جامعة أم القرى
كلية العلوم التطبيقية
قسم العلوم الرياضية

رمز المقرر: 4041101-4
المتطلب السابق: لا يوجد
الفصل الدراسي الثالث 1444هـ

الخطة الدراسية لمقرر: تفاضل وتكامل (Calculus I)

Weeks	Chapters	Sections	Topics
1	Fundamentals	**1.2** Precalculus, 6th Edition, Stewart	**Exponents and Radicals** Examples: *1, 2, 3, 5, 8 and 10.* Exercises: *15, 17, 21, 23, 35, 37, 49, 55 and 57.*
		1.5 Precalculus, 6th Edition, Stewart	**Solving Equations** Examples: *1, 4, 5, 6, 7 and 8.* Exercises: *15, 43, 51, 53, 55, 59, 65, 69, 79 and 80.*
2	Fundamentals	**0.2** Calculus, 9th Edition, Purcell	**Inequalities and Absolute Values** Examples: *1, 2, 3, 4, 5, 8, 9 and 13.* Exercices: *1, 3, 11, 13, 17, 21, 25 and 39.*
		0.3 Calculus, 9th Edition, Purcell	**Distance Formula and Circle Equation** Examples: *1, 2, 3, 5, 6, and 7.* Exercises: *11, 13, 17, 23, 25, 29, 33, 35 and 39-(a,b,c,d).*
3	Functions	**0.5** Calculus, 9th Edition, Purcell	**Functions (Domain, Range, Graphing Functions, etc)** Examples: *1, 2, 4-(a) and 5.* Exercises: *9, 13, 15, 17 and 21.*
		0.6 Calculus, 9th Edition, Purcell	**Operations of Functions** Examples: *1, 2, 3 and 4.* Exercises: *1, 11 and 15.*
4	Functions	**0.7** Calculus, 9th Edition, Purcell	**Trigonometric Functions and Identities** Examples: *5 and 6.* Exercises: *9, 11, 27 and 29.*
	Limits and Continuity	**1.1** Calculus, 9th Edition, Purcell	**Introduction to Limits** Examples: *1, 2 and 5-(Example for one-sided limits).* Exercises: *1, 3, 7, 8, 11 and 43-(a,b).*
5	Limits and Continuity	**1.3** Calculus, 9th Edition, Purcell	**Limits Theorems (Theorems A, B, C and D)** Examples: *1, 2, 3, 4, 5, 6, 7, 8 and 9.* Exercises: *1, 5, 9, 13, 15, 19 and 27.*
		1.5 Calculus, 9th Edition, Purcell	**Limits at Infinity and Infinite Limits** Examples: *1, 2, 3, 5, 6 and 7.* Exercises: *2, 3, 27, 29, 31 and 45.*
6	Limits and Continuity	**1.6** Calculus, 9th Edition, Purcell	**Continuity of Functions** Examples: *1, 2, 3, 4, 5, 6 and 7.* Exercises: *1, 9, 11, 13, 15, 27 and 31.*
	Differentiation	**2.2** Calculus, 9th Edition, Purcell	**Definition of Derivative** Examples: *1, 2, 3, 4, 5 and 6.* Exercises: *1, 3, 5, 7, 11, 13 and 19.*
Midterm Exam			
7	Differentiation	**2.3** Calculus, 9th Edition, Purcell	**Rules for Finding Derivatives** Examples: *1, 2, 3, 4, 5 and 6.* Exercises: *1, 5, 9, 11, 15, 27, 29, 39 and 43.*
		2.4 Calculus, 9th Edition, Purcell	**Derivative of Trigonometric Functions** Examples: *1, 2, 3, 4, 5, 6, 7 and 8.* Exercises: *1, 3, 5, 7, 9, 11, 13, 15, 17 and 19.*
		2.5 Calculus, 9th Edition, Purcell	**Chain Rule** Examples: *1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.* Exercises: *1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23, 33 and 39.*

8	Differentiation	**2.6** Calculus, 9th Edition, Purcell	**Higher Order Derivatives** Examples: *1, 2, 3 and 4.* Exercises: *1, 3, 5, 7, 9, 11 and 19.*
		2.7 Calculus, 9th Edition, Purcell	**Implicit Differentiation** Examples: *1, 2, 3 and 4.* Exercises: *1, 3, 5, 7, 9, and 11.*
These sections must be read by students		**3.1**, **3.2**, **3.3** and **3.5** Calculus, 9th Edition, Purcell	**Maxima and Minima**
			Monotonicity
			Local Maxima and Minima
			Concavity and Inflection Points
			Extending the Concept of Limits to Infinity
			Sketching the Graphs
9	Integration	**3.8** Calculus, 9th Edition, Purcell	**Antiderivatives (Indefinite Integrals)** Examples: *1, 2, 3, 4, 5 and 6.* Exercises: *2, 5, 9, 11, 15, 25, 29 and 33.*
		4.4 Calculus, 9th Edition, Purcell	**Definite Integrals and the Second Fundamental Theorem** Examples: *1, 2, 3, 4, 5, 7, 8, 9, 10 and 12.* Exercises: *1, 3, 5, 7, 9, 11, 14, 17 and 21.*

المراجع (References):

1- Calculus (9th Edition), Dale Varberg, Edwin Purcell and Steven Rigdon, Prentice Hall (2006).

2- Mathematics for preparatory year program (Book1), Oxford University Press (2013).

3- Stewart, James, Lothar Redlin, and Saleem Watson. Precalculus: Mathematics for calculus. Cengage Learning, 2015.

نوع التقييم	الدرجة
مشاركة، واجبات واختبارات قصيرة	10
اختبار منزلي	10
اختبار نصفي	30
اختبار نهائي	50

1.2 EXPONENTS AND RADICALS

$a^3 = a \cdot a \cdot a$

$\sqrt{9} = 3$ $\qquad \sqrt{4} = 2$

$\sqrt[3]{27} = 3$ $\qquad \sqrt[3]{8} = 2$

$\sqrt[4]{81} = 3$ $\qquad \sqrt[4]{16} = 2$

$\sqrt[5]{32} = 2$

$(ab)^5 = a^5 b^5$ $\qquad \sqrt{ab} = \sqrt{a}\,\sqrt{b}$

$\left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$ $\qquad \sqrt{\frac{x^3}{y^2}} = \frac{\sqrt{x^3}}{\sqrt{y^2}}$

خطأ $(a+b)^2 \cancel{=} a^2+b^2$

$(\text{جبل})^0 = 1$ $\qquad$ الاسس الصفري

$$\left(\frac{a^5 y^{-3} x^2}{124^5}\right)^0 = 1$$

الاسس السالب

$x^{-3} = \frac{1}{x^3}$ $\qquad \frac{1}{a^{-5}} = a^5$

$$\left(\frac{a}{b}\right)^{-3} = \left(\frac{b}{a}\right)^3 = \frac{b^3}{a^3}$$

Laws of exponents قواعد الاسس

① الاسس في حاله الضرب تجمع

$$a^m \cdot a^n = a^{m+n} \qquad 2^3 \cdot 2^5 = 2^8$$

② الاسس في حالة القسمه تطرح

$$\frac{a^m}{a^n} = a^{m-n} \qquad \frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2}$$

③ اسس الاسس يتحول الى ضرب

$$(a^m)^n = a^{m \cdot n} \qquad (2^3)^3 = 2^9$$

الاسس الكسريه وتحويلها الى جذور

$$5^{2/3} = \sqrt[3]{5^2} = \left(\sqrt[3]{5}\right)^2$$

$$a^{m/n} = \sqrt[n]{a^m}$$

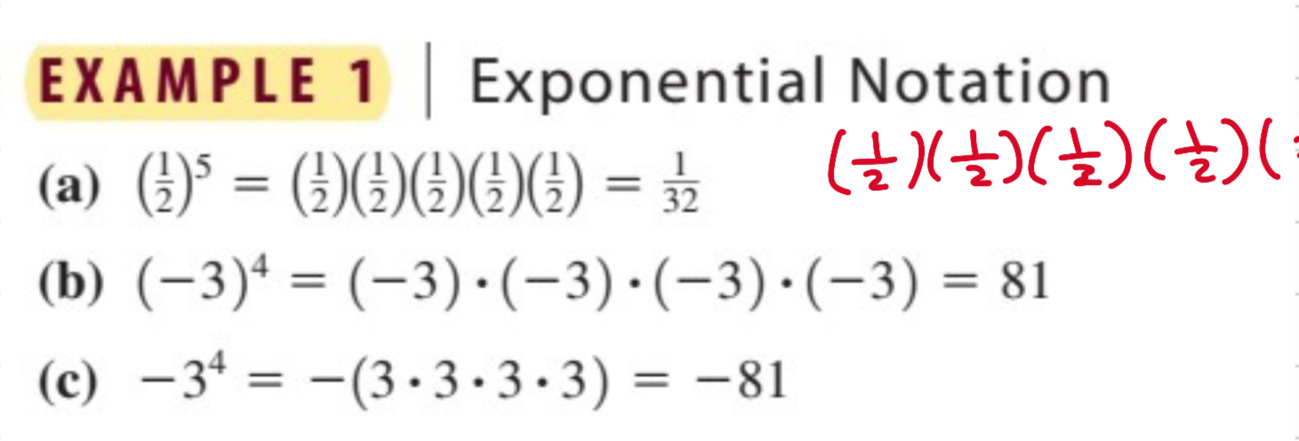
$$9^{1/2} = \sqrt{9} = 3$$

$$\sqrt[3]{a^4} = a^{\frac{4}{3}}$$

$$\sqrt{2^3} = 2^{\frac{3}{2}}$$

$$\sqrt[5]{4} = 4^{\frac{1}{5}}$$

$$64^{-1/3} = \frac{1}{64^{1/3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$$



EXAMPLE 1 | Exponential Notation

(a) $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{32}$

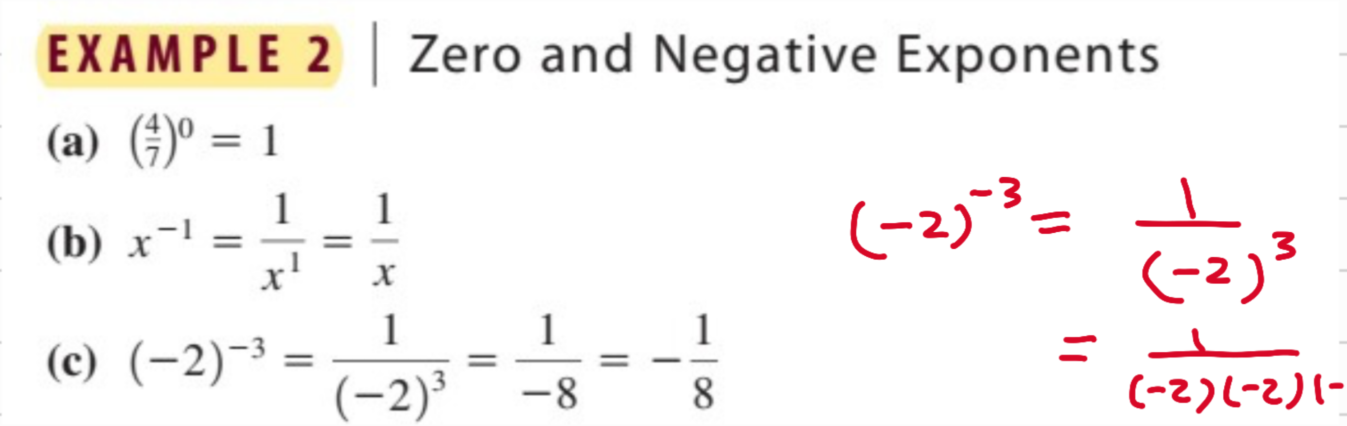
(b) $(-3)^4 = (-3)\cdot(-3)\cdot(-3)\cdot(-3) = 81$

