Chapter 1

Linear Momentum and Collisions

الزخم الخطي التصادمات

Outline

- 9.1 Linear Momentum and Its Conservation
- 9.2 Analysis Model: Isolated System (Momentum)
- 9.3 Analysis Model: Nonisolated System (Momentum)
- 9.4 Collisions in One Dimension
- 9.5 Collisions in Two-Dimensions
- 9.6 The Center of Mass
- 9.7 System of Many Particles
- 9.9 Rocket Propulsion

$\vec{P}$ الزخم الخطي حفظ

9.1 - Linear Momentum and Its Conservation

$\vec{P} = m \times \vec{V}$ الكتلة x السرعة

$k\,m/s$

Definition

- The ***linear momentum*** of a particle, or an object that can be modeled as a particle, of mass m moving with a velocity $\vec{\mathbf{v}}$ is defined to be the product of the mass and velocity:

$$\boxed{\vec{\mathbf{p}} = m\vec{\mathbf{v}}}$$

- The terms momentum and linear momentum will be used interchangeably.

$\vec{P} = m\vec{V}$ $\quad m = \frac{\vec{P}}{\vec{V}}$ $\quad V = \frac{\vec{P}}{m}$

Linear Momentum

كمية متجهة $\vec{p}$

- Linear momentum is a ***vector*** quantity.

اتجاه الزخم نفس اتجاه السرعة

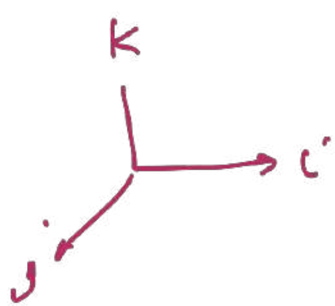
- Its direction is the ***same as the direction*** of the velocity.
- The dimensions of momentum are ML/T (Mass · Length/ Time)

$p = mv = ML/T$

النظام العالمي للوحدات

- The SI units of momentum are kg · m / s
- Momentum can be expressed in component form:

$$p_x = mv_x, \qquad p_y = mv_y, \qquad p_z = mv_z$$



$\vec{P} = P_x i + P_y j + P_z k$

قانون نيوتن الثاني والزخم

$F \rightarrow \boxed{m} \rightarrow a$

$\sum F = ma$

Newton's Second Law and Momentum

p, $\sum F$

- The linear momentum can be related to the resultant force acting on a particle through Newton's second law:

$\sum F = ma$

$\sum F = m \frac{dV}{dt}$

$\sum F = \frac{d\,mv}{dt} = \frac{dP}{dt}$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\boxed{\Rightarrow \sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}}$$

القوة المحصلة المؤثرة على جسم تساوي مشتقة الزخم

$\sum F = \frac{dp}{dt}$

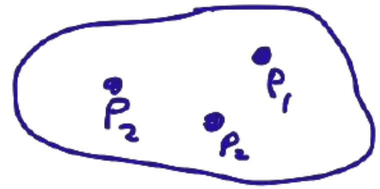
- The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle:

معدل تغير الزخم الخطي المؤثر في جسم يساوي محصلة القوى عليه

  - This is the form in which Newton presented the Second Law.
  - It is a more general form than the one we used previously.
  - This form also allows for mass changes.

طريقة اخرى واكثر شمولا لتمثيل قانون نيوتن الثاني يمكن تطبيقه في حالة تغير الكتلة

Derivation



نظام معزول = قوة الكليه صفر

- Consider an isolated system with two masses:
  - m_1 moves at a velocity $\vec{\mathbf{v}}_1$ due to a force $\vec{\mathbf{F}}_{21}$ exerted on it by m_2.
  - m_2 moves at a velocity $\vec{\mathbf{v}}_2$ due to a force $\vec{\mathbf{F}}_{12}$ exerted on it by m_1.

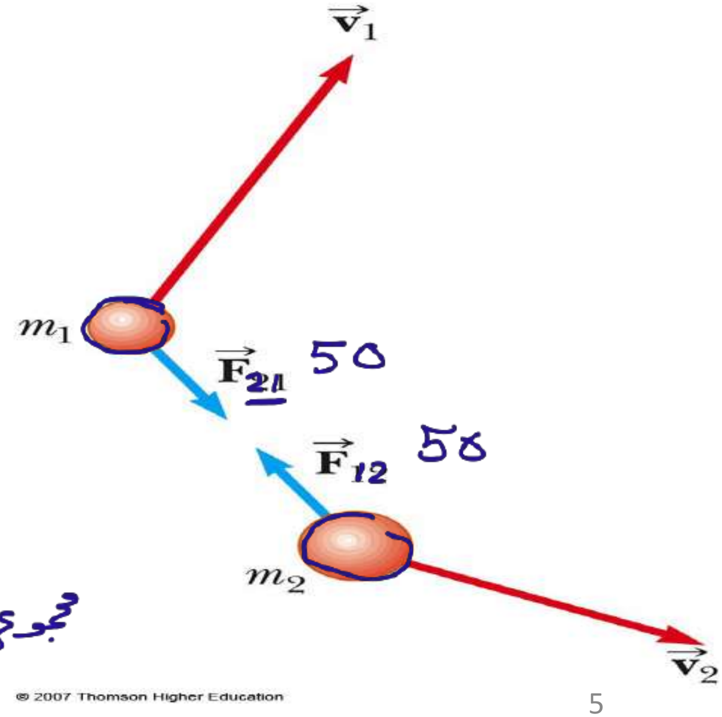
- From Newton's third law:

قوتين متساويتين
متعاكستين

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

$$\Rightarrow \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21} = 0$$

مجموع القوى الكليه بالنظام المعزول = صفر



Derivation, cont.

$F_{12} + F_{21} = 0$

- From Newton's second law:

$$m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 = 0$$

$$\Rightarrow m_1\frac{d\vec{\mathbf{v}}_1}{dt} + m_2\frac{d\vec{\mathbf{v}}_2}{dt} = 0$$

$$\Rightarrow \frac{d(m_1\vec{\mathbf{v}}_1)}{dt} + \frac{d(m_2\vec{\mathbf{v}}_2)}{dt} = 0$$

مشتقة الثابت = صفر

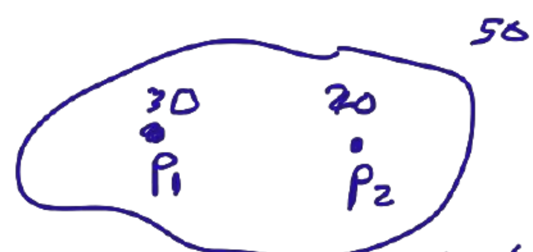
$$\Rightarrow \boxed{\frac{d}{dt}}(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2) = 0$$

$$\Rightarrow m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = \text{constant}$$

$P_1 + P_2 =$ ثابت

- Hence, for an isolated system, the sum of the quantity $m\vec{\mathbf{v}}$ for each particle is conserved. → محفوظ

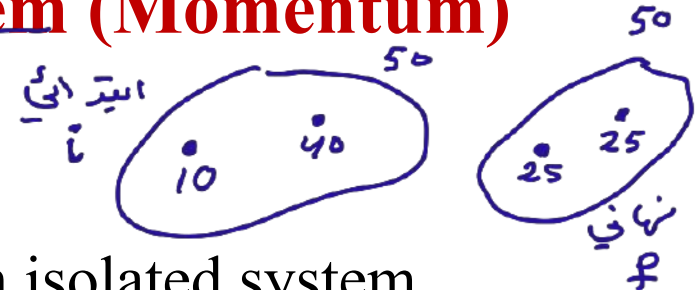
- This is the reason for the definition of linear momentum.



مجموع الزخم في نظام معزول = ثابت

نظام معزول

9.2 Analysis Model: Isolated System (Momentum)



حفظ الزخم

Conservation of Linear Momentum

- Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

$$\vec{\mathbf{p}}_{\text{tot}} = \text{constant}$$

ΣP

مجموع الزخم دائماً ثابت

- The momentum ***of the system is conserved***, not necessarily the momentum of an individual particle.

الزخم للنظام يكون ثابت وليس بالضرورة للأجسام

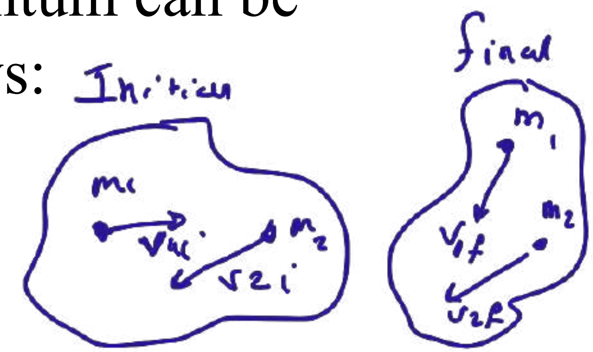
- This also tells us that the total momentum of an isolated system equals its initial momentum or final momentum.

كلي ابتدائي النهائي مقدار ثابت

$$\vec{\mathbf{p}}_{\text{tot}} = \vec{\mathbf{p}}_{\text{tot}\,i} = \vec{\mathbf{p}}_{\text{tot}\,f} = \text{constant}$$

Conservation of Linear Momentum, cont.

- For two particles, conservation of momentum can be expressed mathematically in various ways:
  - Vector form:



$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$P_{tot\,i} = P_{tot\,f}$

$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$

قبل   بعد

  - Component form:

$$\begin{cases} p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx} \\ p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy} \\ p_{1iz} + p_{2iz} = p_{1fz} + p_{2fz} \end{cases}$$

$P_{1iy} + P_{2iy} = P_{1fy} + P_{2fy}$

- Remember: Conservation of momentum can be applied to systems with ***any number of particles.***

يمكن تطبيق العبارات على أي عدد من الاجسام

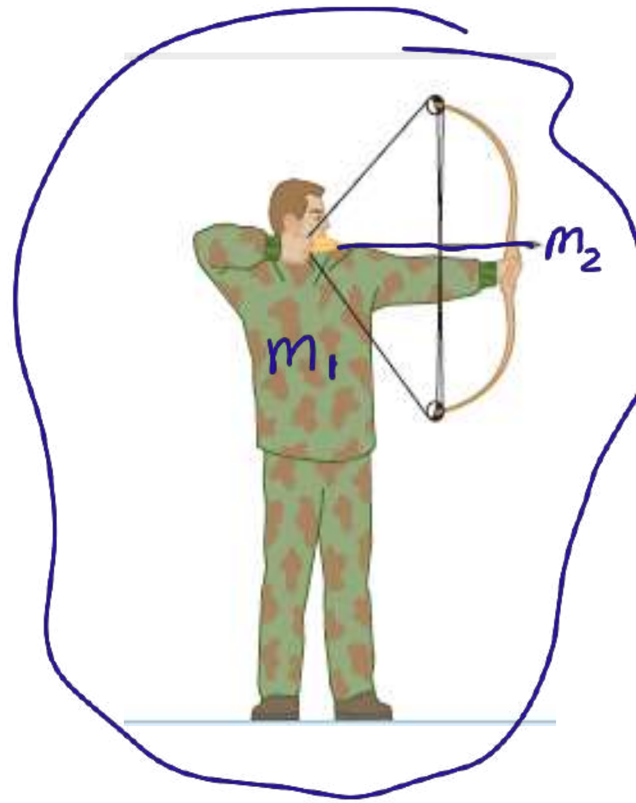
Example 9.1: The Archer

A 60-kg archer stands at rest on frictionless ice and fires a 0.030-kg arrow horizontally at 85 m/s. With what velocity does the archer move across the ice after firing the arrow?

