0.2

متباينات

Inequalities and Absolute Values

equation $\quad 2x+1=5 \quad 2x=4 \quad x=2$

Inequalities $\quad 2x+1<5$

الحل يكون على شكل فترة Interval

$2x+1<5$

$2x<4 \qquad x<2$

الحل هو جميع الارقام اقل من 2

Interval Notation $\quad \{x : 5 \le x\}$

x حيث ان x اكبر او تساوي 5

$\{x : 1 < x < 5\}$

$(1, 5)$

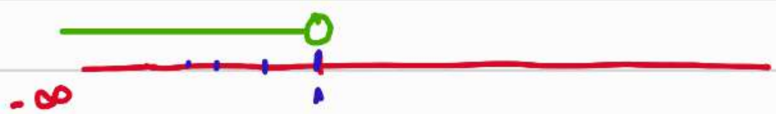
$\{x : 1 \le x \le 5\}$

$[1, 5]$

$\{x : 1 \le x < 5\}$

$[1, 5)$

$x<1$



$(-\infty, 1)$

$x \ge 5$

$[5, \infty)$

حل المتباينات

* حل المتباينة هو نفس حل المعادلة الخطية الا في حالة الضرب
او القسمة على عدد سالب تقلب الاشارة

$-2x + 1 \geq 10$

$\frac{-2x}{-2} \geq \frac{9}{-2}$

$x \leq -4.5$

$0 \leq x + 3 \leq 6$

$-3 \leq x \leq 3$

EXAMPLE 1 Solve the inequality $2x - 7 < 4x - 2$ and show the graph of its solution set.

$2x - 7 < 4x - 2$

$2x - 4x < -2 + 7$

$(-2.5, \infty)$

$\frac{-2x}{-2} < \frac{5}{-2}$

$x > -2.5$

EXAMPLE 2 Solve $-5 \leq 2x + 6 < 4$.

$-5 \leq 2x + 6 < 4$

$-5 - 6 \leq 2x < 4 - 6$

$\frac{-11}{2} \leq \frac{2x}{2} < \frac{-2}{2}$

بالقسمة على 2

$-5.5 \leq x < -1$

$[-5.5, -1)$

EXAMPLE 3 Solve the quadratic inequality $x^2 - x < 6$.

① تحليل المعادلة التربيعية

$x^2 - x - 6 < 0$

$x^2 - x - 6 = 0$

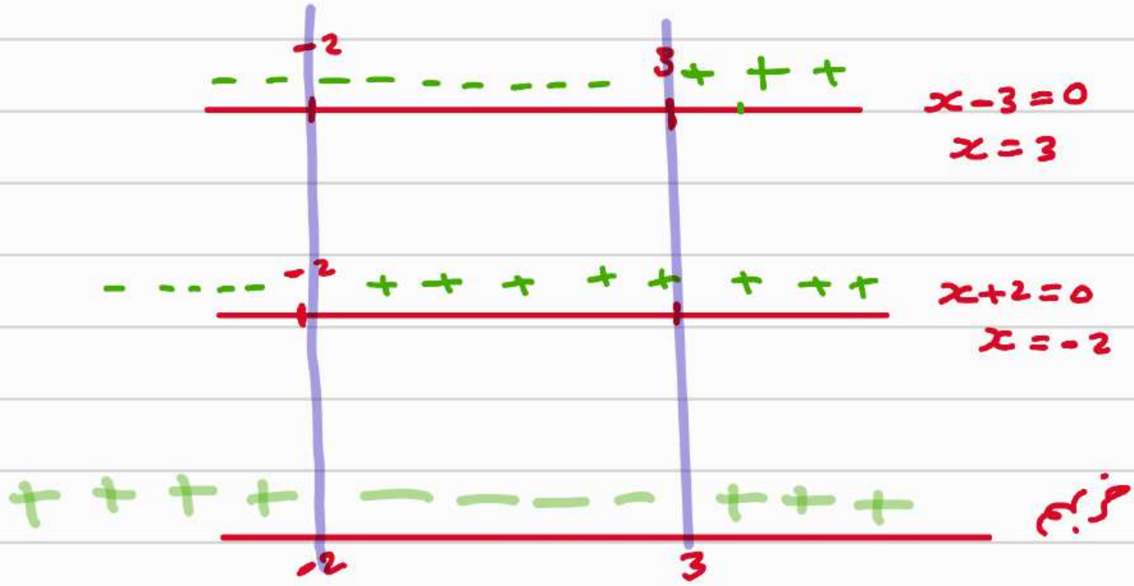
$(x - 3)(x + 2) = 0$

$x = 3 \qquad x = -2$

$(x-3)(x+2) < 0$

② حاصل ضرب الحدين يجب ان يكون سالب

③ نرسم الحدين ونحفظ الاعداد واصفار الاشارة test point



الحل هو الفترة السالبة $(-2, 3)$

EXAMPLE 4 Solve $3x^2 - x - 2 > 0$.