(c) $-3^4 = -(3\cdot3\cdot3\cdot3) = -81$

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{32}$

$(-3)^4 = (-3)(-3)(-3)(-3) = 81$

$-3^4 = -(3\times3\times3\times3) = -81$



EXAMPLE 2 | Zero and Negative Exponents

(a) $\left(\frac{4}{7}\right)^0 = 1$

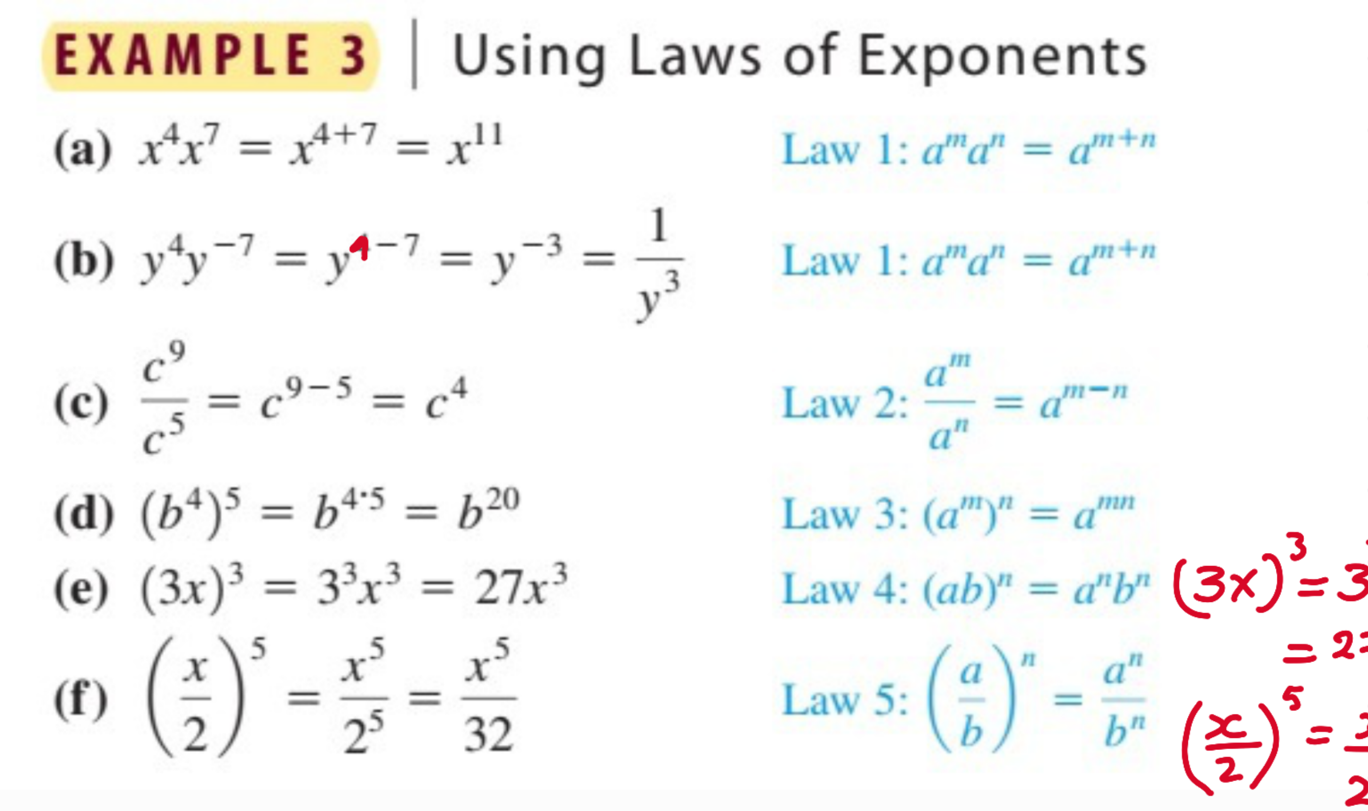
(b) $x^{-1} = \frac{1}{x^1} = \frac{1}{x}$

(c) $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$

$$(-2)^{-3} = \frac{1}{(-2)^3}$$

$$= \frac{1}{(-2)(-2)(-2)} = \frac{1}{-8}$$

$$\frac{1}{-8} = \frac{-1}{8} = -\frac{1}{8}$$



EXAMPLE 3 | Using Laws of Exponents

(a) $x^4x^7 = x^{4+7} = x^{11}$ — Law 1: $a^ma^n = a^{m+n}$

(b) $y^4y^{-7} = y^{4-7} = y^{-3} = \frac{1}{y^3}$ — Law 1: $a^ma^n = a^{m+n}$

(c) $\frac{c^9}{c^5} = c^{9-5} = c^4$ — Law 2: $\frac{a^m}{a^n} = a^{m-n}$

(d) $(b^4)^5 = b^{4\cdot5} = b^{20}$ — Law 3: $(a^m)^n = a^{mn}$

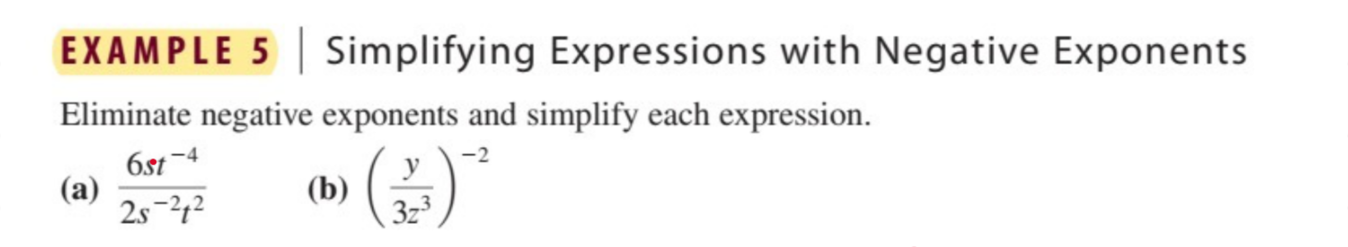
(e) $(3x)^3 = 3^3x^3 = 27x^3$ — Law 4: $(ab)^n = a^nb^n$

(f) $\left(\frac{x}{2}\right)^5 = \frac{x^5}{2^5} = \frac{x^5}{32}$ — Law 5: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$$(3x)^3 = 3^3x^3$$

$$= 27x^3$$

$$\left(\frac{x}{2}\right)^5 = \frac{x^5}{2^5} = \frac{x^5}{32}$$



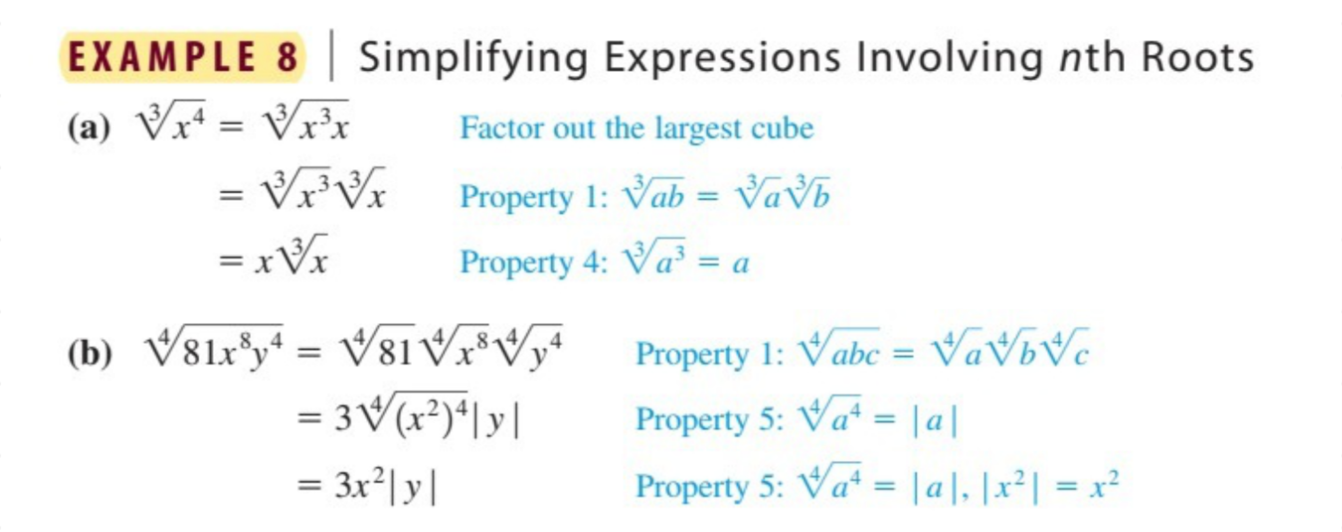
EXAMPLE 5 | Simplifying Expressions with Negative Exponents

Eliminate negative exponents and simplify each expression.

(a) $\dfrac{6st^{-4}}{2s^{-2}t^2}$ **(b)** $\left(\dfrac{y}{3z^3}\right)^{-2}$

a) $\dfrac{6st^{-4}}{2s^{-2}t^2} = 3\, s^{1--2}\, t^{-4-2} = 3\, s^3\, t^{-6} = \dfrac{3s^3}{t^6}$

b) $\left(\dfrac{y}{3z^3}\right)^{-2} = \left(\dfrac{3z^3}{y}\right)^2 = \dfrac{3^2(z^3)^2}{y^2} = \dfrac{9z^6}{y^2}$



EXAMPLE 8 | Simplifying Expressions Involving *n*th Roots

(a) $\sqrt[3]{x^4} = \sqrt[3]{x^3x}$ Factor out the largest cube

$= \sqrt[3]{x^3}\sqrt[3]{x}$ Property 1: $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$

$= x\sqrt[3]{x}$ Property 4: $\sqrt[3]{a^3} = a$

(b) $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\sqrt[4]{x^8}\sqrt[4]{y^4}$ Property 1: $\sqrt[4]{abc} = \sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}$

$= 3\sqrt[4]{(x^2)^4}\,|y|$ Property 5: $\sqrt[4]{a^4} = |a|$

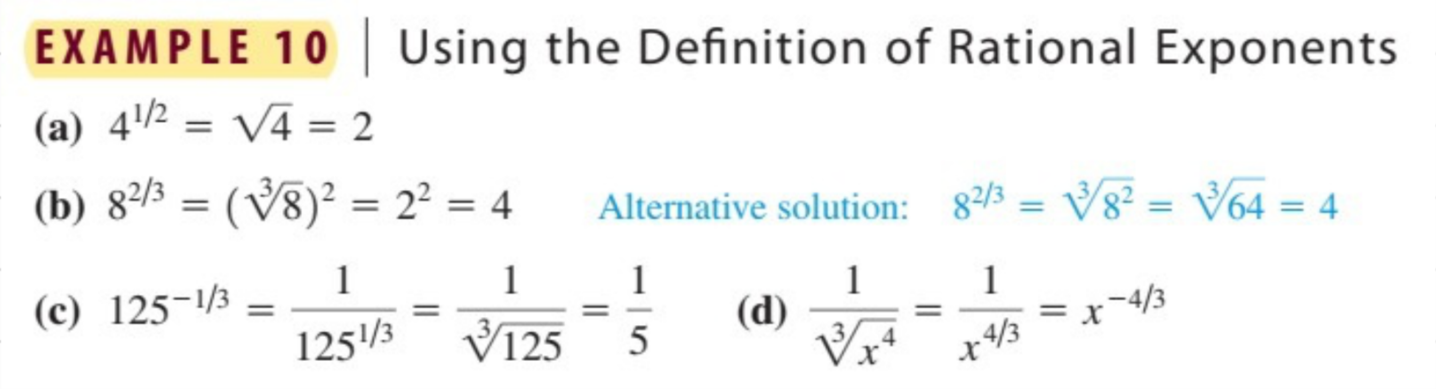
$= 3x^2|y|$ Property 5: $\sqrt[4]{a^4} = |a|$, $|x^2| = x^2$

a) $\sqrt[3]{x^4} = \sqrt[3]{x^3x^1} = \sqrt[3]{x^3}\cdot\sqrt[3]{x}$

$= x^{\frac{3}{3}}\ x^{\frac{1}{3}} = x\ x^{\frac{1}{3}} = x\sqrt[3]{x}$

b) $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\cdot\sqrt[4]{x^8}\cdot\sqrt[4]{y^4}$

$3\ x^{8/4}\ y^{4/4} = 3x^2y$



EXAMPLE 10 | Using the Definition of Rational Exponents

(a) $4^{1/2} = \sqrt{4} = 2$

(b) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$ Alternative solution: $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

(c) $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$ **(d)** $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$

a) $4^{1/2} = \sqrt{4} = 2$

b) $8^{2/3} = \sqrt[3]{8^2} =$

$\rightarrow \sqrt[3]{64} = 4$

$\rightarrow (\sqrt[3]{8})^2 = 2^2 = 4$

c) $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

$5 \times 5 \times 5$

d) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$

1.2 EXERCISES

15–24 ■ Evaluate each expression.