Given

m_1=60 kg, $v_{1i} = 0$, v_{1f} =?

m_2=0.030 kg, $v_{2i} = 0$, $v_{2f} = 85$m/s



Solution

From the conservation of momentum

$$m_1 \underbrace{\cancel{v_{1i}}}_{=0} + m_2 \underbrace{\cancel{v_{2i}}}_{=0} = m_1 v_{1f} + m_2\, v_{2f}$$

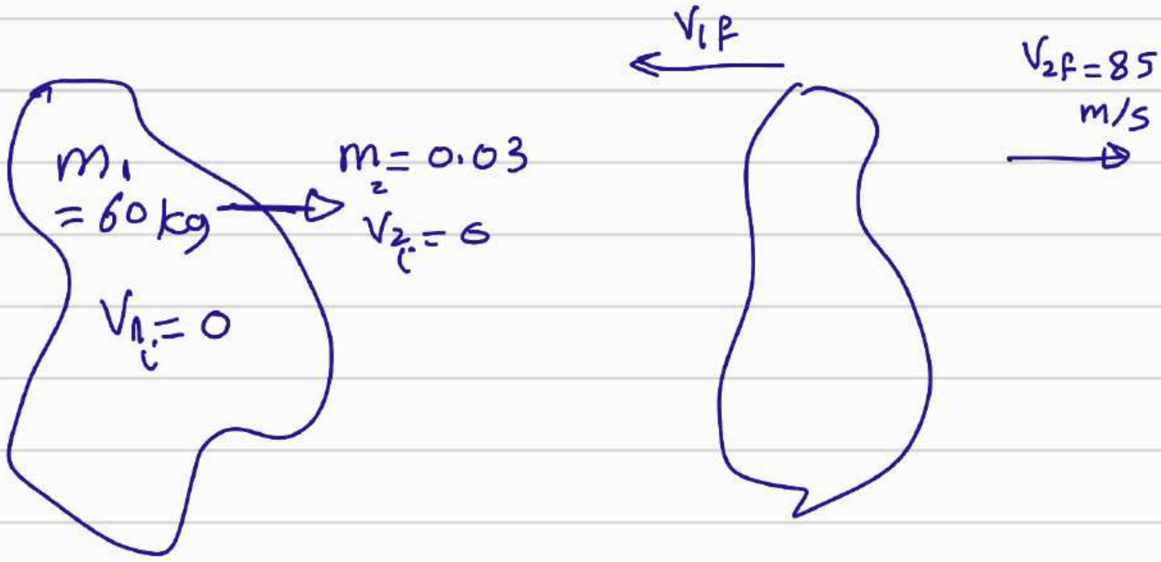
$$0 = m_1 v_{1f} + m_2\, v_{2f}$$

$$m_1 v_{1f} = -m_2\, v_{2f}$$

$$v_{1f} = \frac{-m_2\, v_{2f}}{m_1} = \frac{-(0.03)(85)}{60}$$

$$\Rightarrow \boxed{v_{1f} = -0.042\text{m/s}}$$

9-1



Initial   final

$$P_{tot\,i} = P_{tot\,f}$$

$$\cancel{m_1 v_{1i}} + \cancel{m_2 v_{2i}} = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = 60(v_{1f}) + 0.03(85)$$

$$-\frac{(0.03)(85)}{60} = \frac{\cancel{60}(v_{1f})}{\cancel{60}}$$

$$v_{1f} = \frac{-(0.03)(85)}{60} = -0.0425 \text{ m/s}$$

نظام غير معزول

9.3 Analysis Model: Nonisolated System (Momentum)

Derivation

- Knowing the change in momentum caused by a force is useful in solving some types of problems. From Newton's Second Law:

$$\sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\Sigma F = \frac{dP}{dt}$$

$$\Rightarrow d\vec{\mathbf{p}} = \sum \vec{\mathbf{F}}\ dt$$

$$\int dP = \int \Sigma F\, dt$$

- Integrating,

$$P_f - P_i = \int \Sigma F\, dt$$

impulse ∴ I

التغير في P

$$\int_{\vec{\mathbf{p}}_i}^{\vec{\mathbf{p}}_f} d\vec{\mathbf{p}} = \int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\ dt$$

$$I = \int \Sigma F\, dt$$

$$\Rightarrow \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \Delta\vec{\mathbf{p}} = \int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\ dt$$

Impulse الدفع $\vec{I}$

- The integral is defined as the impulse, $\vec{\mathbf{I}}$, of the force acting on an object over the time interval $\Delta t = t_f - t_i$

$$\boxed{\vec{\mathbf{I}} = \int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\, dt}$$

- The impulse of the force acting on a particle equals the change in the momentum of the particle.

$P = kg\,m/s$

$I = kg\,m/s$

الدفع = التغير في الزخم

$$\boxed{\vec{\mathbf{I}} = \Delta \vec{\mathbf{p}}}$$

- This expresses the ***impulse-momentum theorem*** which is equivalent to Newton's Second Law

قاعده الدفع والزخم

$\vec{v}$

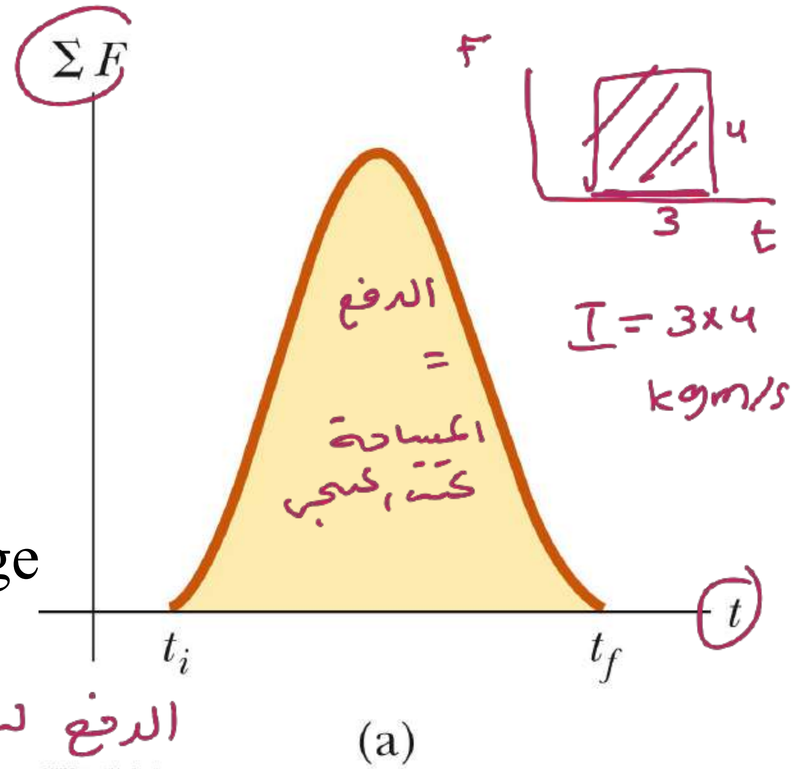
$\vec{P}$

$\vec{I}$

Impulse, cont.

الدفع: كمية متجهة تساوي المساحة تحت منحنى t و ΣF واتجاهه نفس اتجاه p

- Impulse is a ***vector*** quantity.
  - The magnitude of the impulse is equal to the ***area under the force-time curve***.
  - The direction of impulse is the same as the direction of change in momenta.
- The force may vary with time.
- Dimensions of impulse is the ***same as momentum***.
- Impulse is not a property of the particle, but a measure of the change in momentum of the particle.



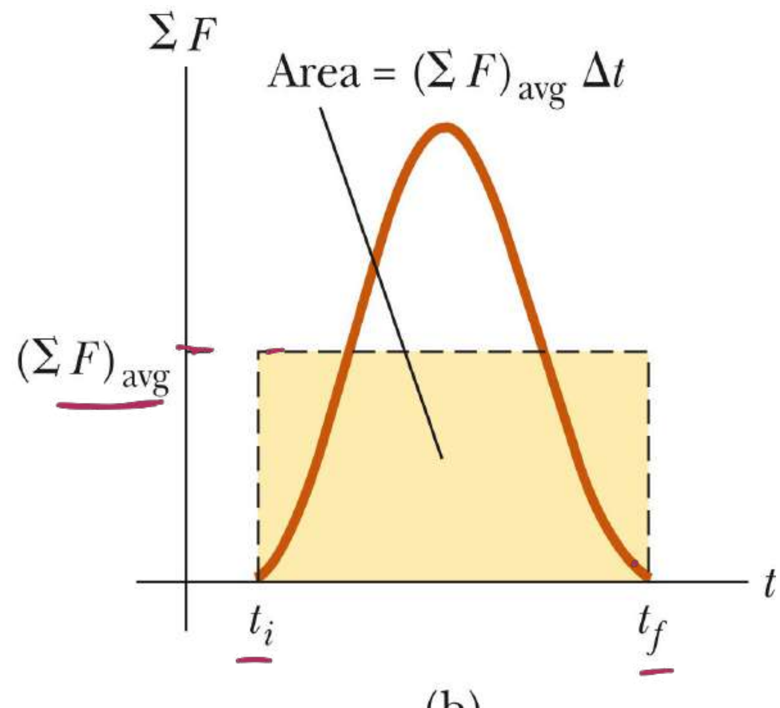
(a)

الدفع ليس خاصية للجسم لكنه يعبر عن تغير الزخم

Impulse and Time-Averaged Force

- Because the net force imparting an impulse to a particle can generally vary in time, it is convenient to define a time-averaged net force:

$$\left(\sum \vec{\mathbf{F}}\right)_{avg} \equiv \frac{1}{\Delta t}\int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\, dt$$



(b)

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- The impulse is then expressed as

$$\vec{\mathbf{I}} = \left(\sum \vec{\mathbf{F}}\right)_{avg} \Delta t$$

- This would give the same impulse as the time-varying force does.

$$I = \int \Sigma F\, dt = \Sigma F_{avg} \Delta t = F \Delta t$$

Impulse Approximation

- In Impulse Approximation, it is assumed that ***one force*** acting on a particle acts for a short time but is much greater than any other force present.

- This approximation is especially useful in analyzing collisions. تصادم

- This force will be called the ***impulsive force.*** The impulse will then be calculated by:

$$\boxed{\vec{\mathbf{I}} = \vec{\mathbf{F}}\,\Delta t}$$

- The particle is assumed to move very little during the collision. Hence, $\vec{\mathbf{p}}_i$ and $\vec{\mathbf{p}}_f$ represent the momenta
لحظياً ***immediately*** before and after the collision.