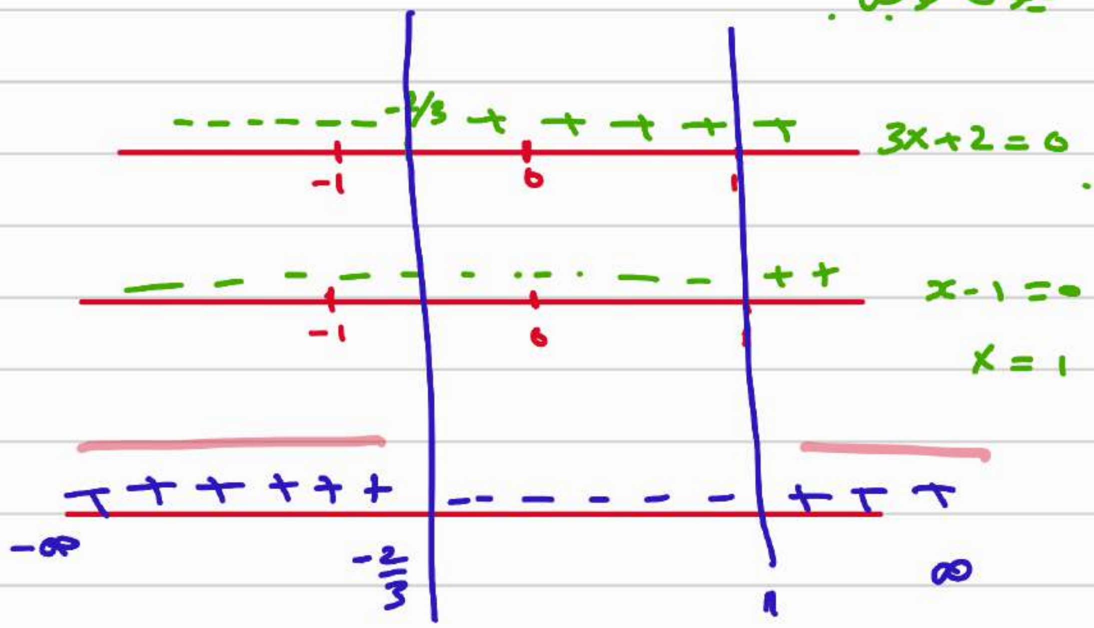
$(3x + 2)(x - 1) > 0$

حاصل ضرب الحدين يجب ان يكون موجب.

