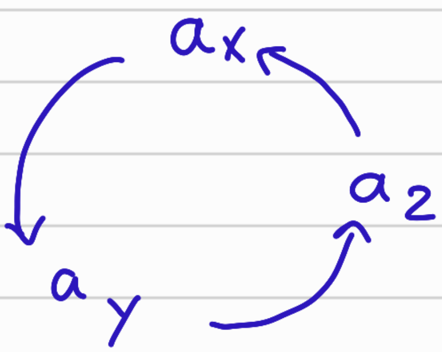
اذا كان المتجهين متوازيين الضرب الاتجاهي صفر

$a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$

نتيجة الضرب الاتجاهي متجه ثالث عمودي ع كلا المتجهين

عكس عقارب الساعة +

مع عقارب الساعة −



$a_x \times a_y = a_z$ $\qquad$ $a_z \times a_x = a_y$

$a_y \times a_z = a_x$

الضرب الثلاثي

$$\bar{A}\cdot(\bar{B}\times\bar{C}) = \bar{B}\cdot(\bar{C}\times\bar{A}) = \bar{C}\cdot(\bar{A}\times\bar{B})$$

$$\bar{A}\times(\bar{B}\times\bar{C}) = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B})$$

طريقة حساب مركبة متجه باتجاه متجه آخر

Find the component $\bar{A}$ along $\bar{B}$

$$A_B = (A\cdot B)\frac{\vec{B}}{|B|^2}$$

حساب المتجه العمودي على كل من المتجهين A , B

$\bar{A}\times\bar{B} \Rightarrow$ عمودي على كل منهما

حساب المتجه العمودي على المتجهين A و B والذي طوله 1

$$\hat{a}_{A\times B} = \frac{\bar{A}\times\bar{B}}{|A\times B|}$$

1.11 Given that

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z$$
$$\mathbf{Q} = 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{R} = -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

find: (a) $|\mathbf{P} + \mathbf{Q} - \mathbf{R}|$, (b) $\mathbf{P} \cdot \mathbf{Q} \times \mathbf{R}$, (c) $\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R}$, (d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R})$, (e) $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$, (f) $\cos \theta_{PR}$, (g) $\sin \theta_{PQ}$.

(a) $|\mathbf{P} + \mathbf{Q} - \mathbf{R}|$

1. Compute:

$$\mathbf{P} + \mathbf{Q} - \mathbf{R} = (2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z) + (4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z) - (-\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$$

$$= (2 + 4 + 1)\mathbf{a}_x + (-1 + 3 - 1)\mathbf{a}_y + (-2 + 2 - 2)\mathbf{a}_z = 7\mathbf{a}_x + 1\mathbf{a}_y - 2\mathbf{a}_z$$

2. Compute the magnitude:

$$|\mathbf{P} + \mathbf{Q} - \mathbf{R}| = \sqrt{(7)^2 + (1)^2 + (-2)^2} = \sqrt{49 + 1 + 4} = \sqrt{54} = 7.35$$

(b) $\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{R})$

1. Compute $\mathbf{Q} \times \mathbf{R}$:

$$\mathbf{Q} \times \mathbf{R} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = \mathbf{a}_x(3 \cdot 2 - 2 \cdot 1) - \mathbf{a}_y(4 \cdot 2 - (-1) \cdot 2) + \mathbf{a}_z(4 \cdot 1 - (-1) \cdot 3)$$

$$= 4\mathbf{a}_x - 10\mathbf{a}_y + 7\mathbf{a}_z$$

2. Compute $\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{R})$:

$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{R}) = (2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z) \cdot (4\mathbf{a}_x - 10\mathbf{a}_y + 7\mathbf{a}_z)$$

$$= (2)(4) + (-1)(-10) + (-2)(7) = 8 + 10 - 14 = 4$$

(c) $(\mathbf{Q} \times \mathbf{P}) \cdot \mathbf{R}$

1. Compute $\mathbf{Q} \times \mathbf{P}$:

$$\mathbf{Q} \times \mathbf{P} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} = \mathbf{a}_x(3 \cdot -2 - 2 \cdot -1) - \mathbf{a}_y(4 \cdot -2 - 2 \cdot 2) + \mathbf{a}_z(4 \cdot -1 - 3 \cdot 2)$$

$$= -4\mathbf{a}_x + 12\mathbf{a}_y - 10\mathbf{a}_z$$

2. Compute $(\mathbf{Q} \times \mathbf{P}) \cdot \mathbf{R}$:

$$(-4\mathbf{a}_x + 12\mathbf{a}_y - 10\mathbf{a}_z) \cdot (-\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$$

$$= (-4)(-1) + (12)(1) + (-10)(2) = 4 + 12 - 20 = -4$$

(d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R})$

1. From above:
   - $\mathbf{P} \times \mathbf{Q} = -4\mathbf{a}_x - 12\mathbf{a}_y - 8\mathbf{a}_z$
   - $\mathbf{Q} \times \mathbf{R} = 4\mathbf{a}_x - 10\mathbf{a}_y + 7\mathbf{a}_z$
2. Compute the dot product:

$$(-4\mathbf{a}_x - 12\mathbf{a}_y - 8\mathbf{a}_z) \cdot (4\mathbf{a}_x - 10\mathbf{a}_y + 7\mathbf{a}_z)$$

$$= (-4)(4) + (-12)(-10) + (-8)(7) = -16 + 120 - 56 = 206$$

(e) $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$

1. Compute:

$$(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R}) = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ -4 & -12 & -8 \\ 4 & -10 & 7 \end{vmatrix}$$

$$= \mathbf{a}_x((-12)(7) - (-8)(-10)) - \mathbf{a}_y((-4)(7) - (-8)(4)) + \mathbf{a}_z((-4)(-10) - (-12)(4))$$

$$= 16\mathbf{a}_x + 12\mathbf{a}_y + 8\mathbf{a}_z$$

(f) $\cos\theta_{\mathbf{PR}}$

1. Compute:

$$\cos\theta_{\mathbf{PR}} = \frac{\mathbf{P} \cdot \mathbf{R}}{|\mathbf{P}||\mathbf{R}|}$$

2. Dot product:

$$\mathbf{P} \cdot \mathbf{R} = (2)(-1) + (-1)(1) + (-2)(2) = -2 - 1 - 4 = -7$$

3. Magnitudes:

$$|\mathbf{P}| = \sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3, \quad |\mathbf{R}| = \sqrt{(-1)^2 + (1)^2 + (2)^2} = \sqrt{6}$$

4. Substitute:

$$\cos\theta_{\mathbf{PR}} = \frac{-7}{3\sqrt{6}} = -0.953$$

$$\sin\theta_{\mathbf{PQ}} = \frac{|\mathbf{P} \times \mathbf{Q}|}{|\mathbf{P}||\mathbf{Q}|}$$

$$\sin\theta_{\mathbf{PQ}} = \frac{16.12}{3\sqrt{29}} = \frac{16.12}{16.15} \approx 0.998$$