الدفع: التغير في الزخم

Problem Analysis

معزول

Isolated system

$$\vec{\mathbf{p}}_{tot} = \text{constant}$$

ثابت

$$\sum \vec{\mathbf{F}}_{ext} = 0$$

القوة الخارجية = صفر

(لا يوجد قوة خارجية) غير معزول

Non-isolated system

الزخم سوف يتغير

$\Delta P = P_f - P_i$

$\Delta P = I$

Impluse

$$\vec{\mathbf{I}} = \Delta \vec{\mathbf{p}}_{tot}$$

قوة لحظية

Impulsive Force

$$\vec{\mathbf{F}} = \frac{\vec{\mathbf{I}}}{\Delta t}$$

متوسط

Average Force

$$\left(\sum \vec{\mathbf{F}}\right)_{avg} = \frac{\vec{\mathbf{I}}}{\Delta t}$$

$I = \Delta P = P_f - P_i = mv_f - mv_i$

$I = F\Delta t$

Example 9.3: How Good Are the Bumpers?

In a particular crash test, an automobile of mass 1500 kg collides with a wall, as shown in Figure. The initial and final velocities of the automobile are $\vec{\mathbf{v}}_i = -15.0\hat{\mathbf{i}}$ m/s and $\vec{\mathbf{v}}_f = 2.60\hat{\mathbf{i}}$ m/s, respectively.

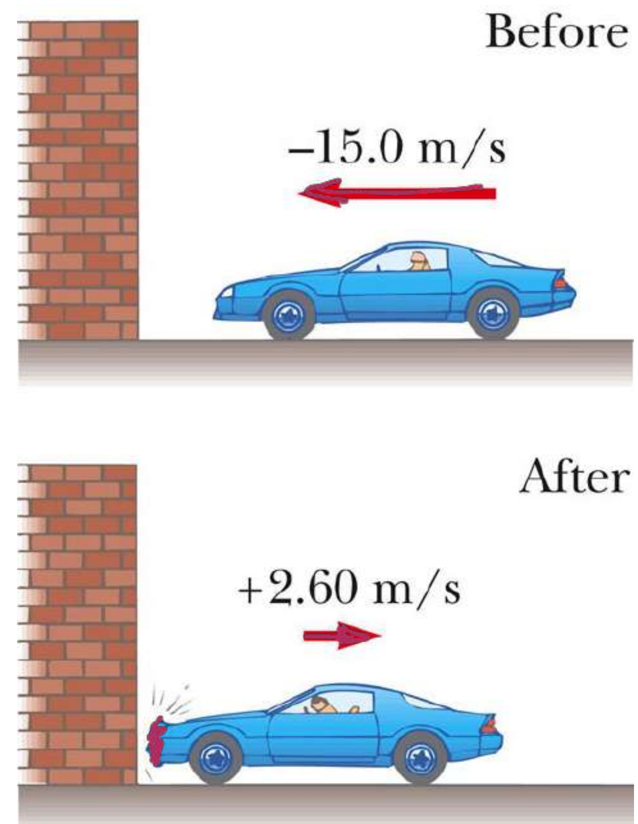
a) If the collision lasts for 0.150 s, find the impulse caused by the collision and the average force exerted on the automobile.

I

F

b) What if the car did not rebound from the wall? Suppose the final velocity of the car is zero and the time interval of the collision remains at 0.150 s. Would that represent a larger or a smaller net force on the car?

اعاده السوال لكن السرعه النهائية 0



(a)

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$m = 1500\,kg$
$\vec{v}_i = -15\hat{i}\ m/s$ $\qquad v_f = 2.6\hat{i}\ m/s$
$\Delta t = 0.15$

a) $I = \Delta P = P_f - P_i = m\vec{v}_f - m\vec{v}_i$

$$I = m(\vec{v}_f - \vec{v}_i)$$

$$I = 1500(2.6\hat{i} + 15\hat{i})$$

$$= 1500(17.6\hat{i}) = 26400\hat{i}\ kgm/s$$

$$2.6 \times 10^4\hat{i}$$

$$I = F\Delta t \qquad F = \frac{I}{\Delta t}$$

$$F = \frac{26400\hat{i}}{0.150} = 176000\hat{i}\ N$$

$$17.6 \times 10^4\hat{i}\ N$$

b) $m = 1500$
$v_i = -15\hat{i}$ $\qquad v_f = 0$ $\qquad \Delta t = 0.15$

$$I = P_f - P_i = m(\vec{v}_f - \vec{v}_i)$$

$$= 1500(0 - -15\hat{i})$$

$$= 2.25 \times 10^4\hat{i}\ kgm/s$$

$$F = \frac{I}{\Delta t} = \frac{1.5 \times 10^5}{0.15} = 1.5 \times 10^5\hat{i}\ N$$

Solution:

Given

m=1500 kg

$\vec{\mathbf{v}}_i = -15.0\hat{\mathbf{i}}$ m/s

$\vec{\mathbf{v}}_f = 2.60\hat{\mathbf{i}}$ m/s,

$t = 0.150$s

a) First we calculate the impulse:

$$
\begin{aligned}
\vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} \\
&= \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i \\
&= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i \\
&= m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i) \\
&= 1500(2.60\hat{\mathbf{i}} + 15\hat{\mathbf{i}}) \\
&= 1500(17.6\hat{\mathbf{i}}) \\
&= [2.64\times10^4\hat{\mathbf{i}}] \text{ kg.m/s}
\end{aligned}
$$

The average force:

$$
\begin{aligned}
\vec{\mathbf{F}}_{\text{avg}} &= \frac{\vec{\mathbf{I}}}{\Delta t} \\
&= \frac{2.64\times10^4\hat{\mathbf{i}}}{0.150} \\
&= 1.76\times10^5\hat{\mathbf{i}} \text{ N}
\end{aligned}
$$

b) The impulse when $\vec{\mathbf{v}}_f = 0$:

$$
\begin{aligned}
\vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} \\
&= \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i \\
&= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i \\
&= -m\vec{\mathbf{v}}_i \\
&= -1500(-15\hat{\mathbf{i}}) \\
&= [2.25\times10^4\hat{\mathbf{i}}]\ \text{kg.m/s}
\end{aligned}
$$

The average force:

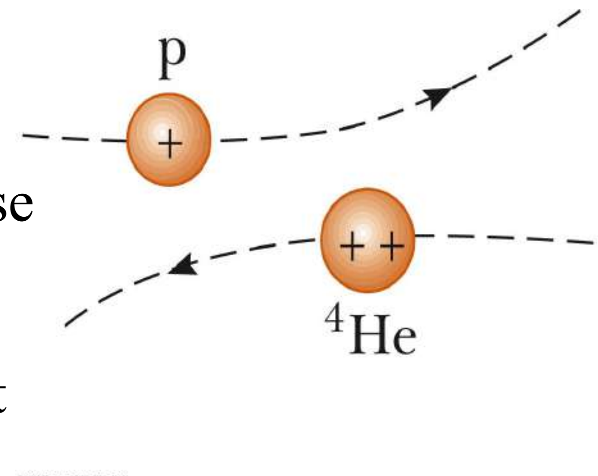
$$
\begin{aligned}
\vec{\mathbf{F}}_{\text{avg}} &= \frac{\vec{\mathbf{I}}}{\Delta t} \\
&= \frac{2.25\times10^4\hat{\mathbf{i}}}{0.150} \\
&= 1.5\times10^5\hat{\mathbf{i}}\ \text{N}
\end{aligned}
$$

Comparing the two forces, we conclude that the force is smaller in case b.

9.4 Collisions in One Dimension

التصادم في بعد واحد

Characteristics



- We use the term collision to represent an event during which ***two particles*** come close to each other and ***interact*** by means of forces
  - May involve physical contact, but must be generalized to include cases with interaction without physical contact.
- The time interval during which the velocity changes from its initial to final values is assumed to be short.

نهمل القوى الخارجية

- The interaction forces are assumed to be much greater than any external forces present.
  - This means the ***impulse approximation can be used.***

$I = F\Delta T$

Types of Collisions

تصادم مرن

Elastic collision

$P_i = P_f$
$K_i = K_f$

momentum and kinetic energy are conserved

الزخم والطاقة محفوظة

تصادم تام المرونة

- Perfectly elastic collisions occur on a microscopic level.
- In macroscopic collisions, only approximately elastic collisions actually occur, some energy is lost to deformation, sound, etc.

في العالم الجاهري لا يحدث تصادم تام المرونة

سيضيع جزء من الطاقة على شكل صوت حرارة

تصادم غير مرن

Inelastic collision

الزخم محفوظ لكن الطاقة لا

kinetic energy is not conserved
momentum is conserved

$P_i = P_f$
$K_f \neq K_i$

تلتصق الاجسام مع بعض بعد التصادم

- If the objects stick together after the collision, it is a ***perfectly inelastic*** collision.

عديم المرونة

Momentum is conserved in all collisions

تصادم عديم المرونة

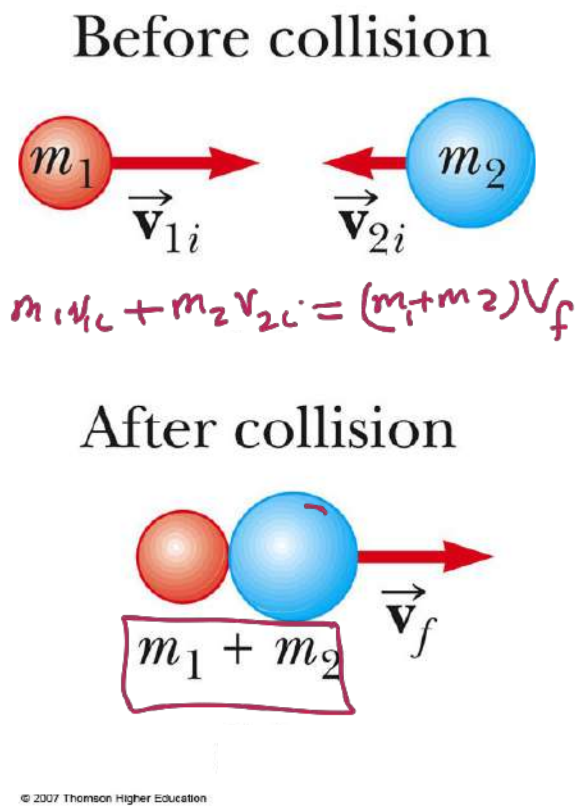
Perfectly Inelastic Collisions

- Since the objects stick together, they share the ***same velocity after*** the collision.