15. (a) -3^2 $= -(3^2) = -(3\times3) = -9$

17. (a) $\left(\frac{5}{3}\right)^0 2^{-1}$ $= 1\times 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

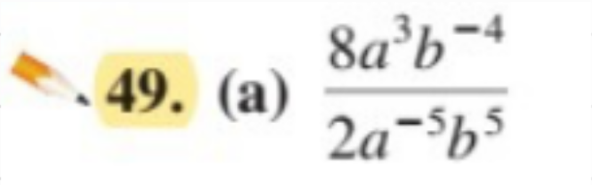
21. (a) $\sqrt{\frac{4}{9}}$ $= \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

23. (a) $\left(\frac{4}{9}\right)^{-1/2}$ $= \left(\frac{9}{4}\right)^{1/2} = \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

35–40 ■ Simplify each expression.

35. (a) x^8x^2 $= x^{10}$

37. (a) $\frac{y^{10}\cancel{y^0}}{y^7}$ $= y^{10-7} = y^3$

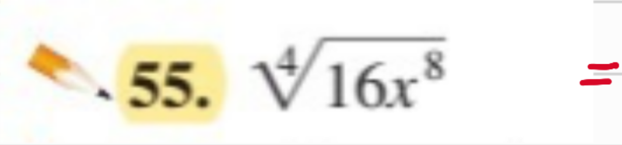


49. (a) $\frac{8a^3b^{-4}}{2a^{-5}b^5}$

$$= 4a^{3--5}\, b^{-4-5}$$

$$= 4a^8\, b^{-9}$$

$$= \frac{4a^8}{b^9}$$

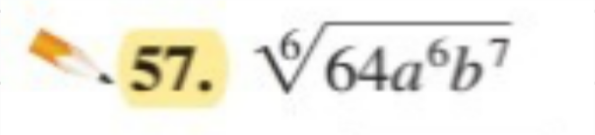


55. $\sqrt[4]{16x^8}$

$$= \sqrt[4]{16} \cdot \sqrt[4]{x^8}$$

$$= 2 \cdot x^{8/4}$$

$$= 2x^2$$



57. $\sqrt[6]{64a^6b^7}$

$$= \sqrt[6]{64}\ \sqrt[6]{a^6}\ \sqrt[6]{b^7}$$

$$= 2\ a^{6/6}\ b^{7/6}$$

$$= 2ab^{7/6}$$

1.5 EQUATIONS

المعادلات

Solve the linear equation

معادلات خطية

$$3x + 1 = 7$$
$$3x = 7 - 1$$
$$\frac{\cancel{3}x}{\cancel{3}} = \frac{6}{3}$$
$$x = 2$$

$$10x - 2 = 12 + 3x$$
$$10x - 3x = 12 + 2$$
$$\frac{\cancel{7}x}{\cancel{7}} = \frac{14}{7}$$
$$x = 2$$

Quadratic equations

معادلات تربيعية

$$x^2 - 3 = 13$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = \pm 4$$

$$\sqrt{(x-2)^{\cancel{2}}} = \sqrt{100}$$
$$x - 2 = \pm 10$$
$$x - 2 = +10 \qquad x - 2 = -10$$
$$x = 12 \qquad x = -8$$

$$ax^2 + bx + c = 0$$

تحليل

اكمال مربع

القانون العام

Solve the Quadratic equation

$$x^2 - 6x - 7 = 0$$

Factorization التحليل الطريقة ①

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0 \quad \begin{cases} \to x - 7 = 0 & \boxed{x = 7} \\ \to x + 1 = 0 & \boxed{x = -1} \end{cases}$$

Completing the square اكمال المربع الطريقة ②

$$x^2 - \overset{b}{6x} - \overset{c}{7} = 0$$

$$x^2 - 6x + 9 = 7 + 9$$

$$\sqrt{(x-3)^2} = \sqrt{16}$$

$$x - 3 = \pm 4$$

$x - 3 = 4$ | $x - 3 = -4$
$x = 3 + 4$ | $x = -4 + 3$
$\boxed{x = 7}$ | $\boxed{x = -1}$

① نقسم على معامل x^2
② ننقل c الى اليمين
③ نجمع للطرفين $\left(\frac{b}{2}\right)^2$
$\left(-\frac{6}{2}\right)^2 = 9$
④ نحول الطرف اليسار الى مربع كامل
⑤ حل المعادلة

Quadratic Formula حل المعادلة بالقانون طريقة ③

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$x^2 - 6x - 7 = 0$$

$a = 1 \qquad b = -6 \qquad c = -7$

$$x = \frac{+6 \pm \sqrt{(-6)^2 - 4 \times 1 \times -7}}{2(1)}$$

$$x = \frac{+6 \pm \sqrt{36 + 28}}{2}$$

$$x = \frac{+6 \pm \sqrt{64}}{2} = \frac{6 \pm 8}{2}$$

$$x = \frac{6+8}{2} = 7 \qquad x = \frac{6-8}{2} = -1$$

$$\boxed{x = 7} \qquad \boxed{x = -1}$$

Discriminant (D) مميز المعادلة

$$D = b^2 - 4ac$$

$D =$ موجبه

المعادله لها حلين حقيقين

$D > 0$

Two distinct real solution

$D = 0$

هناك حل واحد فقط

there is one real solution

$D =$ سالبه

لا يوجد حلول حقيقيه

$D < 0$

no real slootion

EXAMPLE 1 | Solving a Linear Equation

Solve the equation $7x - 4 = 3x + 8$.

$$7x - 3x = 8 + 4$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$\boxed{x = 3}$$

تحليل

EXAMPLE 4 | Solving a Quadratic Equation by Factoring

Solve the equation $x^2 + 5x = 24$.

$ax^2 + bx + c = 0$

$$x^2 + 5x - 24 = 0$$

نبحث عن عددين ضربهم -24

ومجموعهم +5

8 -3

$$(x + 8)(x - 3) = 0$$

$x + 8 = 0$ $x - 3 = 0$

$\boxed{x = -8}$ $\boxed{x = 3}$

EXAMPLE 5 | Solving Simple Quadratics

Solve each equation.

(a) $x^2 = 5$ **(b)** $(x - 4)^2 = 5$

a) $\sqrt{x^2} = \sqrt{5}$ $x = \pm\sqrt{5}$

b) $\sqrt{(x-4)^2} = \sqrt{5}$ $x - 4 = \pm\sqrt{5}$

$x - 4 = +\sqrt{5}$ $x - 4 = -\sqrt{5}$

$x = \sqrt{5} + 4$ $x = -\sqrt{5} + 4$

حل المعادلة باكمال المربع

EXAMPLE 6 | Solving Quadratic Equations by Completing the Square

Solve each equation.

(a) $x^2 - 8x + 13 = 0$ **(b)** $3x^2 - 12x + 6 = 0$

a) $x^2 - 8x + 16 = -13 + 16$

$\left(\frac{-8}{2}\right)^2 = 16$

$(x-4)^2 = 3$

$\sqrt{(x-4)^2} = \sqrt{3}$

$(x-4) = \pm\sqrt{3}$

$x - 4 = +\sqrt{3}$

$x = \sqrt{3} + 4$

$x - 4 = -\sqrt{3}$

$x = -\sqrt{3} + 4$

---

b) $3x^2 - 12x + 6 = 0$

① نقسم كل المعادلة على 3

$x^2 - 4x + 2 = 0$

$\left(\frac{-4}{2}\right)^2 = 4$

$x^2 - 4x + 4 = -2 + 4$

$(x-2)^2 = 2$

$\sqrt{(x-2)^2} = \sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$x - 2 = \sqrt{2}$

$x = \sqrt{2} + 2$

$x - 2 = -\sqrt{2}$

$x = -\sqrt{2} + 2$

EXAMPLE 7 | Using the Quadratic Formula

استخدم القانون العام

Find all solutions of each equation.

(a) $3x^2 - 5x - 1 = 0$ (b) $4x^2 + 12x + 9 = 0$ (c) $x^2 + 2x + 2 = 0$

$\underbrace{3}_{a}x^2 \underbrace{-5}_{b}x \underbrace{-1}_{c}$

a) $a = 3$ $b = -5$ $c = -1$

$$x = \frac{--5 \pm \sqrt{(-5)^2 - (4\times3\times-1)}}{2\times3} = \frac{5 \pm \sqrt{25+12}}{6}$$

$$x = \frac{5 \pm \sqrt{37}}{6} \quad \begin{cases} \rightarrow x = \dfrac{5+\sqrt{37}}{6} \\ \rightarrow x = \dfrac{5-\sqrt{37}}{6} \end{cases}$$

---

b) $a = 4$ $b = 12$ $c = 9$

zero

$$x = \frac{-12 \pm \sqrt{12^2 - (4\times4\times a)}}{8} = \frac{-12 \pm \sqrt{144-144}}{8}$$

$$x = \frac{-12}{8} = -\frac{3}{2}$$

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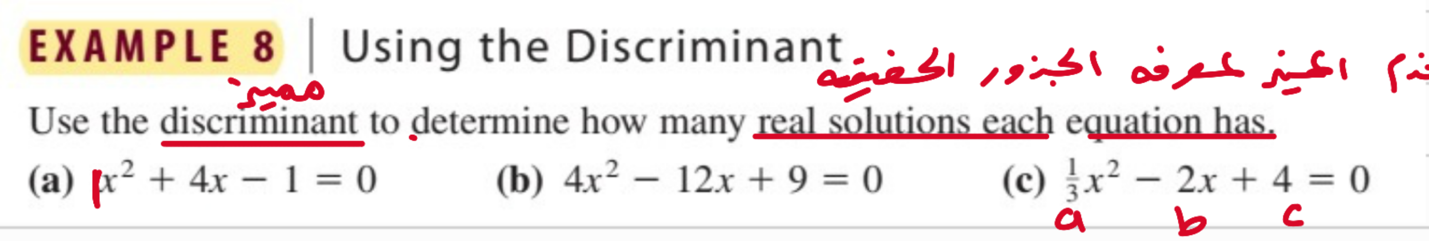
c) $a = 1$ $b = 2$ $c = 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4\times1\times2}}{2(1)} = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$x = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{-1}}{2}$$

$$x = \frac{-\cancel{2} \pm \cancel{2}\sqrt{-1}}{\cancel{2}} = -1 \pm \sqrt{-1}$$

يعني
عدد تخيلي
غير حقيقي



EXAMPLE 8 | Using the Discriminant استخدم المميز لمعرفة الجذور الحقيقيه

مميز

Use the discriminant to determine how many real solutions each equation has.