$3x + 2 = 0$

$3x = -2$

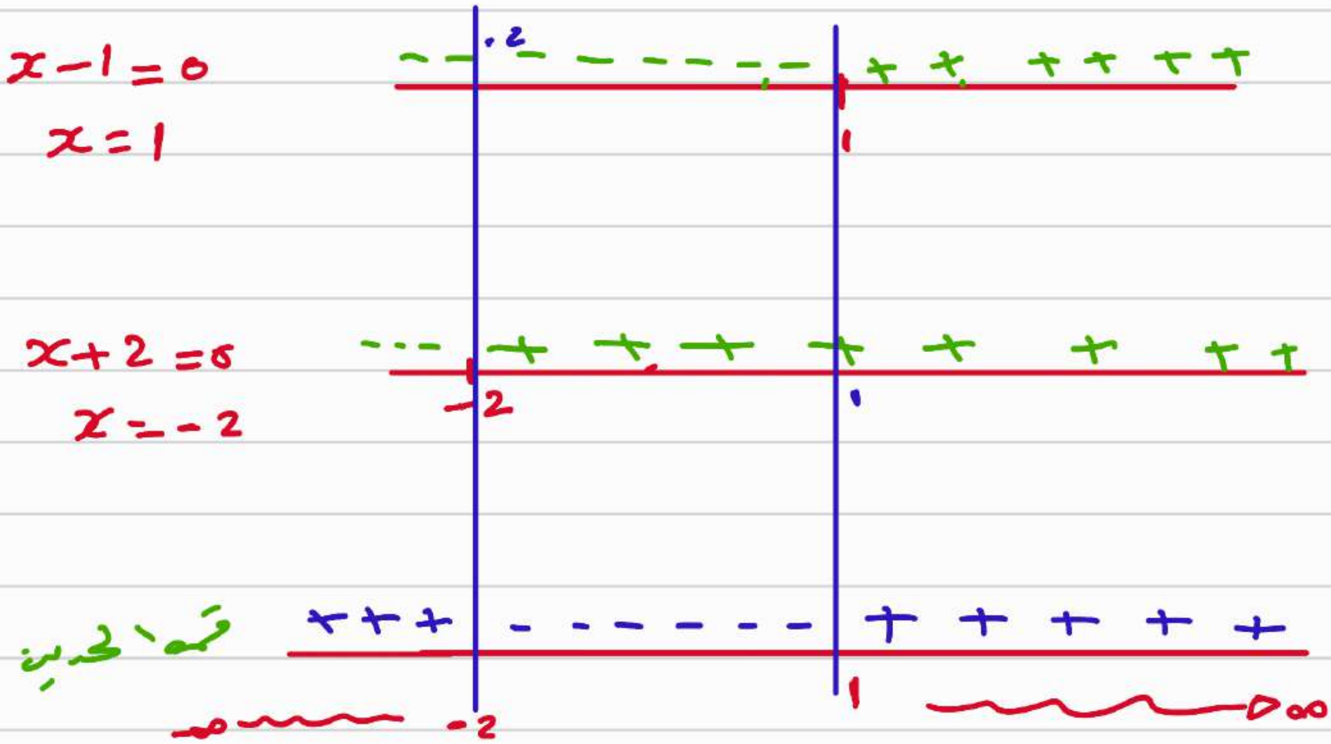
$x = -\frac{2}{3}$



الحل $(-\infty, -\frac{2}{3}) \cup (1, \infty)$

EXAMPLE 5 Solve $\dfrac{x-1}{x+2} \geq 0.$

قسمة الحدين يجب ان تكون صفر او موجبه



فترات الحل

$(-\infty,-2) \cup [1,\infty)$

يجب استثناء الحالة التي يكون فيها المقام 0

$x+2=0$

نحذف -2 من الحل $\boxed{x=-2}$

Absolute Value

| القيمة المطلقة |

$|5| = 5$ $\qquad$ $|-5| = 5$

خصائص القيمة المطلقة

* $|ab| = |a||b| \Rightarrow |3x| = |3||x| = 3|x|$

* $\left|\frac{a}{b}\right| = \frac{|a|}{|b|} \Rightarrow \left|\frac{-4}{b}\right| = \frac{|-4|}{|b|} = \frac{4}{|b|}$

* $|a+b| \leq |a| + |b|$ $\qquad$ $|-3+5| \ ? \ |-3| + |5|$
  $2 \leq 3 + 5$

* $|a-b| \geq \big||a| - |b|\big|$ $\qquad$ $|5-3| \ ? \ 5 - 3$
  $8 \geq 2$

مهم جداً

$|x| < a \Leftrightarrow -a < x < a$

$|x| < 5 \Leftrightarrow -5 < x < 5$

$|x| > a \Leftrightarrow x < -a \qquad x > a$

$|x| > 5 \Leftrightarrow x < -5 \ , \ x > 5$

EXAMPLE 8 Solve the inequality $|x - 4| < 2$ and show the solution set on the real line. Interpret the absolute value as a distance.

$$|x-4| < 2 \iff -2 < x-4 < 2$$

$$-2+4 < x < 2+4$$

$$2 < x < 6 \qquad (2, 6)$$

EXAMPLE 9 Solve the inequality $|3x - 5| \geq 1$ and show its solution set on the real line.

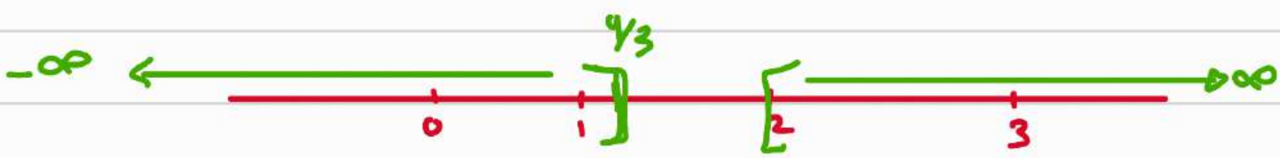
$$|3x-5| \geq 1$$

$$3x-5 \leq -1 \quad \text{or} \quad 3x-5 \geq 1$$

$$3x \leq -1+5 \qquad 3x \geq 6$$

$$3x \leq 4 \qquad x \geq \frac{6}{3}$$

$$x \leq \frac{4}{3} \quad \text{or} \quad x \geq 2$$



$$\left(-\infty, \frac{4}{3}\right] \cup [2, \infty)$$

EXAMPLE 13 Solve $x^2 - 2x - 4 \le 0$.

نبحث عن الفترات التي تكون اقل من او تساوي 0

$x^2 - 2x - 4 = 0 \qquad a = 1 \qquad b = -2 \qquad c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{5} = 2.1$

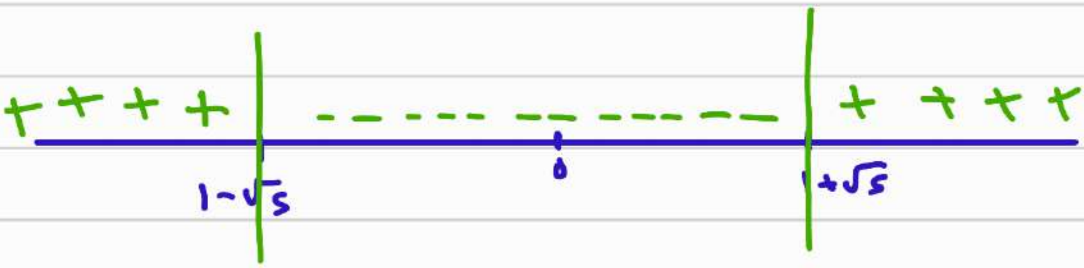
$$= \frac{--2 \pm \sqrt{4 - (4 \times 1 \times -4)}}{2}$$

$$= \frac{2 \pm \sqrt{20}}{2} = \frac{2}{2} \mp \frac{\sqrt{4 \times 5}}{2}$$

$$\boxed{x = 1 \pm \sqrt{5}}$$

$1 + \sqrt{5} \approx 3.1$

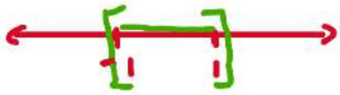
$1 - \sqrt{5} \approx -1.1$

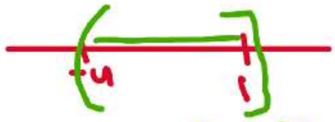


$[1 - \sqrt{5}, 1 + \sqrt{5}]$

Problem Set 0.2

1. Show each of the following intervals on the real line.

(a) $[-1, 1]$ 

(b) $(-4, 1]$ 

(c) $(-4, 1)$ 

(d) $[1, 4]$ 

(e) $[-1, \infty)$ 

(f) $(-\infty, 0]$ 

In each of Problems 3–26, express the solution set of the given inequality in interval notation and sketch its graph.