الجسمين التصقو وتحركو بنفس السرعة

- From the conservation of momenta:
- $m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$

$$\boxed{\Rightarrow \vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}}{m_1 + m_2}}$$



$m_1v_{1i} + m_2v_{2i} = (m_1+m_2)V_f$

قانون لحساب السرعة النهائية بعد تصادم عديم المرونة

$K = \frac{1}{2} m v^2$

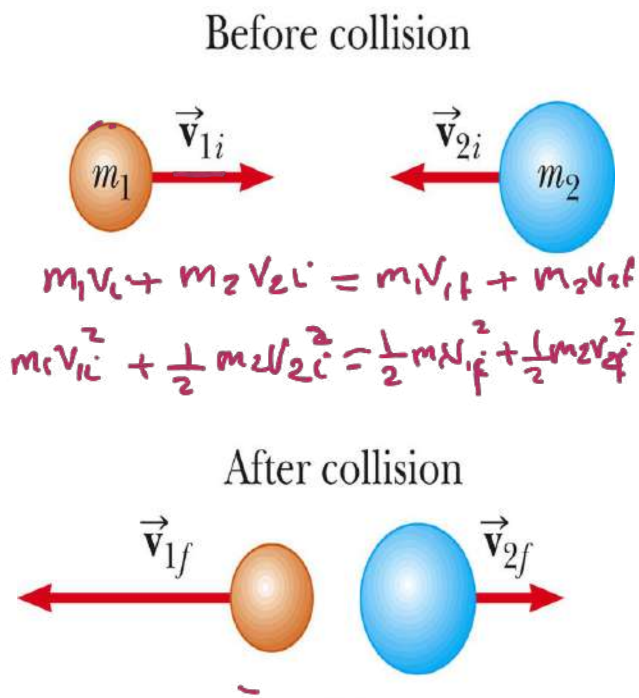
$K_i = K_f$

Elastic Collisions   تصادم مرن

- Both momentum and kinetic energy are conserved.

$$m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i} = m_1 \vec{\mathbf{v}}_{1f} + m_2 \, \vec{\mathbf{v}}_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 \, v_{2f}^2$$



$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$

$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

- Since we are working in one dimension:

$$\boxed{\begin{aligned} m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 \, v_{2f}^2 \end{aligned}}$$

الإشارات المناسبة

- It is important to use the ***appropriate signs*** for v: positive if the particle move to the right and negative if it moves to the left.

Elastic Collisions, Analysis

- Typically, there are two unknowns to solve for and so we need two equations.

- The kinetic energy equation can be difficult to use. With some algebraic manipulation, a different equation can be used:

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

- This equation, along with conservation of momentum, can be used to solve for the two unknowns. If these two unknows are the final velocities:

قوانين حساب
السرعات النهائيه
في حالة التصادم
اعرفها

احفظ

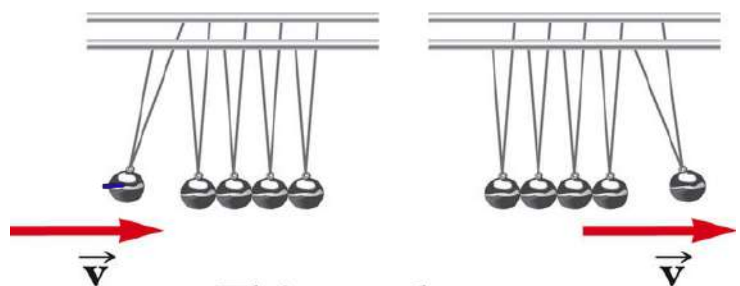
$$\boxed{\begin{aligned} v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \\ v_{2f} &= \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \end{aligned}}$$

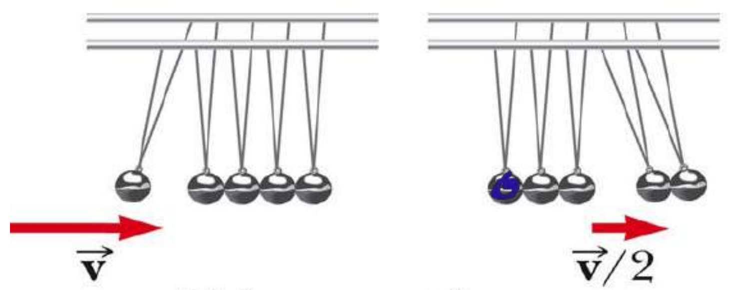
- It ***can only be used*** with a one-dimensional, elastic collision between two objects.

باتجاه واحد مرن

Example 9.4: The Executive Stress Reliever

An ingenious device that illustrates conservation of momentum and kinetic energy is shown in Figure. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost- elastic collision between it and ball 2, ball 5 moves out, as shown in Figure (b). If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1,as in Figure (c)?





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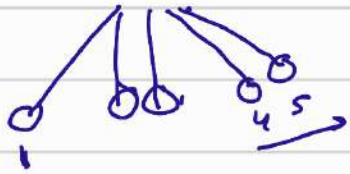
$m_1 = m \qquad v_{1i} = v \qquad v_{f1} = 0$

$m_2 = 2m \qquad v_{2i} = 0 \qquad v_{f2} = \frac{v}{2}$



قانون حفظ الزخم   تصادم مرن

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m v_{1i} + 0 = 0 + 2m\, v_{2f}$$

$$\cancel{m} v = 2\cancel{m}\, v_{2f}$$

$$v = \cancel{2}\, \frac{v}{\cancel{2}}$$

$$v = v$$

زخم

قانون حفظ الطاقة

$$\frac{1}{2} m_1 v_{1i}^2 + \cancel{\frac{1}{2} m_2 v_{2i}^2} = \frac{1}{2} m_1 \cancel{v_{1f}^2} + \frac{1}{2} m_2 v_{2f}^2$$

$$\cancel{\frac{1}{2}}\cancel{m} v^2 = \cancel{\frac{1}{2}}\, 2\cancel{m} \left(\frac{v}{2}\right)^2$$

$$v^2 = \frac{2v^2}{4}$$

$$v^2 = \frac{v^2}{2}$$

لا يمكن حدوث هذه الحاله بسبب اختلاف السرعة

Solution:

Given:

$m_1 = m\,, \qquad v_{1i} = v, \qquad v_{1f} = 0$

$m_2 = 2m, \qquad v_{2i} = 0\,, \qquad v_{2f} = v/2$

We investigate both the conservation of momentum and energy:

$$m_1 v_{1i} + m_2 \cancelto{0}{v_{2i}} = m_1 \cancelto{0}{v_{1f}} + m_2\, v_{2f}$$

$$\Rightarrow \cancel{m} v_{1i} = 2\cancel{m} v_{2f}$$

$$\Rightarrow v = 2\left(\frac{v}{2}\right)$$

$$\Rightarrow v = v$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} \cancelto{0}{m_2 v_{2i}^2} = \frac{1}{2} m_1 \cancelto{0}{v_{1f}^2} + \frac{1}{2} m_2\, v_{2f}^2$$

$$m\cancel{v}_{1i}^2 = 2\cancel{m}v_{1f}^2$$

$$v^2 = 2\frac{v^2}{2^2}$$

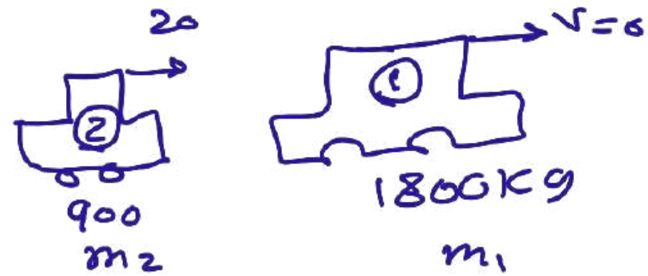
$$\Rightarrow v^2 \neq \frac{v^2}{2}$$

The motion described cannot happen because it violates the conservation of energy.

Example 9.5: Carry Collision Insurance

An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car. The two cars become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

Solution:



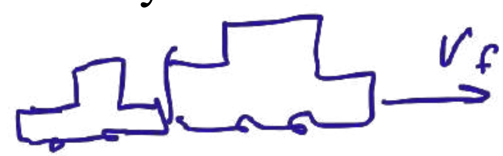
Given:

$m_1 = 900$ kg $\quad v_{1i} = 20\ m/s$

$m_2 = 1800$ kg $\quad , \quad v_{2i} = 0, \quad v_f = ??$

The magnitude of the total momentum of the system before the collision is equal to that of the smaller car because the larger car is initially at rest.

$P_{tot\,i} = P_{tot\,f}$



$$P_i = P_f$$

$m_1 v_{1i} + m_2 V_{2i} = (m_1 + m_2) V_f$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$

$900 \times 20 = (1800 + 900) V_f$

$$v_f = \frac{m_1\, v_{1i}}{m_1 + m_2} = \frac{(900)(20)}{900 + 1800}$$

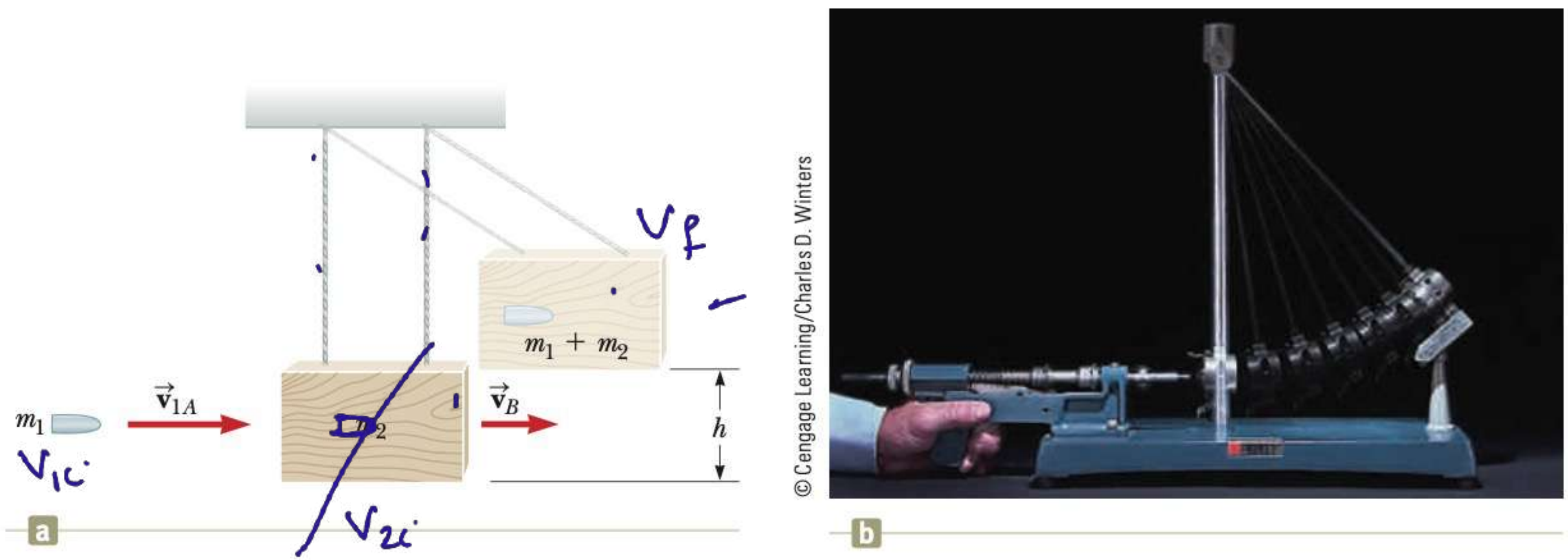
$V_f = \dfrac{900 \times 20}{(1800 + 900)} = 6.67\ m/s$

$$= 6.67\ \text{m/s}$$

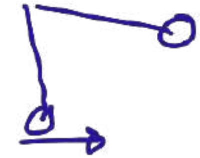
Example 9.6: The Ballistic Pendulum

The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we determine the speed of the projectile from a measurement of h?