(a) $1x^2 + 4x - 1 = 0$ (b) $4x^2 - 12x + 9 = 0$ (c) $\frac{1}{3}x^2 - 2x + 4 = 0$

a b c

a) $D = b^2 - 4ac = 16 - 4(1)(-1) = 16 + 4 = 20$

$20 > 0$ هناك حلان حقيقيان

Two distinct real solution

b) $D = (-12)^2 - 4(4)(9) = 144 - 144 = 0$

$D = 0$ هناك حل واحد

one real solutio

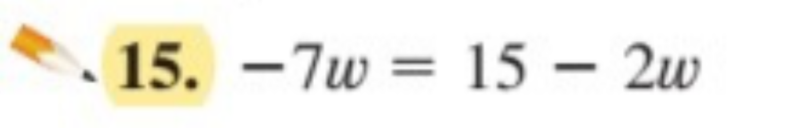
c) $D = (-2)^2 - (4)\left(\frac{1}{3}\right)(4) = \frac{3\times 4}{3\times 1} - \frac{16}{3}$

$= \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}$

$D < 0$ سالبة

No real solutions

1.5 EXERCISES

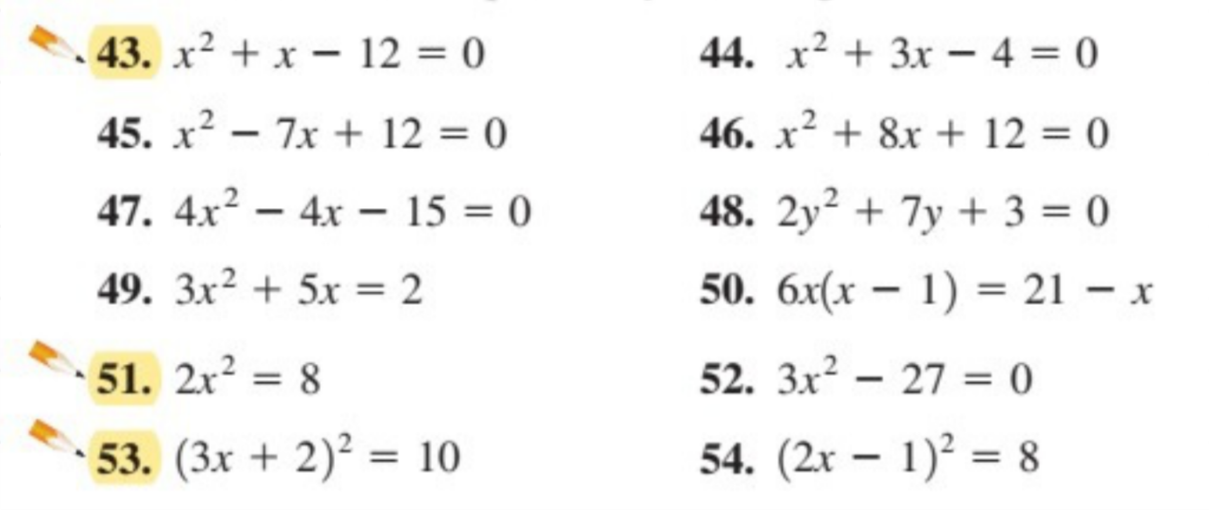


15. $-7w = 15 - 2w$

$$-7W + 2W = 15$$

$$-5W = 15 \qquad W = \frac{15}{-5} = \boxed{-3}$$

تحليل



43–54 ■ Solve the equation by factoring.

43. $x^2 + x - 12 = 0$

44. $x^2 + 3x - 4 = 0$

45. $x^2 - 7x + 12 = 0$

46. $x^2 + 8x + 12 = 0$

47. $4x^2 - 4x - 15 = 0$

48. $2y^2 + 7y + 3 = 0$

49. $3x^2 + 5x = 2$

50. $6x(x - 1) = 21 - x$

51. $2x^2 = 8$

52. $3x^2 - 27 = 0$

53. $(3x + 2)^2 = 10$

54. $(2x - 1)^2 = 8$

43) $x^2 + x - 12 = 0$

$$(x+4)(x-3) = 0$$

$x + 4 = 0$ $\qquad$ $x - 3 = 0$

$\boxed{x = -4}$ $\qquad$ $\boxed{x = 3}$

51) $\frac{2x^2}{2} = \frac{8}{2}$

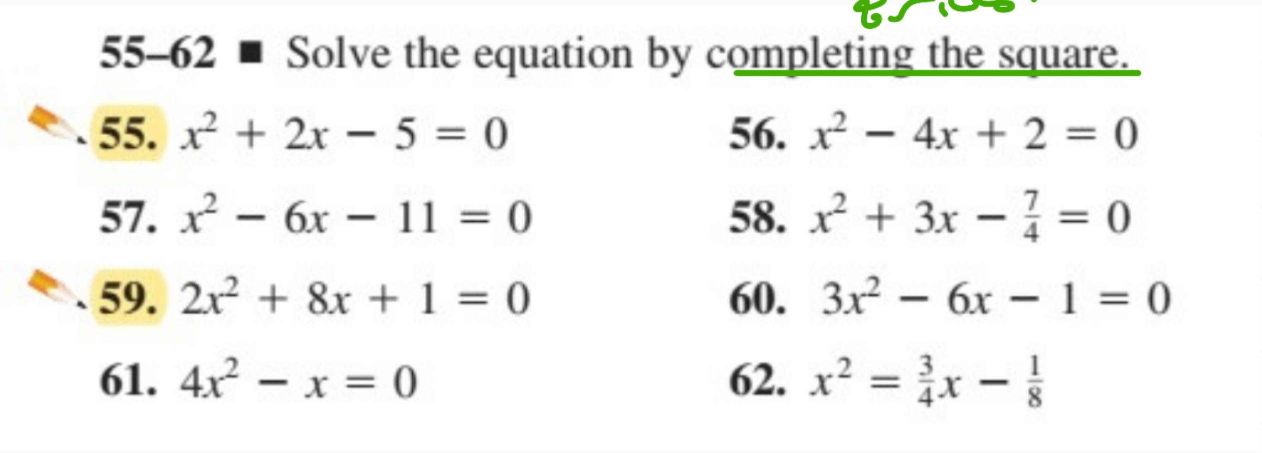
$$\sqrt{x^2} = \sqrt{4}$$

$$\boxed{x = \pm 2}$$

53) $\sqrt{(3x+2)^2} = \sqrt{10}$ $\qquad$ $3x + 2 = \pm\sqrt{10}$

$$\frac{3x}{3} = \frac{\pm\sqrt{10} - 2}{3} \qquad \boxed{x = \frac{\pm\sqrt{10} - 2}{3}}$$

اكمال مربع



55–62 ■ Solve the equation by completing the square.

55. $x^2 + 2x - 5 = 0$ **56.** $x^2 - 4x + 2 = 0$

57. $x^2 - 6x - 11 = 0$ **58.** $x^2 + 3x - \frac{7}{4} = 0$

59. $2x^2 + 8x + 1 = 0$ **60.** $3x^2 - 6x - 1 = 0$

61. $4x^2 - x = 0$ **62.** $x^2 = \frac{3}{4}x - \frac{1}{8}$

$\left(\frac{b}{2}\right)^2$

55) $x^2+2x-5=0$ $x^2+2x+1 = 5+1$

$x^2+2x+1=6$ $\sqrt{(x+1)^2}=\sqrt{6}$

$x+1=\pm\sqrt{6}$ $x=+\sqrt{6}-1$

$x=-\sqrt{6}+1$

59) $\frac{2x^2}{2}+\frac{8x}{2}+\frac{1}{2}=0$ $x^2+4x+\frac{1}{2}=0$

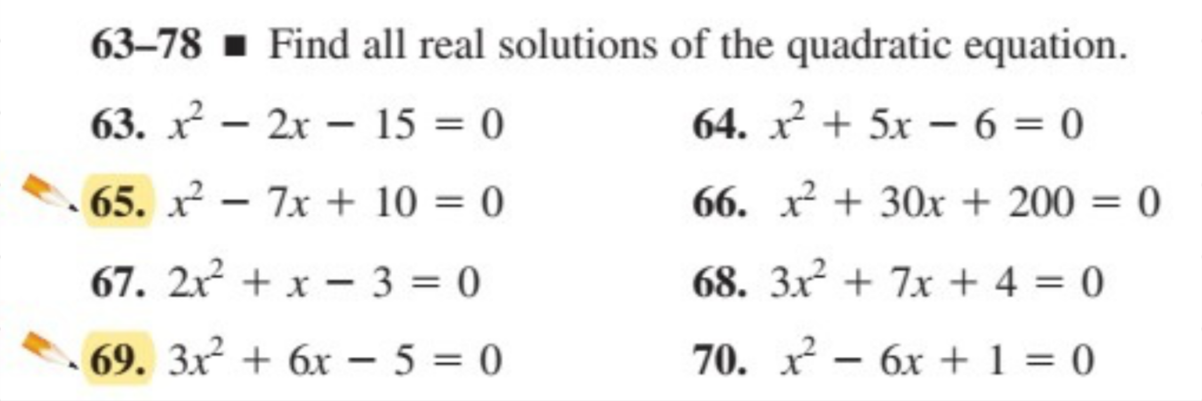
$\left(\frac{b}{2}\right)^2$

$x^2+4x+4 = -\frac{1}{2}+4$

$\sqrt{(x+2)^2} = \sqrt{\frac{7}{2}}$ $x+2=\pm\sqrt{\frac{7}{2}}=\pm\sqrt{\frac{14}{4}}$

$x+2 = \frac{\pm\sqrt{14}}{2}$

$x = \frac{\pm\sqrt{14}}{2}-2$



63–78 ■ Find all real solutions of the quadratic equation.