3. $x - 7 < 2x - 5$

4. $3x - 5 < 4x - 6$

$-7 + 5 < 2x - x$

$-2 < x$

$x > -2$

$[-2, \infty)$

11. $x^2 + 2x - 12 < 0$

12. $x^2 - 5x - 6 > 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm \sqrt{4 - 4 \times 1 \times (-12)}}{2a} = \dfrac{-2 \pm \sqrt{52}}{2}$

$x = -1 \pm \dfrac{\sqrt{52}}{2} = -1 \pm \dfrac{\sqrt{13}\cancel{\sqrt{4}}}{\cancel{2}} = -1 \pm \sqrt{13}$

$(-1-\sqrt{13}\ ,\ -1+\sqrt{13})$

13. $2x^2 + 5x - 3 > 0$

14. $4x^2 - 5x - 6 < 0$

15. $\dfrac{x+4}{x-3} \le 0$

16. $\dfrac{3x-2}{x-1} \ge 0$

17. $\dfrac{2}{x} < 5$

18. $\dfrac{7}{4x} \le 7$

13) $2x^2+5x-3>0$

$(2x-1)(x+3)>0$

$2x-1=0$ $\quad$ $x+3=0$

$2x=1$ $\quad$ $x=-3$

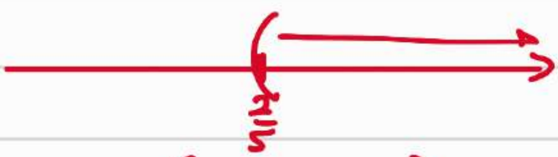
$x=\frac{1}{2}$

$(-\infty,-3)\ \cup\ (\frac{1}{2},\infty)$

17) $\dfrac{2}{x}<5$ $\quad$ نضرب ب x

$2<5x$ $\quad$ $5x>2$ $\quad$ $x>\frac{2}{5}$

$x-\frac{2}{5}>0$



$(2/5\ ,\infty)$

21. $(x + 2)(x - 1)(x - 3) > 0$

22. $(2x + 3)(3x - 1)(x - 2) < 0$

23. $(2x - 3)(x - 1)^2(x - 3) \geq 0$

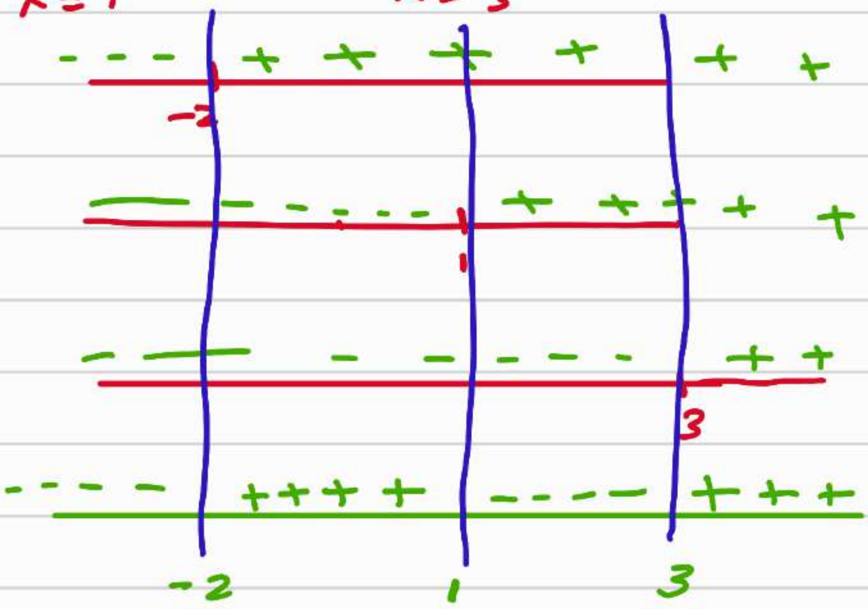
24. $(2x - 3)(x - 1)^2(x - 3) > 0$

25. $x^3 - 5x^2 - 6x < 0$ **26.** $x^3 - x^2 - x + 1 > 0$

21) $(x+2)(x-1)(x-3) > 0$

$x+2=0$, $x=-2$ $x-1=0$, $x=1$ $x-3=0$, $x=3$

$(-2,1) \cup (3,\infty)$



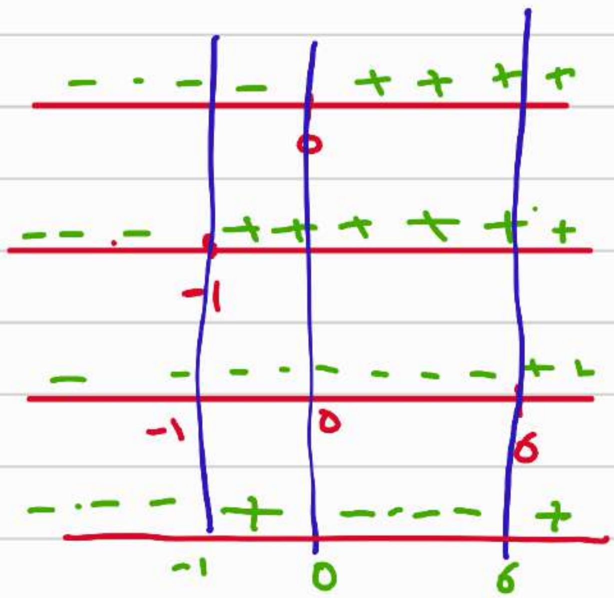
25) $x^3 - 5x^2 - 6x < 0$

$x(x^2-5x-6) < 0$

$x(x+1)(x-6) < 0$

$x=0$ $x=-1$ $x=6$

$(-\infty,-1) \cup (0,6)$