$$V_f = \frac{m_1 v_{1i} + \cancel{m_2 v_{2i}}}{m_1 + m_2} \quad \text{zer}$$



طاقة الحركة عند اخفض نقطة = طاقة الوضع عند اعلى نقطة



وضع

طاقة حركة

Solution: $Mgh = \frac{1}{2}M v_f^2$

Because the projectile imbeds in the block, we can categorize the collision between them as perfectly inelastic.

$$v_f = \frac{m_1\, v_{1i}}{m_1 + m_2}$$

$$v_f = \frac{m_1 v_{i1}}{(m_1 + m_2)}$$

the total kinetic energy

$$K_f = \frac{1}{2}(m_1 + m_2)\, v_f^2$$

$$k = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_{1i}}{(m_1 + m_2)}\right)^2$$

Substitute the value of v_f into K_f expression

$$k = \frac{1}{2}\cancel{(m_1 + m_2)}\,\frac{m_1^2 v_{1i}^2}{(m_1 + m_2)^{\cancel{2}}}$$

$$K_f = \frac{m_1^2 v_{1i}^2}{2(m_1 + m_2)}$$

$$K_o = \frac{m_1^2 v_1^2}{2(m_1 + m_2)}$$

This kinetic energy of the system immediately after the collision is *less* than the initial kinetic energy of the projectile as is expected in an inelastic collision.

$$\frac{m_1^2 v_1^2}{2(m_1 + m_2)} = (m_1 + m_2)\, g\, h$$

$$\sqrt{v_1^2} = \sqrt{\frac{2(m_1+m_2)^2 g h}{m_1^2}}$$

We define the gravitational potential energy of the system

$$U_f = 0$$

$$V_1 = \sqrt{2}\,\frac{(m_1+m_2)}{m_1}\sqrt{gh}$$

$$U_C = (m_1 + m_2)gh.$$

Apply the conservation of mechanical energy principle to the system:

$$Kf + Uf = K_C + U_C$$

$$\boxed{V_1 = \sqrt{2gh}\,\frac{(m_1+m_2)}{m_1}}$$

$$\frac{m_1^2 v_{1i}^2}{2(m_1+m_2)} + 0 = 0 + (m_1 + m_2)gh$$

Solve for v_{1i}:

$$v_{\mathbf{1}\boldsymbol{i}} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$

التصادم في بعدين

9.5 Collisions in Two Dimension

Two-Dimensional Collisions

$P_{tot\,i\,x} = P_{tot\,f\,x}$
$P_{tot\,i\,y} = P_{tot\,f\,y}$

- The momentum is conserved in each of the directions x and y.

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2\, v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2\, v_{2fy}$$

- Use subscripts for the velocity in the following order:

$$v_{1ix}$$

Identifying the object

Indicating initial or final values

The velocity component

If the collision is elastic, use conservation of kinetic energy as a second equation

- Remember, the simpler equation derived previously can only be used for one-dimensional situations.

$A_y = A\sin\theta$

$A_x = A\cos\theta$

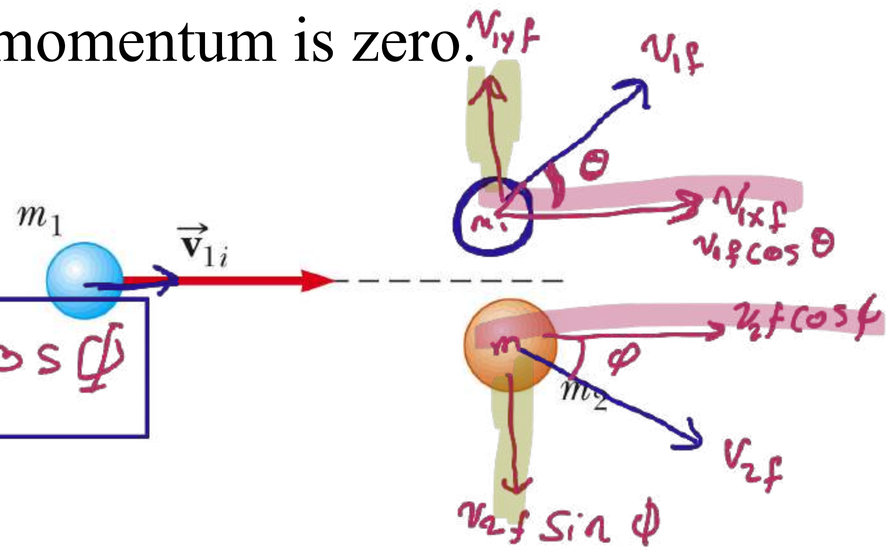
Two-Dimensional Collision, Example

- Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest.
- In the x-direction, the initial momentum is $m_1 v_{1i}$.
- In the y-direction, the initial momentum is zero.

قانون حفظ الزخم في x

$P_{xi} = P_{xf}$

$$\boxed{m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi}$$



(a) Before the collision

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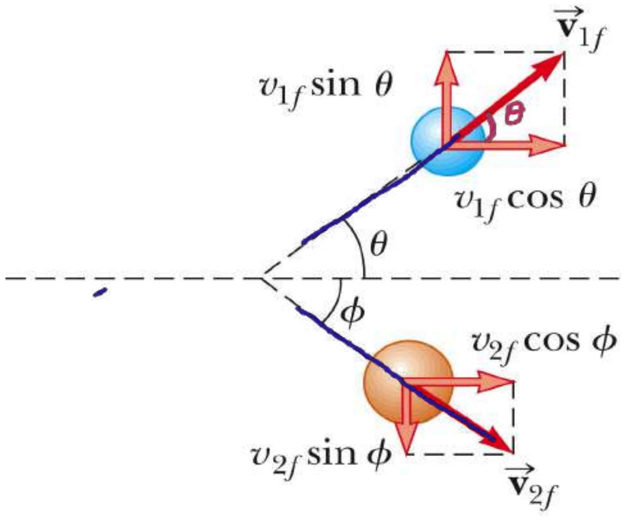
قانون حفظ الزخم في y

$P_{yi} = P_{yf}$

$$\boxed{0 = m_1 v_{1f}\sin\theta + m_2 v_{2f}\sin\phi}$$

Two-Dimensional Collision, Example, cont.



(b) After the collision

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- After the collision, the momentum in the:
  - x-direction is:
    $$m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi$$
  - y-direction is:
    $$m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi$$
- The conservation of momentum in the x and y direction gives:

$$m_1v_{1i} = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi$$

$$0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi$$

- If the collision is elastic, apply the kinetic energy equation.

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

معادله الطاقه
سوف تكون معادله
واحدة لأن
الطاقه قياسية

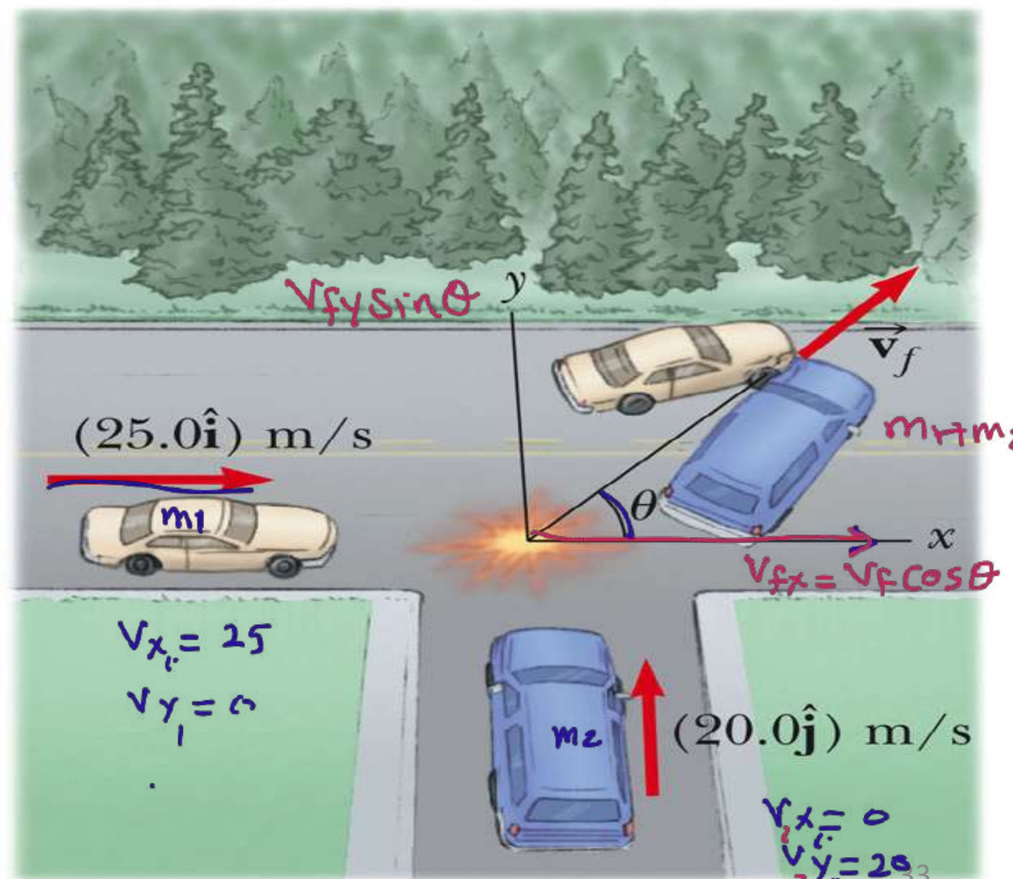
- This is an example of a ***glancing collision***

Example 9.8: Collision at an Intersection

A 1500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

Given:

$m_1 = 1500\text{kg}$
$m_2 = 2500\text{kg}$
$v_{1ix} = 25 \text{ m/s}$
$v_{1iy} = 0$
$v_{2ix} = 0$
$v_{2iy} = 20\text{m/s}$



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طريقة:

حفظ الزخم باتجاه x

$$m_1 V_{1ix} + \cancel{m_2 V_{2ix}} = (m_1+m_2) V_f \cos\theta$$

$$m_1 V_{1ix} = (m_1+m_2) V_f \cos\theta$$

$$\boxed{1500(25) = (1500+2500) V_f \cos\theta} \quad \text{...(1)}$$

حفظ الزخم باتجاه y

$$\cancel{m_1 V_{1iy}} + m_2 V_{2iy} = (m_1+m_2) V_f \sin\theta$$

$$m_2 V_{2iy} = (m_1+m_2) V_f \sin\theta$$

$$\boxed{(2500)20 = (1500+2500) V_f \sin\theta}$$

$$37500 = 4000\, \underline{V_f} \cos\theta \quad \text{...(1)}$$

$$50000 = 4000\, V_f \sin\theta \quad \text{...(2)}$$

نقسم المعادلتين
2/1

$$\frac{50000}{37500} = \frac{\cancel{4000}\, \cancel{V_f}}{\cancel{4000}\, \cancel{V_f}} \frac{\sin\theta}{\cos\theta}$$