63. $x^2 - 2x - 15 = 0$

64. $x^2 + 5x - 6 = 0$

65. $x^2 - 7x + 10 = 0$

66. $x^2 + 30x + 200 = 0$

67. $2x^2 + x - 3 = 0$

68. $3x^2 + 7x + 4 = 0$

69. $3x^2 + 6x - 5 = 0$

70. $x^2 - 6x + 1 = 0$

65) $x^2 - 7x + 10 = 0$

$$(x-5)(x-2)=0$$

$x - 5 = 0$

$x = 5$

$x - 2 = 0$

$x = 2$

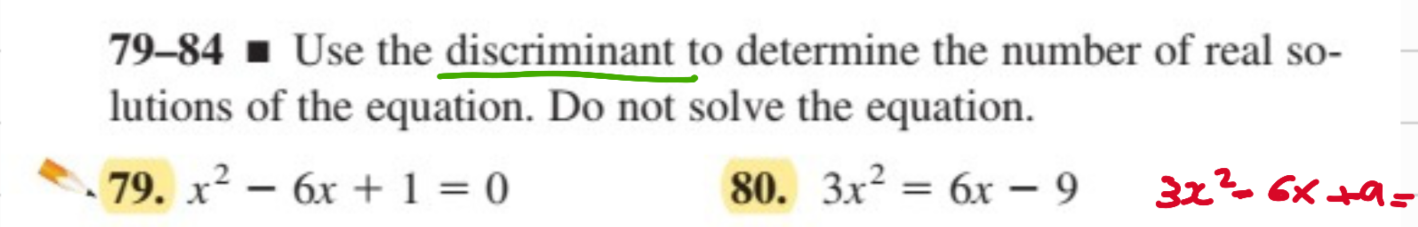
69) $3x^2 + 6x - 5$ $\quad a = 3 \quad b = 6 \quad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36 - (4 \times 3 \times -5)}}{6} = \frac{-6 \pm \sqrt{36+60}}{6}$$

$$= \frac{-6 \mp \sqrt{96}}{6} = \frac{-6}{6} \pm \frac{\sqrt{96}}{6}$$

$$= -1 \pm \frac{\sqrt{96}}{6}$$



79–84 ■ Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

79. $x^2 - 6x + 1 = 0$ **80.** $3x^2 = 6x - 9$

$3x^2 - 6x + 9 = 0$

$D = b^2 - 4ac$ → $+$ حلين

→ 0 حل واحد

→ $-$ لا يوجد حل

79) $b^2 - 4ac = (-6)^2 - (4 \times 1 \times 1)$

$36 - 4 = 32$

there are Two distinct real solution

8) $a = 3$ $b = -6$ $c = 9$

$D = (-6)^2 - (4 \times 3 \times 9)$

$36 - 108 = -72$

No real solution

0.2

متباينات

Inequalities and Absolute Values

equation $2x+1=5$ $2x=4$ $x=2$

Inequalities $2x+1<5$ الحل تكون على شكل فترة Interval

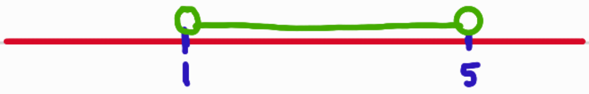
$2x+1<5$

$2x<4$ $x<2$

الحل هو جميع الارقام اقل من 2

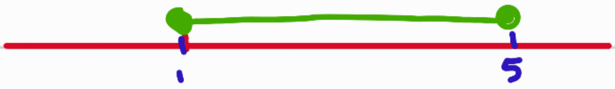
Interval Notation $\{x : 5 \leq x\}$ x حيث ان x اكبر او تساوي 5

$\{x : 1 < x < 5\}$



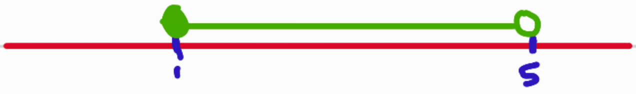
$(1, 5)$

$\{x : 1 \leq x \leq 5\}$



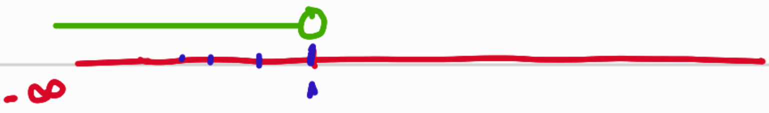
$[1, 5]$

$\{x : 1 \leq x < 5\}$



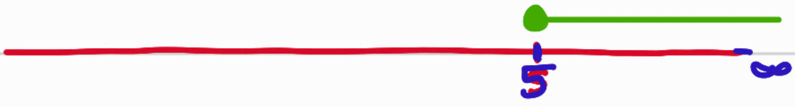
$[1, 5)$

$x < 1$



$(-\infty, 1)$

$x \geq 5$



$[5, \infty)$

حل المتباينات

* حل المتباينة هو نفس حل المعادلة الخطية الا في حالة الضرب او القسمة على عدد سالب تقلب الاشارة

$-2x + 1 \geq 10$

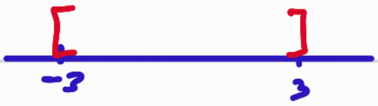
$\frac{-2x}{-2} \geq \frac{9}{-2}$

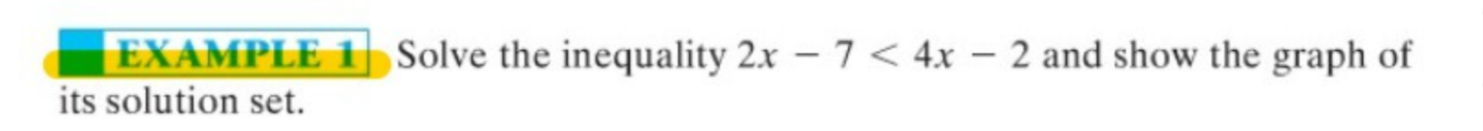
$x \leq -4.5$

* * *

$0 \leq x + 3 \leq 6$

$-3 \leq x \leq 3$



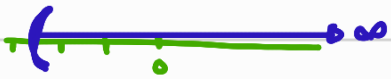

EXAMPLE 1 Solve the inequality $2x - 7 < 4x - 2$ and show the graph of its solution set.

$2x - 7 < 4x - 2$

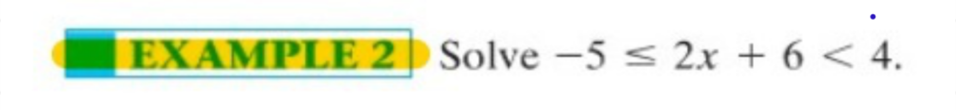
$2x - 4x < -2 + 7$

$\frac{-2x}{-2} < \frac{5}{-2}$

$x > -2.5$



$(-2.5, \infty)$


EXAMPLE 2 Solve $-5 \leq 2x + 6 < 4$.

$-5 \leq 2x + 6 < 4$

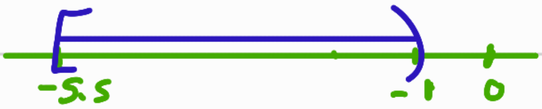
$-5 - 6 \leq 2x < 4 - 6$

$\frac{-11}{2} \leq \frac{2x}{2} < \frac{-2}{2}$

بالقسمة على 2

$-5.5 \leq x < -1$

$[-5.5 , -1)$



EXAMPLE 3 Solve the quadratic inequality $x^2 - x < 6$.

① تحليل للمعادلة لتربيعة

$x^2 - x - 6 \overset{=}{\cancel{<}} 0$

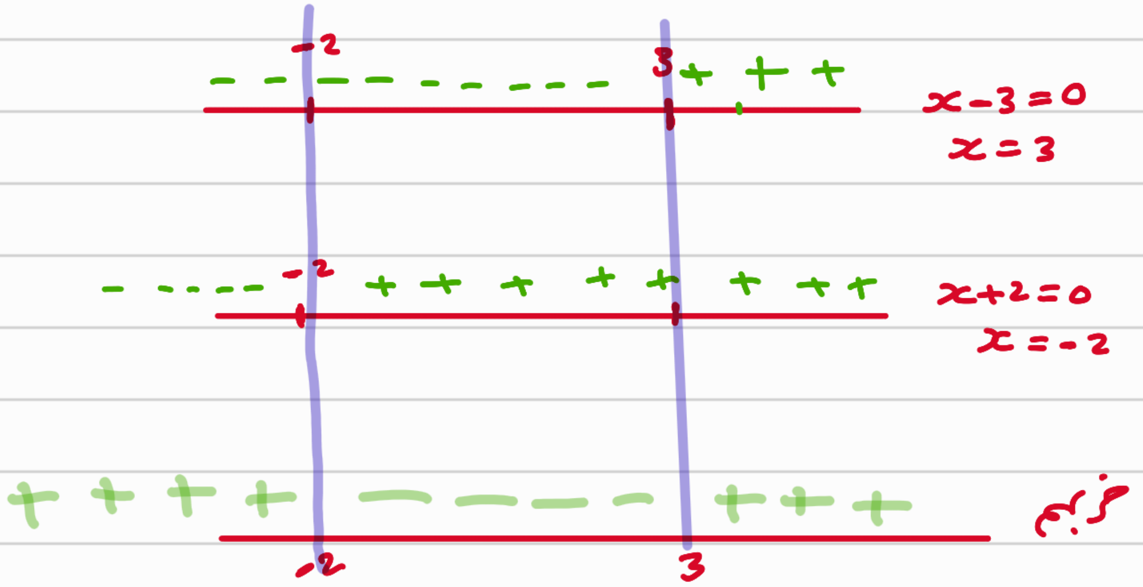
$x^2 - x - 6 = 0$

$(x - \underset{x=3}{3})(x + \underset{x=-2}{2}) = 0$

$(x-3)(x+2) < 0$

② حاصل ضرب الحدين يجب ان يكون سالب

③ نرسم الحدين على خط الاعداد و اختبار test point



-2
3
$x-3=0$
$x=3$
-2
$x+2=0$
$x=-2$
تجميع
-2
3

الحل هو الفترة السالبه $(-2,3)$

EXAMPLE 4 Solve $3x^2 - x - 2 > 0$.

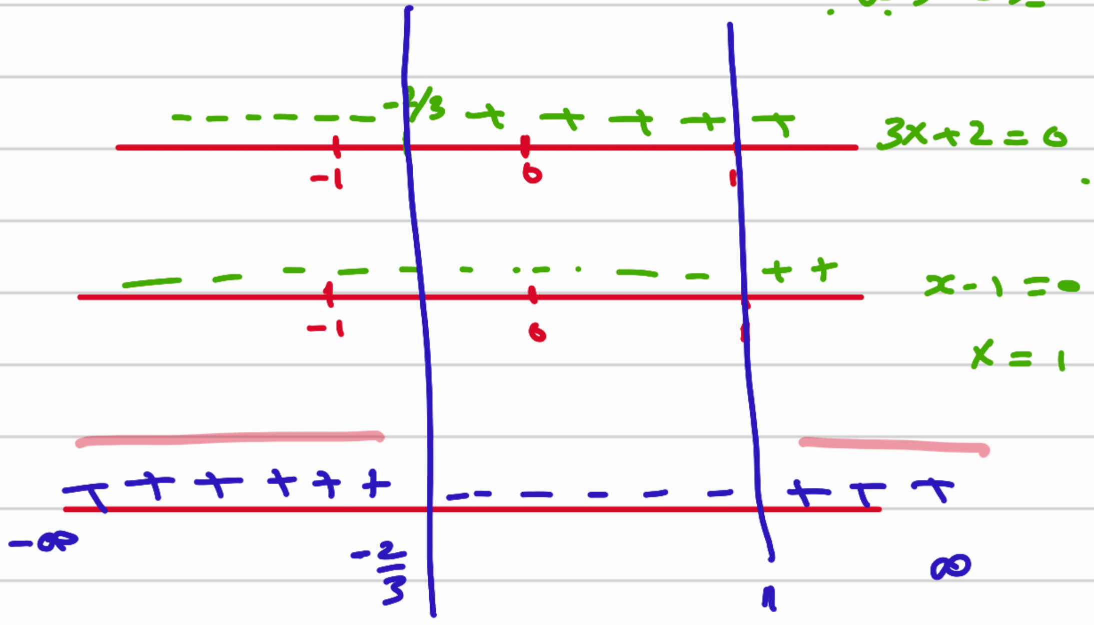
$(3x + 2)(x - 1) > 0$

✓ حاصل ضرب الحدين يجب ان يكون موجب.