In Problems 35–44, find the solution sets of the given inequalities.

35. $|x-2| \geq 5$

36. $|x+2| < 1$

37. $|4x+5| \leq 10$

38. $|2x-1| > 2$

39. $\left|\dfrac{2x}{7} - 5\right| \geq 7$

40. $\left|\dfrac{x}{4} + 1\right| < 1$

$\dfrac{2x}{7} - 5 \leq -7$

$\dfrac{2x}{7} \leq -2$

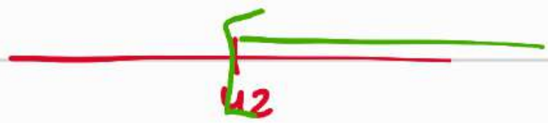
$x \leq -\dfrac{2 \times 7}{2}$

$x \leq -7$

$\dfrac{2x}{7} - 5 \geq 7$

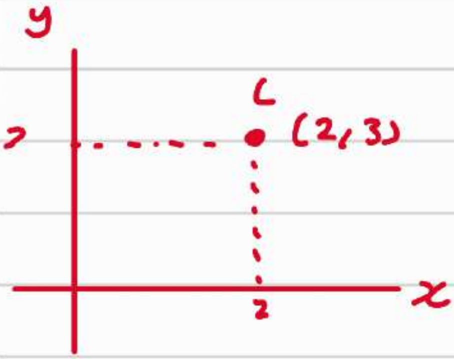
$\dfrac{2x}{7} \geq 12$

$x \geq 42$

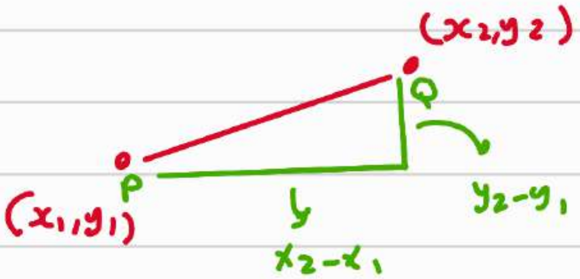


$(-\infty, -7] \cup [42, \infty)$

0.3 The Rectangular Coordinate System



(Distance) المسافه بين نقطتين



$$\boxed{d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$c = \sqrt{a^2 + b^2}$$

اوجد المسافة

EXAMPLE 1 Find the distance between

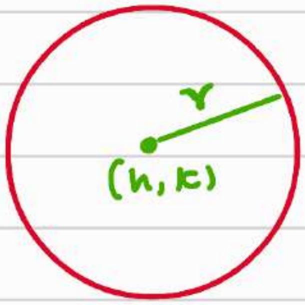
(a) $P(-2, 3)$ and $Q(4, -1)$ — $x_1\ y_1$, $x_2\ y_2$

(b) $P(\sqrt{2}, \sqrt{3})$ and $Q(\pi, \pi)$ — $x_1\ y_1$, $x_2\ y_2$

a) $d = \sqrt{(4 - -2)^2 + (-1 - 3)^2} = \sqrt{36 + 16} = \sqrt{52}$

b) $d = \sqrt{(\pi - \sqrt{2})^2 + (\pi - \sqrt{3})^2} = 2.23$

The equation of circle معادله الدائره

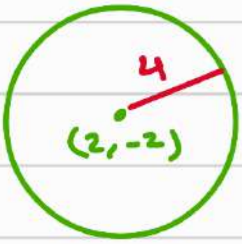


$$\boxed{(x - h)^2 + (y - k)^2 = r^2}$$

مثال

$$(x - 2)^2 + (y - -2)^2 = 16$$

$h = 2 \qquad k = -2 \qquad r = \sqrt{16} = 4$



EXAMPLE 2 Find the standard equation of a circle of radius 5 and center (1, −5). Also find the y-coordinates of the two points on this circle with x-coordinate 2.