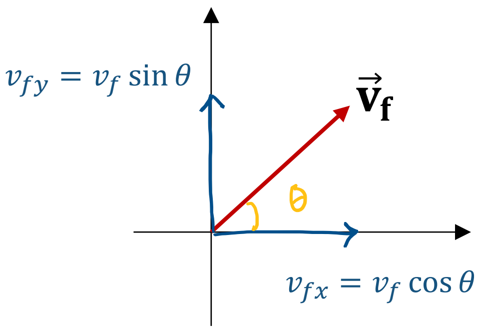
$$\tan\theta = 1.33 \qquad \theta = \tan^{-1}(1.33)$$

$$\theta = 53.1$$

نعوض في معادلة من المعادلات

$$V_f = \frac{37500}{4000(\cos 53.1)} = 15.6\ m/s$$

Solution:



The collision is perfectly inelastic:

$$v_{1fx} = v_{2fx} = v_{fx} = v_f \cos\theta$$
$$v_{1fy} = v_{2fy} = v_{fy} = v_f \sin\theta$$

Conservation of momuntem on x

$$m_1 v_{1ix} + m_2 \cancelto{=0}{v_{2ix}} = m_1 v_{1fx} + m_2\, v_{2fx}$$

$$m_1 v_{1ix} = (m_1 + m_2) v_f \cos\theta$$

$$(1500)(25) = (1500 + 2500) v_f \cos\theta$$

$$3.75 \times 10^4 = 4000\, v_f \cos\theta \quad (1)$$

Conservation of momuntem on y

$$m_1 \cancelto{=0}{v_{1iy}} + m_2 v_{2iy} = m_1 v_{1fy} + m_2\, v_{2fy}$$

$$m_2 v_{2iy} = (m_1 + m_2) v_f \sin\theta$$

$$(2500)(20) = (1500 + 2500) v_f \sin\theta$$

$$5 \times 10^4 = 4000 v_f \sin\theta \quad (2)$$

$A = 3i + 4j$

$|A| = \sqrt{3^2 + 4^2} = 5$ | $\theta = \tan^{-1}\frac{4}{3}$

$=$

Dividing equations (2) on (1):

$$\frac{5\times10^4 = \cancel{4000}v_f \sin\theta}{3.75\times10^4 = \cancel{4000}\, v_f \cos\theta}$$

$$\Rightarrow 1.33 = \tan\theta$$

$$\Rightarrow \theta = \tan^{-1} 1.33$$

$$\Rightarrow \boxed{\theta = 53.1°}$$

From equation (2):

$$5\times10^4 = 4000 v_f \sin\theta$$

$$\Rightarrow v_f = \frac{5\times10^4}{4000 \sin 53.1°}$$

$$\boxed{\Rightarrow v_f = 15.6 \mathrm{m/s}}$$ ✓

تمرين

$V_{1i} = 25i$ $V_{2i} = 20j$

$m_1V_{1i} + m_2V_{2i} = (m_1 + m_2)\, v_f$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)}$$

$$v_f = \frac{1500(15i) + 2500(20j)}{(1500 + 2500)}$$

$$v_f = \frac{37500i + 50000j}{4000}$$

$$v_f = \frac{37500}{4000} i + \frac{50000}{4000} j$$

$$\boxed{v_f = 9.375 i + 12.5 j} \rightarrow$$ مقدار

$$v_f = \sqrt{9.375^2 + 12.5^2} = 15.6$$

$$\theta = \tan^{-1} \frac{12.5}{9.375} = 53.1^\circ$$

Vector method:

Given:

$\vec{\mathbf{v}}_{1i} = 25\hat{\mathbf{i}}$ m/s

$\vec{\mathbf{v}}_{2i} = 20\hat{\mathbf{j}}$ m/s

$\vec{\mathbf{v}}_{1f} = \vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_f = ?$

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\,\vec{\mathbf{v}}_{2f}$$

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\,\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}}{m_1 + m_2}$$

$$\vec{\mathbf{v}}_f = \frac{(1500)25\hat{\mathbf{i}} + (2500)20\hat{\mathbf{j}}}{1500 + 2500}$$

$$\vec{\mathbf{v}}_f = \frac{3.75\times10^4\hat{\mathbf{i}} + 5\times10^4\hat{\mathbf{j}}}{4000}$$

$$\vec{\mathbf{v}}_f = 9.375\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}}$$

Magnitude:

$$\Rightarrow v_f = \sqrt{9.375^2 + 12.5^2}$$

$$\boxed{\Rightarrow v_f = 15.6\text{m/s}}$$

Direction:

$$\Rightarrow \theta = \tan^{-1}\left(\frac{12.5}{9.375}\right)$$

$$\boxed{\Rightarrow \theta = 53.1^\circ}$$

Definition

- There is a special point in a system (a group of particles or an extended object.), called the ***center of mass.***

مركز الكتلة

انتقاليه

- When an external force is applied to the center of mass, the system moves ***translationally*** as if the force were applied to a single particle of mass *M* located at the center of mass
  - $\boxed{M}$ is the total mass of the system

- One method to determine the center of mass is by applying forces at different points. The center of mass is the only point that allows the system to move translationally and not rotationally.

لتحديد نقطة مركز الكتلة: هي النقطة التي اذا اثرت مجموعة قوى على الجسم
لا يتحرك الا بحركة انتقالية وليست دورانية

9.6 The Center of Mass

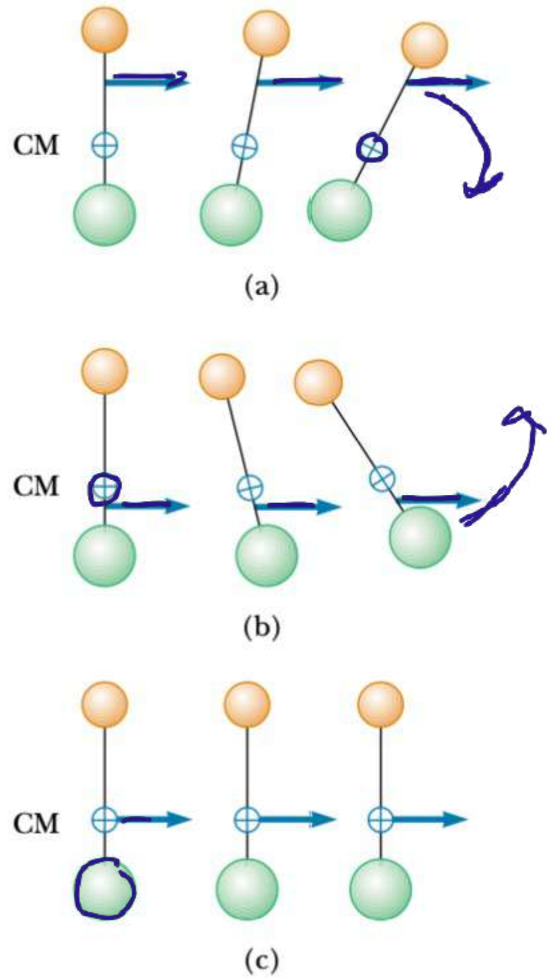
Example

- Consider a mechanical system consisting of a pair of particles that have different masses and are connected by a light, rigid rod.

- The center of mass of the system is located somewhere on the line joining the particles and is closer to the particle having the larger mass.

  - A force above the CM causes the system to rotate clockwise (a).
  - A force below the CM causes the system to rotate counterclockwise (b).
  - A force at the CM causes the system to move translationally (c).

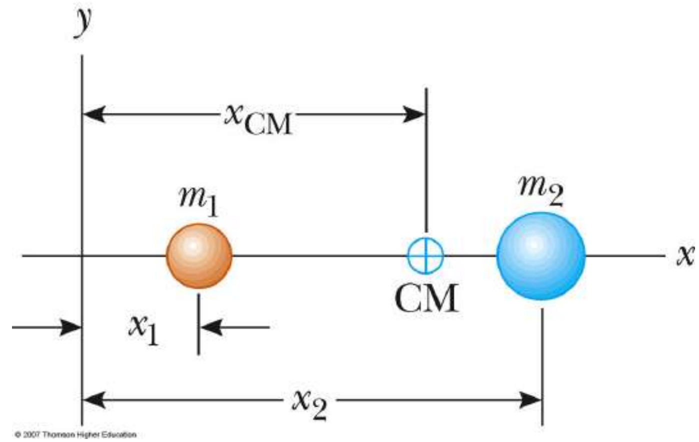
الحركه تكون انتقاليه



Center of Mass for a System of particles, Coordinates

- The x coordinate of the center of mass of the system in Figure is :

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



- We can extend this concept to a system of many particles with masses m_i in three dimensions. The x coordinate of the center of mass of n particles is defined to be:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_i^n m_i x_i}{\sum_i^n m_i} = \frac{\sum_i^n m_i x_i}{M}$$

$$\boxed{= \frac{1}{M} \sum_i^n m_i x_i}$$

$$\frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{M}$$

$$\frac{1}{M} \sum m_i x_i$$

- $\underline{M}$ is the total mass of the system

- Other coordinates can be calculated similarly:

x_{cm} y_{cm} z_{cm}

$$x_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i x_i\,, \qquad y_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i y_i\,, \qquad z_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i z_i$$

- From the coordinates we can find the center of mass position vector:

$$\boxed{\vec{\mathbf{r}}_{\mathrm{CM}} = x_{\mathrm{CM}}\,\hat{\mathbf{i}} + y_{\mathrm{CM}}\,\hat{\mathbf{j}} + z_{\mathrm{CM}}\,\hat{\mathbf{k}}}$$

$r_{cm} = X_{cm} i + Y_{cm} j + Z_{cm} k$

- Substituting the values of the coordinates:

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i x_i\,\hat{\mathbf{i}} + \frac{1}{M}\sum_i^n m_i y_i\,\hat{\mathbf{j}} + \frac{1}{M}\sum_i^n m_i z_i\,\hat{\mathbf{k}}$$

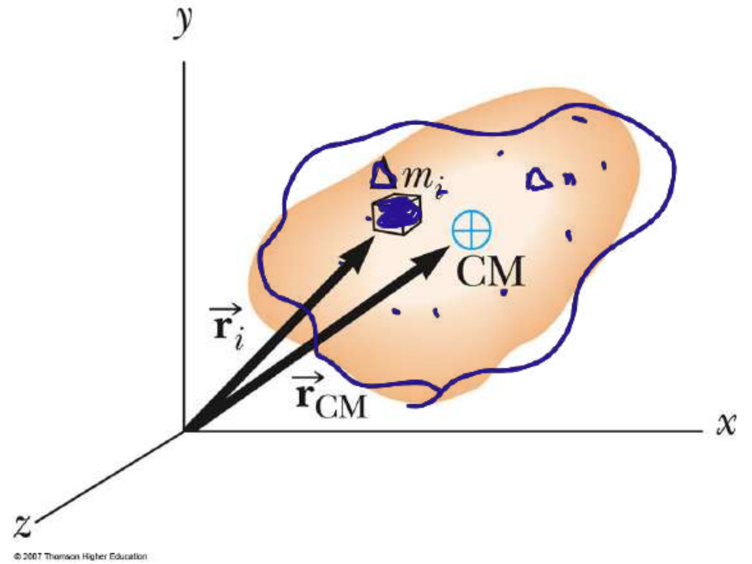
$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i [x_i\,\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}]$$

$$\boxed{\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\sum_i^n m_i\,\vec{\mathbf{r}}_i}$$

$$\vec{\mathbf{r}}_i \equiv x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}]$$