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

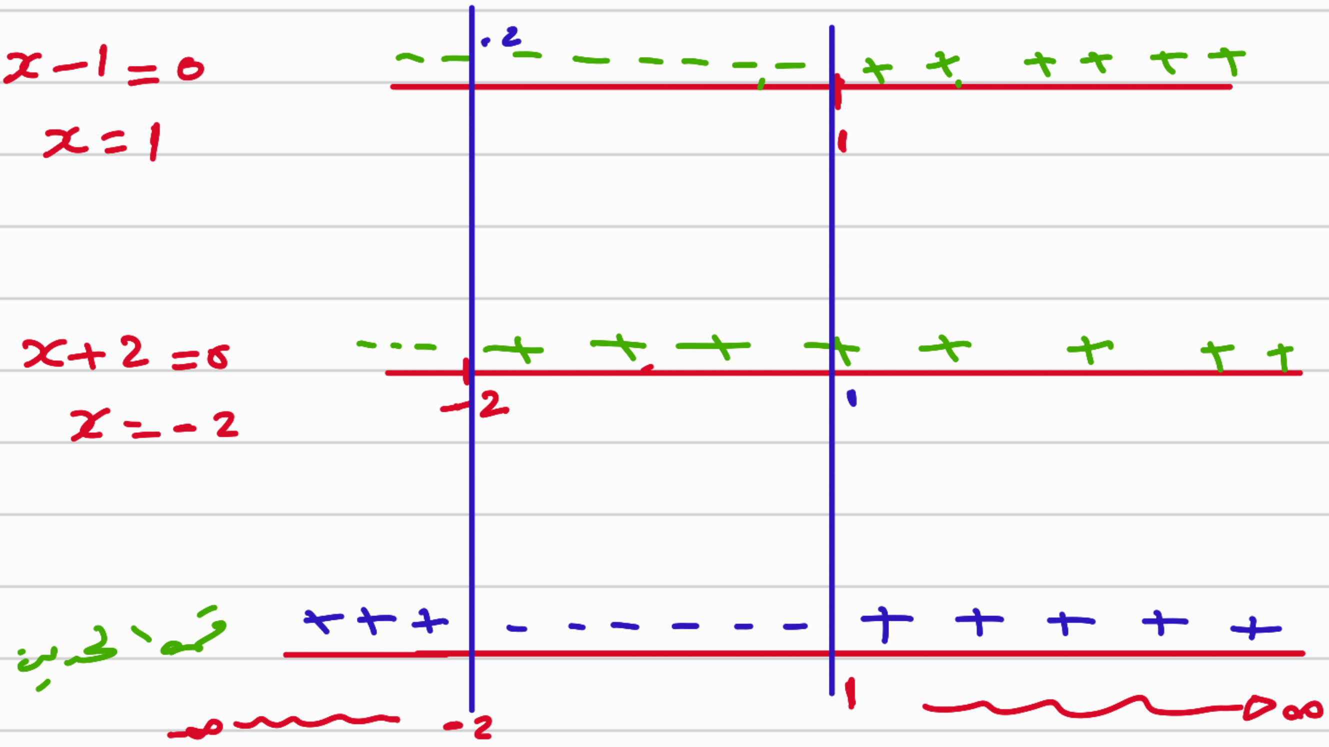


-2/3
-1
0
1
$3x+2=0$
-1
0
1
$x-1=0$
$x=1$
$-\infty$
$-\frac{2}{3}$
1
∞

الحل $(1,\infty) \cup (-\infty,-\frac{2}{3})$

EXAMPLE 5 Solve $\frac{x-1}{x+2} \geq 0.$

قسمة الحدين يجب ان تكون صفرا او موجبه



$x-1=0$
$x=1$
- - - - - - - - - - + + + + + +
1
$x+2=0$
$x=-2$
- - - + + + + + + + + +
-2
1
قسمة الحدين
+ + + - - - - - - - + + + + +
-2
1
-∞
∞

فترات الحل

$(-\infty,-2) \cup [1,\infty)$

يجب استثناء الحالة التي يكون فيها المقام 0

$x+2=0$

نحذف -2 من الحل $\boxed{x=-2}$

Absolute Value

| القيمه المطلقه |

$|5| = 5$ $|-5| = 5$

خصائص القيمه المطلقة

* $|ab| = |a||b| \Rightarrow |3x| = |3||x| = 3|x|$

* $\left|\frac{a}{b}\right| = \frac{|a|}{|b|} \Rightarrow \left|\frac{-4}{b}\right| = \frac{|-4|}{|b|} = \frac{4}{|b|}$

* $|a+b| \leqslant |a| + |b|$ $|-3+5| \; ? \; |-3| + |5|$

$2 \leqslant 3 + 5$

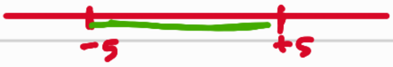
* $|a-b| \geqslant \big||a| - |b|\big|$ $|-5-3| \; ? \; 5 - 3$

$8 \geqslant 2$

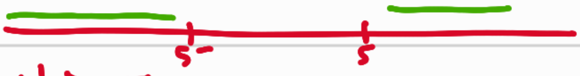
مهم جدا !!

$|x| < a \Leftrightarrow -a < x < a$

$|x| < 5 \Leftrightarrow -5 < x < 5$



$|x| > a \Leftrightarrow x < -a \quad x > a$



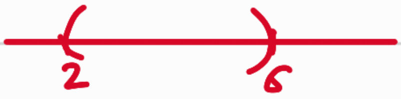
$|x| > 5 \Leftrightarrow x < -5 \,,\ x > 5$

EXAMPLE 8 Solve the inequality $|x-4|<2$ and show the solution set on the real line. Interpret the absolute value as a distance.

$$|x-4|<2 \Longleftrightarrow -2<x-4<2$$

$$-2+4<x<2+4$$

$$2<x<6 \qquad (2,6)$$



EXAMPLE 9 Solve the inequality $|3x-5|\geq 1$ and show its solution set on the real line.

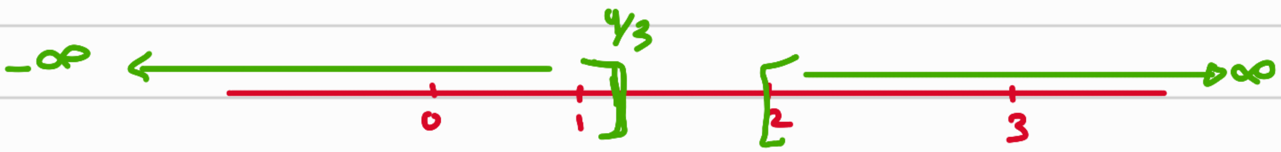
$$|3x-5|\geqslant 1$$

$$3x-5\leqslant -1 \quad \text{or} \quad 3x-5\geqslant 1$$

$$3x\leqslant -1+5 \qquad 3x\geqslant 6$$

$$3x\leqslant 4 \qquad x\geqslant \frac{6}{3}$$

$$x\leqslant \frac{4}{3} \quad \text{or} \quad x\geqslant 2$$



$$(-\infty,\tfrac{4}{3}]\cup[2,\infty)$$

EXAMPLE 13 Solve $x^2 - 2x - 4 \le 0$.

نبحث عن الفترات التي تكون اقل من او تساوي 0

$x^2 - 2x - 4 = 0$ $a = 1$ $b = -2$ $c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{5} = 2.1$

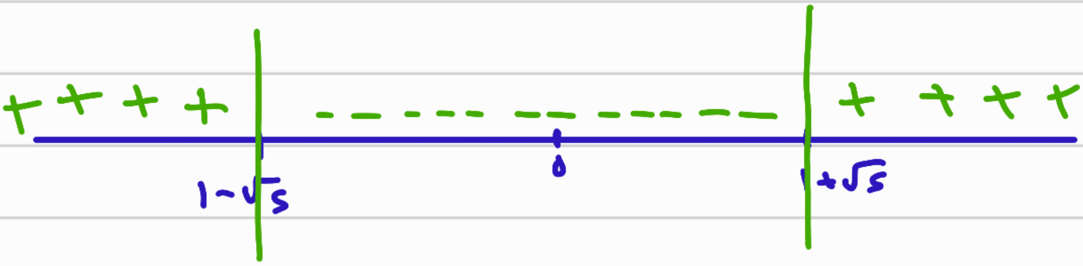
$$= \frac{--2 \pm \sqrt{4 - (4\times1\times-4)}}{2}$$

$$= \frac{2 \pm \sqrt{20}}{2} = \frac{2}{2} \mp \frac{\sqrt{4\times5}}{2}$$

$$\boxed{x = 1 \pm \sqrt{5}}$$

$1 + \sqrt{5} \approx 3.1$

$1 - \sqrt{5} \approx -1.1$

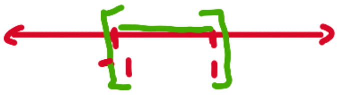


$[1-\sqrt{5} , 1+\sqrt{5}]$

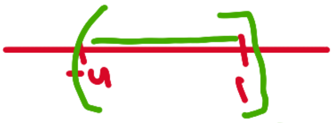
Problem Set 0.2

1. Show each of the following intervals on the real line.

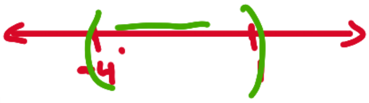
(a) $[-1, 1]$



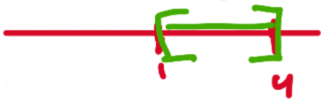
(b) $(-4, 1]$



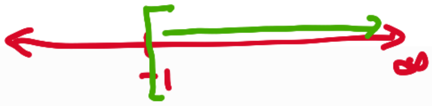
(c) $(-4, 1)$



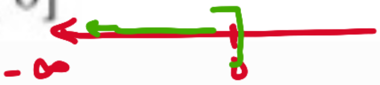
(d) $[1, 4]$



(e) $[-1, \infty)$



(f) $(-\infty, 0]$



In each of Problems 3–26, express the solution set of the given inequality in interval notation and sketch its graph.

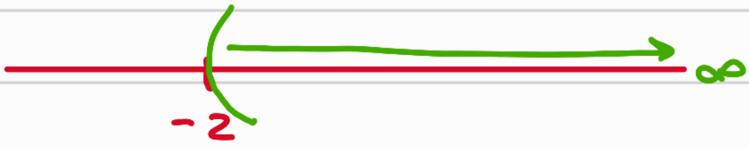
3. $x - 7 < 2x - 5$

4. $3x - 5 < 4x - 6$

$$-7+5 < 2x - x$$

$$-2 < x$$

$$x > -2$$



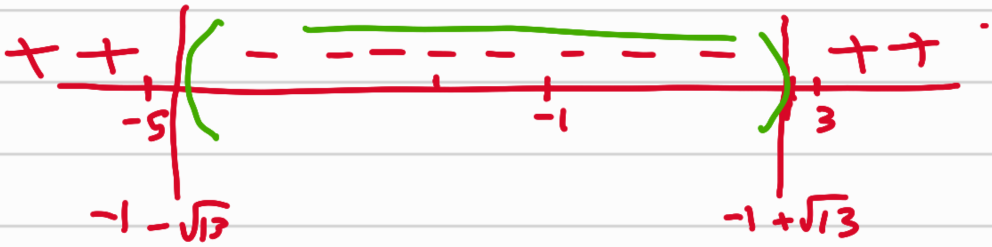
$[-2, \infty)$

11. $x^2 + 2x - 12 < 0$

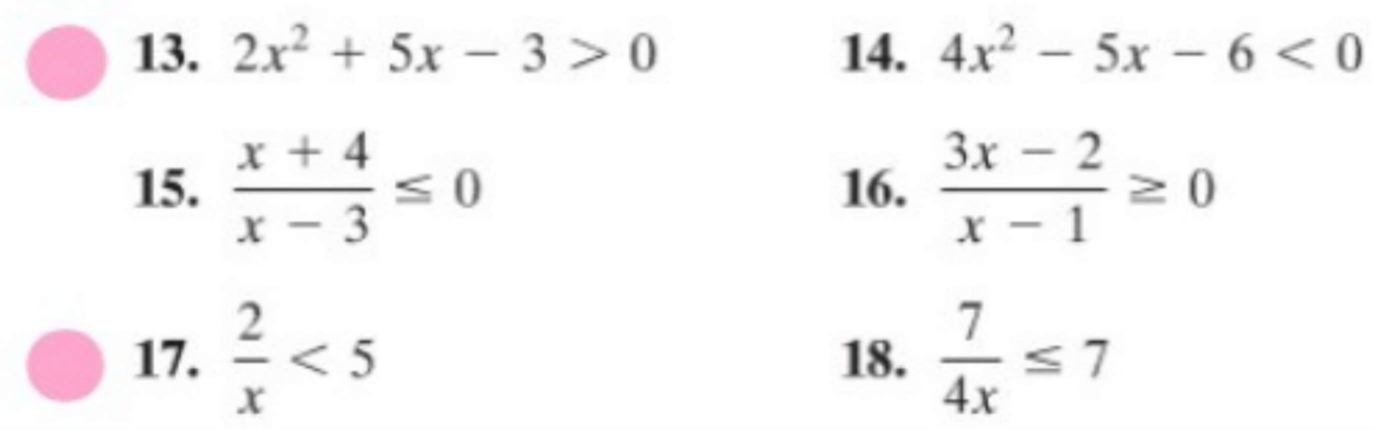
12. $x^2 - 5x - 6 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times (-12)}}{2a} = \frac{-2 \pm \sqrt{52}}{2}$$

$$x = -1 \pm \frac{\sqrt{52}}{2} = -1 \pm \frac{\sqrt{13}\cancel{\sqrt{4}}}{\cancel{2}} = -1 \pm \sqrt{13}$$