$r=5$

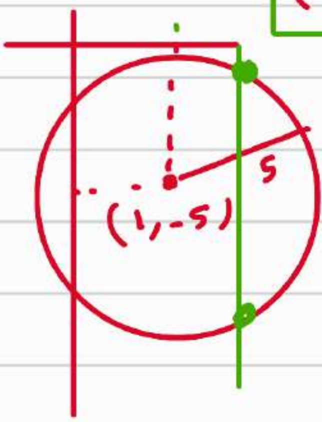
h k

اكتب معادله الدائره

$(1,-5)$ $r=5$

$$(x-h)^2+(y-k)^2=r^2$$

$$\boxed{(x-1)^2+(y+5)^2=25}$$



نعوض قيمه $x=2$ ونحسب قيم y

$$(2-1)^2+(y+5)^2=25$$

$$1+(y+5)^2=25$$

$$\sqrt{(y+5)^2}=\sqrt{24}$$

$$y+5=\pm\sqrt{24}$$

$$y=\pm\sqrt{24}-5$$

$$y=\pm 2\sqrt{6}-5$$

EXAMPLE 3 Show that the equation

$$x^2 - 2x + y^2 + 6y = -6$$

represents a circle, and find its center and radius.

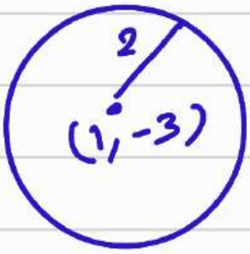
$$(x-h)^2 + (y-k)^2 = r^2$$

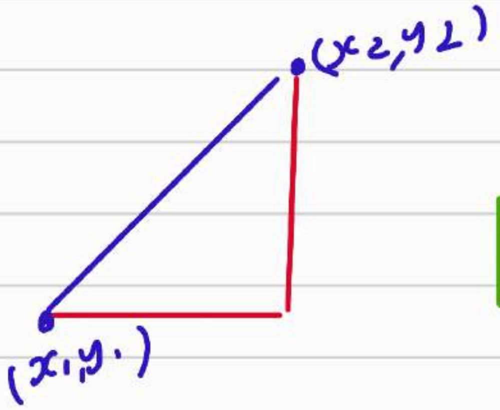
$$\left(\frac{b}{2}\right)^2 \qquad \left(\frac{b}{2}\right)^2$$

$$\underbrace{x^2 - 2x + 1} + \underbrace{y^2 + 6y + 9} = -6 + 1 + 9$$

$$(x-1)^2 + (y+3)^2 = 4$$

$h = 1 \qquad k = -3 \qquad r = 2$





معادلة خط

equation of the line

$$\boxed{y = mx + b}$$

m: slope

b: y intercept
مقطع y

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$\frac{\text{فرق } y}{\text{فرق } x}$ = الميل
slope

$$y = 2x + 1$$

معادله الخط المستقيم

$$\boxed{y - y_1 = m(x - x_1)}$$

$x_1 \; y_1$ $(3,2)$ $\qquad$ $x_2 \; y_2$ $(8,4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{8-3} = \frac{2}{5} = 2.5$$

$$y - y_1 = 2.5(x - x_1)$$

$$y - 2 = 2.5(x - 3)$$

EXAMPLE 5 Find an equation of the line through $(-4, 2)$ and $(6, -1)$.

اكتب معادله الخط

$x_1 \; y_1$ $(-4, 2)$ $\qquad$ $x_2 \; y_2$ $(6, -1)$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{6--4} = \frac{-3}{10}$$

$$y - 2 = -\frac{3}{10}(x + 4)$$

✓ دائماً الميل m هو العدد المضروب بـ x عندما تكون شكل المعادله $y =$

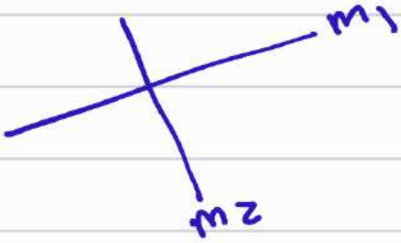
$y = 3x + 5 \qquad m = 3$

$\frac{2y}{2} = \frac{10x}{2} - \frac{2}{2} \qquad y = 5x - 1 \qquad m = 5$

اي خطين متوازيين (Parallel) تكون

لهما نفس قيمة الميل $m_1 = m_2$

اي خطين متعامدين (Perpendicular) تكون



ميل الاول = $-\frac{1}{\text{ميل الثاني}}$

$$m_2 = -\frac{1}{m_1}$$

EXAMPLE 6 Find the equation of the line through (6, 8) that is $\boxed{\text{parallel}}$ to the line with equation $3x - 5y = 11$.

x y

متوازي

حساب ميل خط (m_1)

$$3x - 5y = 11$$

$$y = \frac{3x}{5} + \frac{11}{-5} \qquad m_1 = \frac{3}{5}$$

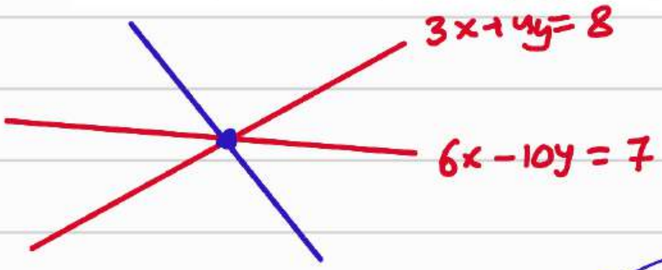
(6, 8) يمر بالنقطة

ميل $m_2 = \frac{3}{5}$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{3}{5}(x - 6)$$