Center of Mass for an Extended Object

- Consider the extended object as a system containing large number of particles each of mass Δm_i and coordinates x_i, y_i and z_i. The Center of mass for the x coordinate for example is approximately:



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$$x_{\mathrm{CM}} \approx \frac{1}{M}\sum_{i}^{n} \Delta m_i x_i$$

- To find the center of mass precisely, we let the number of particles n approaches infinity. Hence, the size of each element Δm_i approaches zero :

$$x_{\mathrm{CM}} = \lim_{\Delta m_i \to 0} \frac{1}{M}\sum_{i}^{n} \Delta m_i x_i = \frac{1}{M}\int x\, dm$$

لحساب لكل جميع
للقطع الصغيره من الكتل نستخدم
التكامل

- Other coordinates can be calculated similarly:

$$\boxed{x_{\mathrm{CM}} = \frac{1}{M}\int x\, dm\,, \qquad y_{\mathrm{CM}} = \frac{1}{M}\int y\, dm\,, \qquad z_{\mathrm{CM}} = \frac{1}{M}\int z\, dm}$$

- The position vector is:

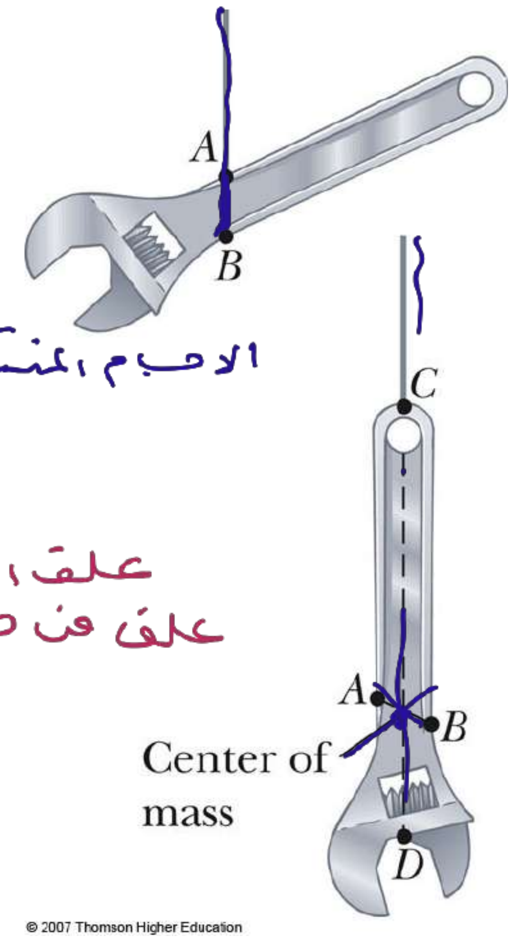
$$\boxed{\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm}$$

الاجسام المتناظره يتم حساب مركز الكتله من الشكل

uniform density

Center of Mass for Symmetric and Irregular Shaped Objects

متناظر　　　　غير منتظم



- The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry
  - If the object has uniform density

الاجسام المتناظره تكون مركز الكتله على محور التناظر بشرط ان تكون الكثافه منتظمه

- To find the center of mass for an irregularly shaped object:
  1. Suspend the object from one point　　علق الجسم
  2. Then suspend from another point　　علق من جهة أخرى
  3. The intersection of the resulting lines is the center of mass

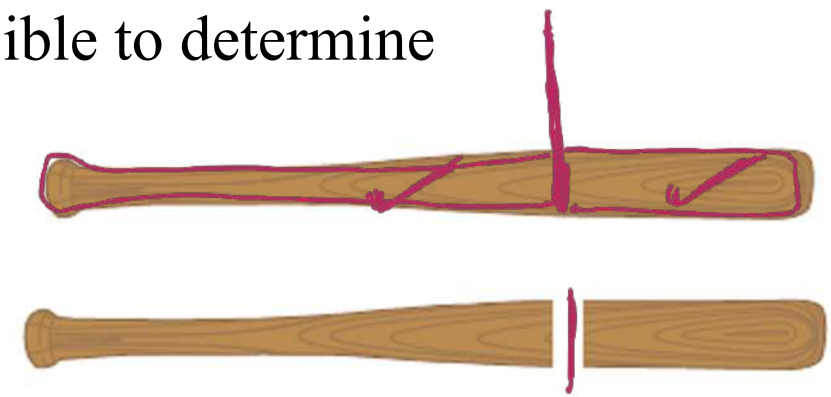
التقاطع هو مركز الكتلة

Quick Quiz 2:

كثافة منتظمة

- A baseball bat of uniform density is cut at the location of its center of mass as shown in Figure. Which piece has the smaller mass?

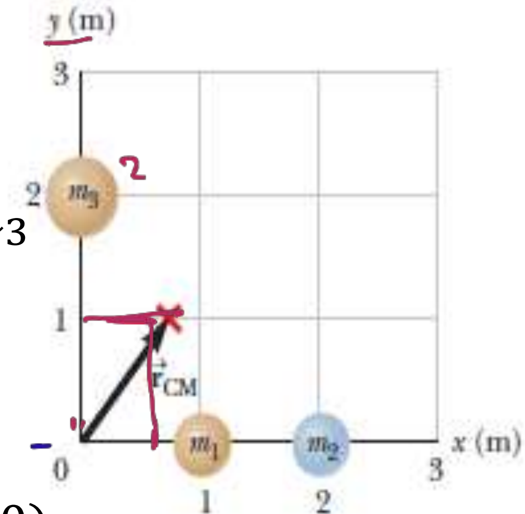
a) the piece on the right
b) the piece on the left
c) both pieces have the same mass
d) impossible to determine



مركز الكتلة

Example 9.10: The Center of Mass of Three Particles

A system consists of three particles located as shown in Figure. Find the center of mass of the system. The masses of the particles are $m_1 = m_2 = 1.0$ kg and $m_3 = 2.0$ kg.



$\boxed{r_{cm} = 0.75\hat{i} + 1\hat{j}}$

$$\vec{\mathbf{r}}_{CM} = x_{CM}\,\hat{\mathbf{i}} + y_{CM}\,\hat{\mathbf{j}}$$

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(1)(1) + (1)(2) + (2)(0)}{1 + 1 + 2} = 0.75\text{m}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (1)(0) + (2)(2)}{1 + 1 + 2} = 1.0\text{ m}$$

$$\Rightarrow \vec{\mathbf{r}}_{CM} = x_{CM}\,\hat{\mathbf{i}} + y_{CM}\,\hat{\mathbf{j}} = [0.75\hat{\mathbf{i}} + 1.0\,\hat{\mathbf{j}}]\text{m}$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{(1)(1) + 1(2) + 2(0)}{1+1+2} = 0.75\text{ m}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3} = \frac{1(0) + 1(0) + 2(2)}{1+1+2} = 1\text{ m}$$

9.6 System of Many Particles

Velocity and Momentum of a System of Particles

سرعه وزخم مركز

- The physical significance of the center of mass is that we can describe the motion of the system in terms of velocity, momentum and acceleration of the center of mass of the system.

سرعه مركز الكتله

- The velocity of the center of mass of a system of particles with a constant total mass M is:

$$V = \frac{d r_{cm}}{dt}$$

$$\vec{\mathbf{v}}_{\mathrm{CM}} = \frac{d\vec{\mathbf{r}}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_{i}^{n} m_i \frac{d\vec{\mathbf{r}}_i}{dt} = \boxed{\frac{1}{M}\sum_{i}^{n} m_i\,\vec{\mathbf{v}}_i}$$

$$V_{cm} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

زخم مركز الكتله

- The momentum can be expressed as

$$P_{cm} = \sum m_i V_i = \sum P_i$$

$$= m_1 V_1 + m_2 V_2 \ldots$$

$$= P_1 + P_2 \ldots$$

$$P = M V_{cm}$$

$$M\vec{\mathbf{v}}_{\mathrm{CM}} = \sum_{i}^{n} m_i\,\vec{\mathbf{v}}_i = \boxed{\sum_{i}^{n} \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\mathrm{tot}}}$$

- The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass

حساب زخم مركز الكتله فقط بجمع الزخم لجميع الاجسام

التسارع القوة

Acceleration and Force of a System of Particles

$a_{cm} = \frac{m_1 a_1 + m_2 a \cdots}{m_1 + m_2 + m_3}$

تسارع مركز الكتلة

- The acceleration of the center of mass can be found by differentiating the velocity with respect to time

$$\vec{\mathbf{a}}_{\mathrm{CM}} = \frac{d\vec{\mathbf{v}}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_{i}^{n} m_i \frac{d\vec{\mathbf{v}}_i}{dt} = \boxed{\frac{1}{M}\sum_{i}^{n} m_i\, \vec{\mathbf{a}}_i.}$$

- The acceleration can be related to a force $\Sigma F = ma$

$$M\vec{\mathbf{a}}_{\mathrm{CM}} = \sum_{i}^{n} m_i\, \vec{\mathbf{a}}_i = \boxed{\sum_{i}^{n} \vec{\mathbf{F}}_i}$$

$F_{cm} = \sum F_i$

- If we sum over all the internal forces, they cancel in pairs and the net force on the system is caused only by the external forces

Newton's Second Law for a System of Particles

- Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

القوه الخارجيه

$$\sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{CM}}$$

القوه المحصله = الكتله الكليه × التسارع

- The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system.

Impulse and Momentum of a System of Particles

$I = \Delta P$

- The impulse imparted to the system by external forces is

نظام معزول $\Sigma F = 0$

$$\boxed{\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}}_{\text{tot}}}$$

$\Delta P = 0$ $P = \text{constan}$

$I = 0$

- The total linear momentum of a system of particles is conserved if no net external force is acting on the system

$$M\vec{\mathbf{v}}_{\text{CM}} = \vec{\mathbf{p}}_{\text{tot}} = \text{constant}\left(\text{when} \sum \vec{\mathbf{F}}_{\text{ext}} = 0\right)$$

Example 9.14: The Exploding Rocket

A rocket is fired vertically upward. At the instant it reaches an altitude of 1000 m and a speed of 300 m/s, it explodes into three equal fragments. One fragment continues to move upward with a speed of 450 m/s following the explosion. The second fragment has a speed of 240 m/s and is moving east right after the explosion. What is the velocity of the third fragment right after the explosion?