+ +
- - - - - - - - - -
+ +
-5
-1
3
$-1-\sqrt{13}$
$-1+\sqrt{13}$

$(-1-\sqrt{13} \quad , \; -1+\sqrt{13})$



13. $2x^2 + 5x - 3 > 0$
14. $4x^2 - 5x - 6 < 0$
15. $\dfrac{x+4}{x-3} \le 0$
16. $\dfrac{3x-2}{x-1} \ge 0$
17. $\dfrac{2}{x} < 5$
18. $\dfrac{7}{4x} \le 7$

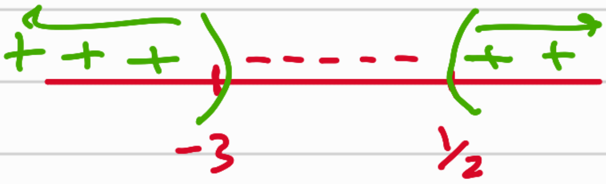
13) $2x^2+5x-3>0$

$(2x-1)(x+3)>0$

$2x-1=0$ $\qquad$ $x+3=0$

$2x=1$ $\qquad$ $x=-3$

$x=\frac{1}{2}$

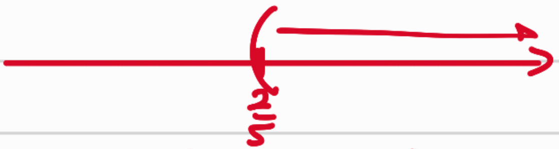


$(-\infty,-3) \cup (\frac{1}{2},\infty)$

17) $\dfrac{2}{x}<5$ $\qquad$ نضرب ب x

$2<5x$ $\qquad$ $5x>2$ $\qquad$ $x>\frac{2}{5}$

$x-\frac{2}{5}>0$



$(2/5 , \infty)$

21. $(x+2)(x-1)(x-3) > 0$

22. $(2x+3)(3x-1)(x-2) < 0$

23. $(2x-3)(x-1)^2(x-3) \geq 0$

24. $(2x-3)(x-1)^2(x-3) > 0$

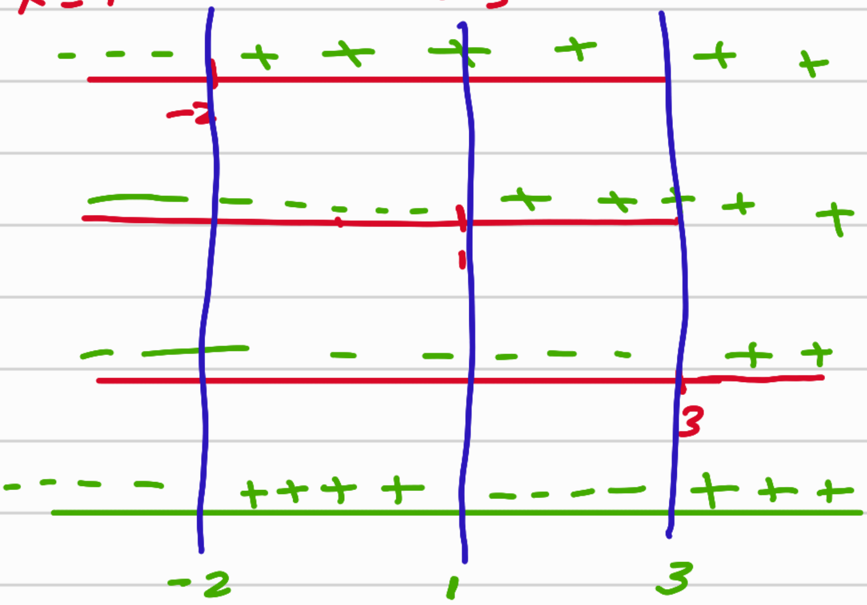
25. $x^3 - 5x^2 - 6x < 0$ **26.** $x^3 - x^2 - x + 1 > 0$

21) $(x+2)(x-1)(x-3) > 0$

$x+2=0$
$x=-2$

$x-1=0$
$x=1$

$x-3=0$
$x=3$



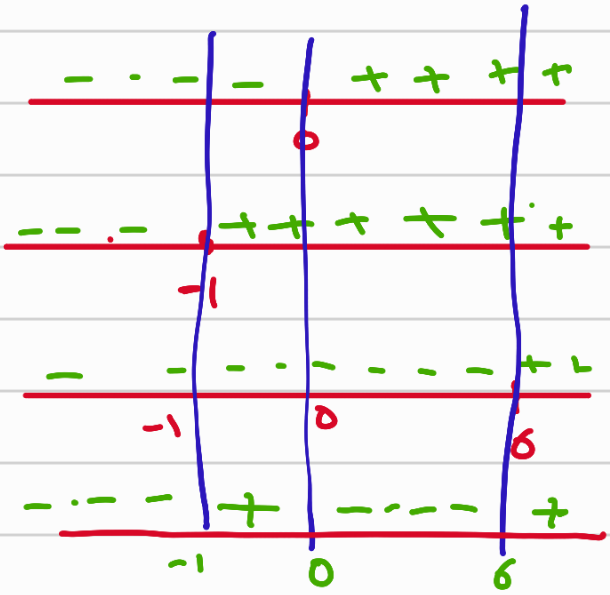
$(-2,1) \cup (3,\infty)$

25) $x^3 - 5x^2 - 6x < 0$

$x(x^2-5x-6) < 0$

$x(x+1)(x-6) < 0$

$x=0$ $x=-1$ $x=6$



$(-\infty,-1) \cup (0,6)$

In Problems 35–44, find the solution sets of the given inequalities.

35. $|x-2| \geq 5$

36. $|x+2| < 1$

37. $|4x+5| \leq 10$

38. $|2x-1| > 2$

39. $\left|\frac{2x}{7}-5\right| \geq 7$

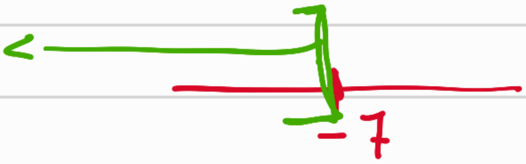
40. $\left|\frac{x}{4}+1\right| < 1$

$\frac{2x}{7}-5 \leq -7$

$\frac{2x}{7} \leq -2$

$x \leq \frac{-2\times 7}{2}$

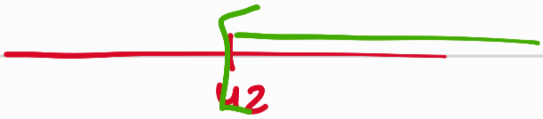
$x \leq -7$



$\frac{2x}{7}-5 \geq 7.$

$\frac{2x}{7} \geq 12$

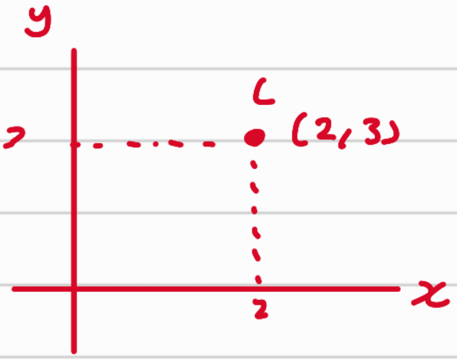
$x \geq 42$



$(-\infty, -7] \cup [42, \infty)$

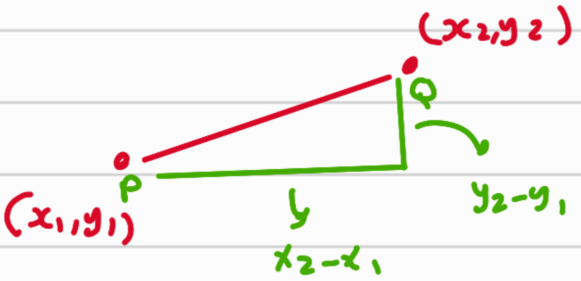
0.3

The Rectangular Coordinate System



(Distance)

المسافه بين نقطتين



$$\boxed{d(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

$$c = \sqrt{a^2+b^2}$$

إيجاد المسافة

EXAMPLE 1 Find the distance between

(a) $P(-2, 3)$ and $Q(4, -1)$ (b) $P(\sqrt{2}, \sqrt{3})$ and $Q(\pi, \pi)$

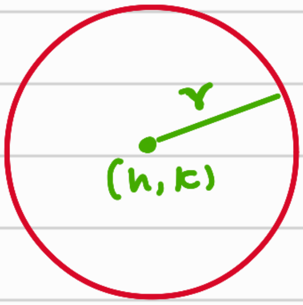
$x_1\ y_1 \quad x_2\ y_2 \qquad x_1\ y_1 \quad x_2\ y_2$

a) $d = \sqrt{(4 - -2)^2 + (-1-3)^2} = \sqrt{36+16} = \sqrt{52}$

b) $d = \sqrt{(\pi - \sqrt{2})^2 + (\pi - \sqrt{3})^2} = 2.23$

The equation of circle

معادله الدائره

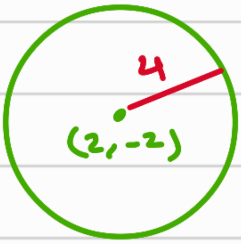


$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

مثال

$(x-2)^2 + (y--2)^2 = 16$

$h = 2 \qquad K = -2 \qquad r = \sqrt{16} = 4$



r=5

EXAMPLE 2 Find the standard equation of a circle of radius 5 and center (1, −5). Also find the y-coordinates of the two points on this circle with x-coordinate 2.

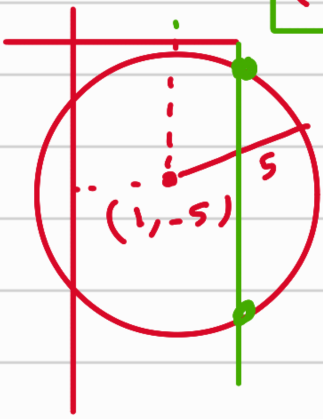
h k

اكتب معادله الدائره

(1, -5)   r=5

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\boxed{(x-1)^2 + (y+5)^2 = 25}$$



نعوض قيمه x=2 لحسب قيم y

$$(2-1)^2 + (y+5)^2 = 25$$

$$1 + (y+5)^2 = 25$$

$$\sqrt{(y+5)^{\cancel{2}}} = \sqrt{24}$$

$$y + 5 = \pm\sqrt{24}$$

$$y = \pm\sqrt{24} - 5$$

$$y = \pm 2\sqrt{6} - 5$$

EXAMPLE 3 Show that the equation

$$x^2 - 2x + \underline{y^2 + 6y} = -6$$

represents a circle, and find its center and radius.