EXAMPLE 7 Find the equation of the line through the point of intersection of the lines with equations $3x + 4y = 8$ and $6x - 10y = 7$ that is perpendicular to the first of these two lines (Figure 16).



$$y - y_1 = m_2(x - x_1)$$

$-\frac{1}{m_1} = m_2$ ميل الخط

$3x + 4y = 8$ $\qquad$ $y = -\frac{3}{4}x + 2$

$m_1 = -\frac{3}{4}$

$$\boxed{m_2 = \frac{4}{3}}$$

$y - y_1 = m(x - x_1$

$y - \frac{1}{2} = \frac{4}{3}(x - 2)$

نسوي معادلة الخطين لحساب نقطة التقاطع

$(3x + 4y = 8) \times -2$

$6x - 10y = 7$

$-6x - 8y = -16$

$-18y = -9$

$y = \frac{-9}{-18} = \boxed{\frac{1}{2}}$

$3x + 4(\frac{1}{2}) = 8$

$3x = 6$ $\qquad$ $\boxed{x = 2}$

Problem Set 0.3

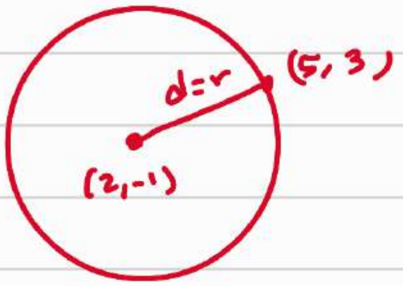
$(x-h)^2+(y-k)^2=r^2$

In Problems 11–16, find the equation of the circle satisfying the given conditions.

11. Center $(\overset{h}{1}, \overset{k}{1})$, radius $\overset{r}{1}$ $\quad (x-1)^2+(y-1)^2=1$

12. Center $(-2, 3)$, radius 4

13. Center $(\overset{h}{2}, \overset{k}{-1})$, goes through $(5, 3)$



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(5-2)^2+(3--1)^2}$$

$$r=d=\sqrt{9+16}=\sqrt{25}=5$$

$$\boxed{(x-2)^2+(y+1)^2=25}$$

In Problems 17–22, find the center and radius of the circle with the given equation.

17. $x^2+2x+10+y^2-6y-10=0$

$\left(\frac{b}{2}\right)^2$

$$\underbrace{x^2+2x+1}+\underbrace{y^2-6y+9}=10-10+1+9$$

$$(x+1)^2+(y-3)^2=10$$

centre $(-1,3)$ $\qquad r=\sqrt{10}$

In Problems 23–28, find the slope of the line containing the given two points.

23. $(1, 1)$ and $(2, 2)$ **24.** $(3, 5)$ and $(4, 7)$

25. $(2, 3)$ and $(-5, -6)$ **26.** $(2, -4)$ and $(0, -6)$

27. $(3, 0)$ and $(0, 5)$ **28.** $(-6, 0)$ and $(0, 6)$

32) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-1} = 1$

25) $m = \frac{-6-3}{-5-2} = \frac{9}{7}$

In Problems 29–34, find an equation for each line. Then write your answer in the form $Ax + By + C = 0$.

29. Through $(2, 2)$ with slope -1

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x - 2)$

$y - 2 = -x + 2$ ✓

$y = -x + 4$ ✓ $y + x - 4 = 0$ ✓

33. Through $(2, 3)$ and $(4, 8)$

34. Through $(4, 1)$ and $(8, 2)$

In Problems 35–38, find the slope and y-intercept of each line.

35. $3y = -2x + 1$ **36.** $-4y = 5x - 6$

33) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{4-2} = \frac{5}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{5}{2}(x - 2)$

$$2y - 6 = 6x - 10$$

$$2y - 6x + 4 = 0$$

مقطع y

In Problems 35–38, find the slope and y-intercept of each line. $y = mx + b$

مقطع y

35. $3y = -2x + 1$ **36.** $-4y = 5x - 6$

35) $3y = -2x + 1$

$$y = -\frac{2}{3}x + \frac{1}{3}$$

y intercep $= \frac{1}{3}$

حل آخر

يمكن حساب b بتعويض $x=0$

39. Write an equation for the line through $(3, -3)$ that is

$x_1 \quad y_1$

(a) $\boxed{\text{parallel}}$ to the line $y = 2x + 5$;

(b) perpendicular to the line $y = 2x + 5$;

(c) parallel to the line $2x + 3y = 6$;

(d) perpendicular to the line $2x + 3y = 6$;

$$y - y_1 = m(x - x_1)$$

$$y + 3 = m_1(x - 3)$$

a) $m_2 = m_1 = 2$

$$y + 3 = 2(x - 3)$$

b) $m_1 = -\frac{1}{m_2} = -\frac{1}{2}$

$$y + 3 = -\frac{1}{2}(x - 3)$$

c) $2x + 3y = 6$ $\qquad y = \frac{6}{3} - \frac{2x}{3}$ $\qquad m = -\frac{2}{3}$