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$$

Given:

$\vec{\mathbf{v}}_\mathbf{i} = 300\hat{\mathbf{j}}$ m/s

$\vec{\mathbf{v}}_{\mathbf{1}f} = 450\hat{\mathbf{j}}$ m/s

$\vec{\mathbf{v}}_{\mathbf{2}f} = 240\hat{\mathbf{i}}$ m/s

$\vec{\mathbf{v}}_{\mathbf{3}f} = ?$

Mass of the rocket:

M

Mass of each fragment:

$M/3$

$$\Rightarrow M\vec{\mathbf{v}}_\mathbf{i} = \frac{M}{3}\vec{\mathbf{v}}_{\mathbf{1}f} + \frac{M}{3}\vec{\mathbf{v}}_{\mathbf{2}f} + \frac{M}{3}\vec{\mathbf{v}}_{\mathbf{3}f}$$

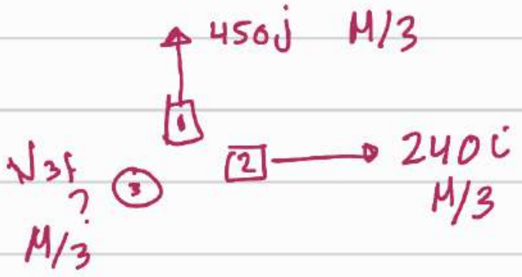
$$\Rightarrow \vec{\mathbf{v}}_\mathbf{i} = \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{1}f} + \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{2}f} + \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{3}f}$$

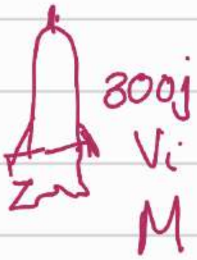
$$\Rightarrow \vec{\mathbf{v}}_\mathbf{i} - \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{1}f} - \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{2}f} = \frac{1}{3}\vec{\mathbf{v}}_{\mathbf{3}f}$$

$$\Rightarrow \vec{\mathbf{v}}_{\mathbf{3}f} = 3\vec{\mathbf{v}}_\mathbf{i} - \vec{\mathbf{v}}_{\mathbf{1}f} - \vec{\mathbf{v}}_{\mathbf{2}f}$$

$$\Rightarrow \vec{\mathbf{v}}_{\mathbf{3}f} = 3(300\hat{\mathbf{j}}) - (450\hat{\mathbf{j}}) - 240\hat{\mathbf{i}}$$

$$\boxed{\Rightarrow \vec{\mathbf{v}}_{\mathbf{3}f} = [-240\hat{\mathbf{i}} + 450\hat{\mathbf{j}}]\text{m/s}}$$





$$P_i = P_f$$

$$\cancel{M} V_i = \frac{\cancel{M}}{3} \cdot V_{1f} + \frac{\cancel{M}}{3} V_{2f} + \frac{\cancel{M}}{3} V_{3f}$$

$$300j = \frac{450}{3} j + \frac{240}{3} i + \frac{V_{3f}}{3}$$

$$900j = 450j + 240i + V_{3f}$$

$$V_{3f} = 900j - 450j - 240i$$

$$V_{3f} = 450j - 240i$$

$$V_{3f} = -240i + 450j$$

m/s

9.8 Rocket Propulsion

Rocket Propulsion

انطلاق الصاروخ

- The operation of a rocket depends upon the law of conservation of linear momentum as applied to a system of particles, where the system is the rocket plus its ejected fuel.
- This system is isolated; therefore, the ejection of the fuel causes the rocket to compensate by accelerating in the opposite direction.
- Note that the mass of the ejected fuel changes with time. We denote it by $\boxed{\Delta m}$.

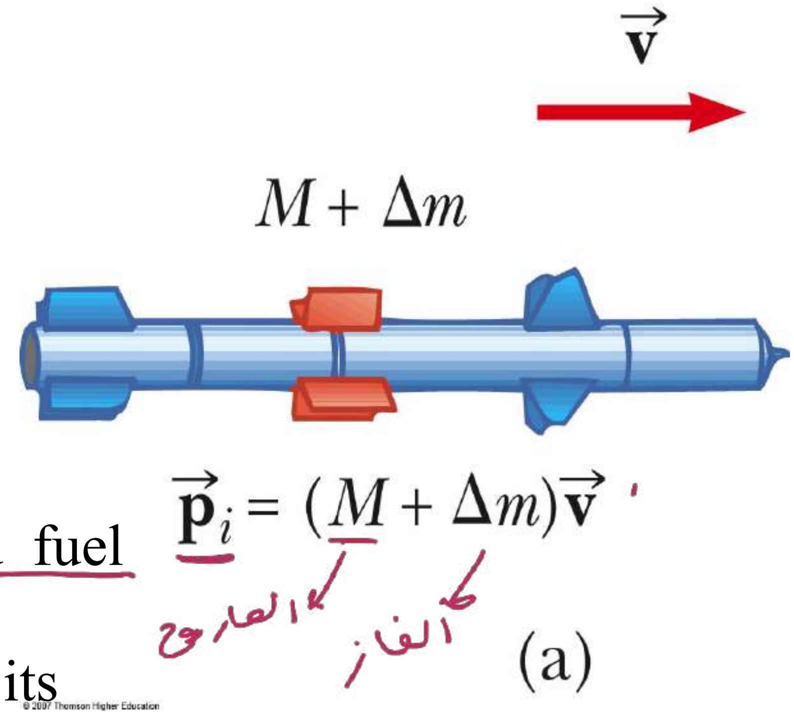
الكتلة تتغير خلال العملية

Δm التغير في الكتلة

Rocket Propulsion

- We use the following symbols:

- M: mass of the rocket.
- Δm: mass of ejected fuel.
- v: the velocity of the rocket.
- v_{e}: velocity of ejected fuel.
- Δv: Change of velocity due to ejected fuel



الكتلة في البداية

- The initial mass of the rocket plus all its fuel is $M + \Delta m$.

- The initial momentum of the system is

الزخم الابتدائي

$$p_i = (M + \Delta m)v$$

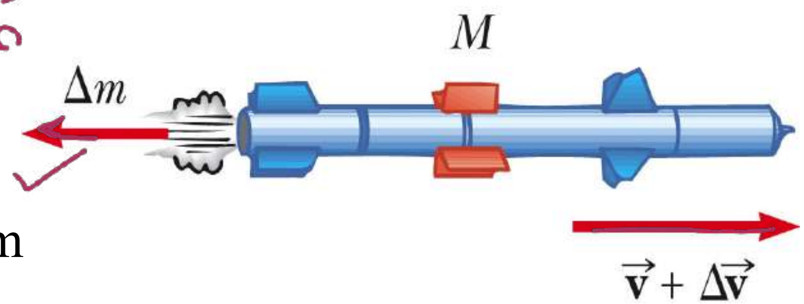
- After the fuel (Δm) is ejected with a velocity v_e, the velocity of the rocket (M) increases to $v + \Delta v$.

الزخم النهائي

سرعة انطلاق الغاز

- The Final momentum of the system is

$$p_f = \underbrace{M(v + \Delta v)}_{\text{جسم الصاروخ}} + \underbrace{\Delta m(v - v_e)}_{\text{الغاز}}$$



(b)

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- From the conservation of linear momentum

$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$$

$$\Rightarrow 0 = M\Delta v - v_e\Delta m$$

$$\Rightarrow M\Delta v = v_e\Delta m$$

- We take the limit as $t \to 0$, this lets $\Delta m \to dm$:

معدل تغير كتله الغاز

ولأن نقصان

$dm = -$

$$M dv = v_e dm$$

- The increase in the ejected mass corresponds to an equal decrease in the rocket mass.

$$\int dv = \int \frac{v_e}{M} dM$$

$$dm = -dM$$

Rocket Propulsion

- Integrating:

$$\Rightarrow dv = -v_e \frac{dM}{M}$$

$$\int_{v_i}^{v_f} dv = -v_e \int_{M_i}^{M_f} \frac{dM}{M}$$

$$\Rightarrow v \Big|_{v_i}^{v_f} = -v_e \ln M \Big|_{M_i}^{M_f}$$

$$\Rightarrow v_f - v_i = v_e[-\ln M_f + \ln M_i]$$

$$\boxed{\Rightarrow v_f - v_i = v_e \ln \frac{M_i}{M_f}}$$

سرعه انطلاق الصاروخ

- The increase in rocket speed is proportional to the speed of the escape gases (v_e).

- The increase in rocket speed is also proportional to the natural log of the ratio M_i/M_f.
  - So, the ratio should be as high as possible, meaning the mass of the rocket should be as small as possible and it should carry as much fuel as possible.

كلما كانت V_e اكبر كانت سرعة الصاروخ اكبر

كلما كانت $\frac{M_i}{M_f}$ كانت السرعة اكبر

Thrust الاندفاع

- The thrust on the rocket is the force exerted on it by the ejected exhaust gases.

$$Thrust = Ma = M\frac{dv}{dt}$$

- From :

$$dv = -v_e\frac{dM}{M}$$

- Multiplied by M and divided by dt, we get

$$M\frac{dv}{dt} = -v_e\frac{dM}{dt}$$

- Combining the two equations, we get

قوة اندفاع الصاروخ

$$\boxed{Thrust = \left|v_e\frac{dM}{dt}\right|}$$

- The expression

معدل احتراق
الغاز
kg/s

$$\frac{dM}{dt}$$

is called the ***burn rate.***

$\frac{M_i}{M_f}$ v_e v_i

Example 9.17:

A rocket moving in space, far from all other objects, has a speed of 3.0×10^3 m/s relative to the Earth. Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of 5.0×10^3 m/s relative to the rocket.

a) What is the speed of the rocket relative to the Earth once the rocket's mass is reduced to half its mass before ignition? عندما تصبح الكتلة نصف الكتلة الاولية

b) What is the thrust on the rocket if it burns fuel at the rate of 50 kg/s? $\frac{dM}{dt}$

Given

$$v_i = 3.0\times10^3 \text{m/s}$$

$$v_e = 5.0\times10^3 \text{m/s}$$

$$M_f = \frac{1}{2} M_i$$

$$\frac{dM}{dt} = 50 \text{ kg/s}$$

Solution:

a) The final speed: $v_f - v_i = v_e \ln \frac{M_i}{M_f}$

$$\Rightarrow v_f = v_i + v_e \ln \frac{M_i}{\frac{1}{2} M_i}$$

$$\Rightarrow v_f = (3.0\times10^3 \text{m/s}) + (5.0\times10^3 \text{m/s}) \ln 2$$

$$\boxed{\Rightarrow v_f = 6.5\times10^3 \text{m/s}}$$

b) $thrus = \left| v_e \frac{dM}{dt} \right|$

Solution:

b) The thrust is :

$= \left| 5\times10^3 \times 50 \right|$

$$Thrust = \left| v_e \frac{dM}{dt} \right|$$

$= 2.5 \times 10^5 \, N$

$$= \left|(5.0\times10^3 \text{m/s})(50 \text{ kg/s })\right|$$

$$\boxed{= 2.4\times10^4 \text{N}}$$

$v_i = 3\times10^3 \, m/s$ $\quad v_e = 5\times10^3$ $\quad \frac{M_i}{M_f} = 2$

a) $v_f - v_i = v_e \ln \frac{M_i}{M_f}$

$v_f - 3\times10^3 = 5\times10^3 \ln 2$

$v_f = 5\times10^3 \ln 2 + 3\times10^3 = 6465.7$

$= 6.46\times10^3$ m/s