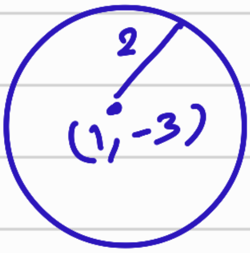
$(x-h)^2+(y-k)^2=r^2$

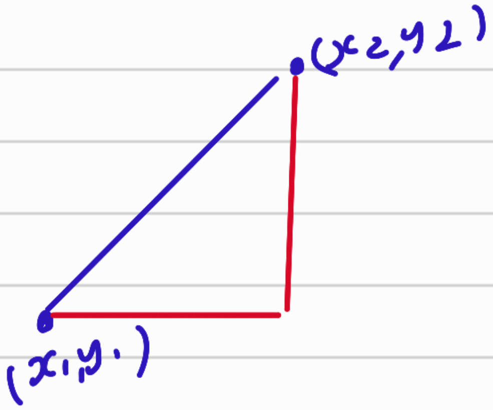
$(\frac{b}{2})^2$ $(\frac{b}{2})^2$

$$\underbrace{x^2-2x+1} + \underbrace{y^2+6y+9} = -6+1+9$$

$$(x-1)^2+(y+3)^2=4$$

$h=1$ $k=-3$ $r=2$





معادلة خط

equation of the line

*

$$\boxed{y = mx + b}$$

m: slope

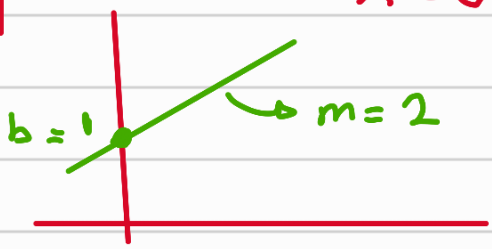
b: y intercept

مقطع y

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$\frac{\text{فرق } y}{\text{فرق } x}$ = الميل

slope



$y=2x+1$

معادله الخط المستقيم

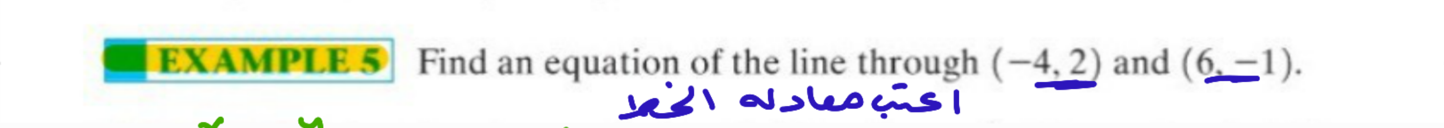
$$\boxed{y - y_1 = m(x - x_1)}$$

$x_1 \; y_1$ $(3, 2)$ $x_2 \; y_2$ $(8, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{8-3} = \frac{2}{5} = 2.5$$

$$y - y_1 = 2.5(x - x_1)$$

$$y - 2 = 2.5(x - 3)$$



EXAMPLE 5 Find an equation of the line through (−4, 2) and (6, −1).

اكتب معادله الخط

$x_1 \; y_1$ $(-4, 2)$ $x_2 \; y_2$ $(6, -1)$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{6--4} = \frac{-3}{10}$$

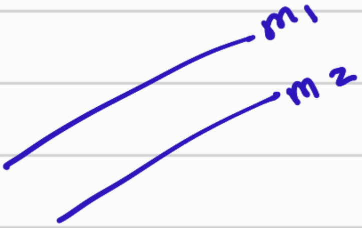
$$y - 2 = -\frac{3}{10}(x + 4)$$

✓ دائماً اخلي m هو العدد المضروب بـ x عندما تكونه شكل المعادله $y =$

$y = 3x + 5$ $m = 3$

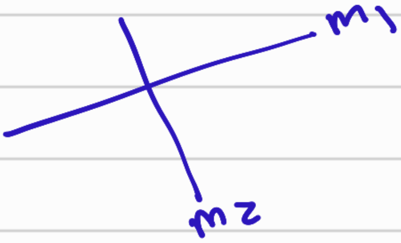
$\frac{2y}{2} = \frac{10x}{2} - \frac{2}{2}$ $y = 5x - 1$ $m = 5$

اي خطين متوازيين (Parallel) يكون



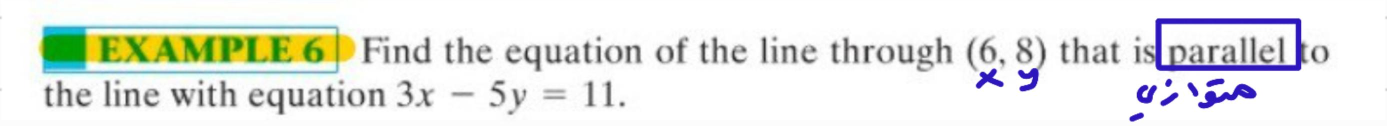
لهما نفس قيمة الميل $m_1 = m_2$

اي خطين متعامدين (Perpendicular) يكون



ميل الاول = $-\frac{1}{\text{ميل الثاني}}$

$$m_2 = -\frac{1}{m_1}$$


EXAMPLE 6 Find the equation of the line through (6, 8) that is $\boxed{\text{parallel}}$ to the line with equation $3x - 5y = 11$.

x y

متوازي

حساب ميل خط (m_1)

$$3x - 5y = 11$$

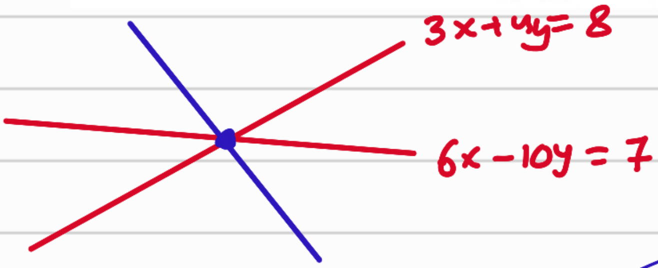
$$y = \frac{3x}{5} + \frac{11}{-5} \qquad m_1 = \frac{3}{5}$$

ميل $m_2 = \frac{3}{5}$ يمر بالنقطة (6, 8)

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{3}{5}(x - 6)$$

EXAMPLE 7 Find the equation of the line through the point of intersection of the lines with equations $3x + 4y = 8$ and $6x - 10y = 7$ that is perpendicular to the first of these two lines (Figure 16).



$$y - y_1 = m_2(x - x_1)$$

$$-\frac{1}{m_1} = m_2$$ ميل الخط

$$3x+4y=8 \qquad y = -\frac{3}{4}x + 2$$

$$m_1 = -\frac{3}{4}$$

$$\boxed{m_2 = \frac{4}{3}}$$

$$y - y_1 = m(x - x_1$$
$$y - \frac{1}{2} = \frac{4}{3}(x-2)$$

نسوي معادله لخطين، لحساب نقطه التقاط

$$(3x+4y=8) \quad \times -2$$
$$6\cancel{x} - 10y = 7$$
$$-\cancel{6x} - 8y = -16$$

$$-18y = -9$$

$$y = \frac{-9}{-18} = \boxed{\frac{1}{2}}$$

$$3x + 4\left(\frac{1}{2}\right) = 8$$

$$3x = 6 \qquad \boxed{x=2}$$

Problem Set 0.3

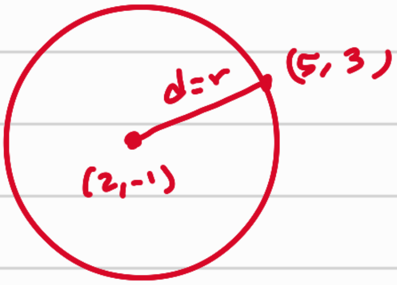
$$(x-h)^2+(y-k)^2=r^2$$

In Problems 11–16, find the equation of the circle satisfying the given conditions.

11. Center $(\overset{h}{\underline{1}}, \overset{k}{\underline{1}})$, radius $\overset{r}{1}$ $(x-1)^2+(y-1)^2=1$

12. Center $(-2, 3)$, radius 4

13. Center $(\overset{h}{2}, \overset{k}{-1})$, goes through $(5, 3)$



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(5-2)^2+(3--1)^2}$$

$$r=d=\sqrt{9+16}=\sqrt{25}=5$$

$$\boxed{(x-2)^2+(y+1)^2=25}$$

In Problems 17–22, find the center and radius of the circle with the given equation.

17. $\underline{x^2+2x}+10+\underline{y^2-6y}-10=0$

$$\left(\frac{b}{2}\right)^2$$

$$\underbrace{x^2+2x+1}+\underbrace{y^2-6y+9}=10-10+1+9$$

$$(x+1)^2+(y-3)^2=10$$

Centre $(-1,3)$ $r=\sqrt{10}$

In Problems 23–28, find the slope of the line containing the given two points.

23. (1, 1) and (2, 2)

24. (3, 5) and (4, 7)

25. (2, 3) and (−5, −6)

26. (2, −4) and (0, −6)

27. (3, 0) and (0, 5)

28. (−6, 0) and (0, 6)

32) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-1} = 1$

25) $m = \frac{-6-3}{-5-2} = \frac{9}{7}$

In Problems 29–34, find an equation for each line. Then write your answer in the form $Ax + By + C = 0$.

29. Through (2, 2) with slope −1

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x - 2)$

$y - 2 = -x + 2$ ✓

✓

$y = -x + 4$ ✓ $y + x - 4 = 0$

33. Through (2, 3) and (4, 8)

34. Through (4, 1) and (8, 2)

In Problems 35–38, find the slope and y-intercept of each line.

35. $3y = -2x + 1$

36. $-4y = 5x - 6$

$y - y_1 = m(x - x_1)$

33). $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{4-2} = \frac{5}{2}$

$y - 3 = \frac{5}{2}(x - 2)$

$$2y - 6 = 6x - 10$$

$$2y - 6x + 4 = 0$$

مقطع y

In Problems 35–38, find the slope and y-intercept of each line. $y = mx + \textcircled{b}$ مقطع y

35. $3y = -2x + 1$ **36.** $-4y = 5x - 6$

35) $3y = -2x + 1$

$$y = -\frac{2}{3}x + \textcircled{\frac{1}{3}}$$ y intercep $= \frac{1}{3}$

حل آخر

يمكن حساب b بنعوض x=0

$x_1 \quad y_1$

39. Write an equation for the line through $(3, -3)$ that is

(a) $\boxed{\text{parallel}}$ to the line $y = 2x + 5$;

(b) perpendicular to the line $y = \textcircled{2}x + 5$;

(c) parallel to the line $\underline{2x + 3y = 6}$;

(d) $\underline{\text{perpendicular}}$ to the line $2x + 3y = 6$;

$$y - y_1 = m(x - x_1)$$

$$y + 3 = m_1(x - 3)$$

---

a) $m_2 = m_1 = 2$

$$y + 3 = 2(x - 3)$$

b) $m_1 = -\frac{1}{m_2} = -\frac{1}{2}$

$$y + 3 = -\frac{1}{2}(x - 3)$$

c) $2x + 3y = 6$ $\quad y = \frac{6}{3} - \frac{2x}{3}$ $\quad m = -\frac{2}{3}$

$$y + 3 = -\frac{2}{3}(x - 3)$$

d) $y = \frac{6}{3} - \frac{2}{3}x$ $\quad m_2 = -\frac{2}{3}$

$m_1 = \frac{3}{2}$

$$y + 3 = \frac{3}{2}(x - 3)$$