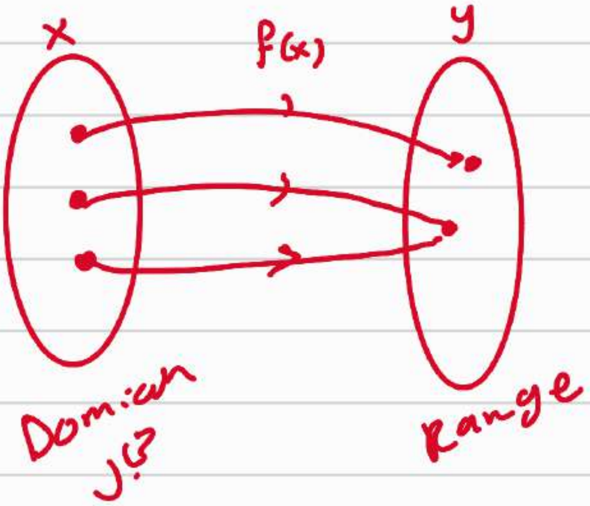
$$y + 3 = -\frac{2}{3}(x - 3)$$

d) $y = \frac{6}{3} - \frac{2}{3}x$ $\qquad m_2 = -\frac{2}{3}$

$m_1 = \frac{3}{2}$

$$y + 3 = \frac{3}{2}(x - 3)$$

0.5 Functions and Their Graphs



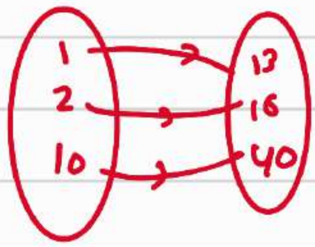
$y = f(x)$

$y = 3x + 10$

$x = 1 \quad y = 3 + 10 = 13$

$x = 2 \quad y = 6 + 10 = 16$

$x = 10 \quad y = 30 + 10 = 40$



EXAMPLE 1 For $f(x) = x^2 - 2x$, find and simplify

(a) $f(4)$ (b) $f(4 + h)$

(c) $f(4 + h) - f(4)$ (d) $[f(4 + h) - f(4)]/h$

$(a+b)^2 = a^2 + 2ab + b^2$

a) $f(4) = 4^2 - 2(4) = 16 - 8 = 8$

b) $f(4+h) = (4+h)^2 - 2(4+h)$

$= 16 + 8h + h^2 - 8 - 2h$

$= h^2 + 6h + 8$

c) $f(4+h) - f(4) = h^2 + 6h + \cancel{8} - \cancel{8} = h^2 + 6h$

d) $\dfrac{f(4+h) - f(4)}{h} = \dfrac{h^2 + 6h}{h} = \dfrac{\cancel{h}(h+6)}{\cancel{h}} = h + 6$

كيفية تحديد المجال لعدد من الدوال

① اذا كانت الدالة كثيرة حدود

مثلا $f(x) = x^5 + 4x^3 - 2x + 1$

المجال والمدى هو $(-\infty , \infty)$ R كل الاعداد

② الدالة الكسرية تكون المجال هو كل الاعداد الحقيقية ماعدا اصفار المقام

$$f(x) = \frac{3}{x+5} \qquad x+5=0 \qquad x=-5$$

المجال هو كل الاعداد ماعدا -5 $R-\{-5\}$

③ في حالة الدالة الجذرية يجب ان يكون ماداخل الجذر اكبر , موجب ماداخل الجذر ≥ 0

$$f(x) = \sqrt{x-2} \qquad x-2 \geq 0 \qquad x \geq 2 \qquad [2,\infty)$$

④ دالة جذرية وكسرية يجب ان يكون ما داخل الجذر اكبر من صفر ولا يساوي صفر

$$f(x) = \frac{1}{\sqrt{x-2}} \qquad x-2 > 0 \qquad x > 2 \qquad (2,\infty)$$

اوجد مجال الدالة:

EXAMPLE 2 Find the natural domains for

(a) $f(x) = 1/(x-3)$ (b) $g(t) = \sqrt{9-t^2}$

(c) $h(w) = 1/\sqrt{9-w^2}$

a) $f(x) = \frac{1}{x-3}$ $x-3=0$ $x=3$

جميع الاعداد ماعدا $x=3$

$\{x : x \neq 3\}$

b) $g(t) = \sqrt{9-t^2}$ $9-t^2 \geq 0$

$9 \geq t^2$ $3 \geq |t|$

$|t| \leq 3$ $\rightsquigarrow$ $-3 \leq t \leq 3$

$[-3, 3]$

c) $h(w) = \frac{1}{\sqrt{9-w^2}}$

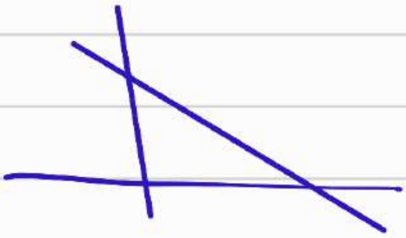
$9-w^2 > 0$ $9 > w^2$

$3 > |w|$ $|w| < 3$

$-3 < w < 3$ $(-3, 3)$

Graphs of the functions

$y = mx + b$ | الداله الخطيه

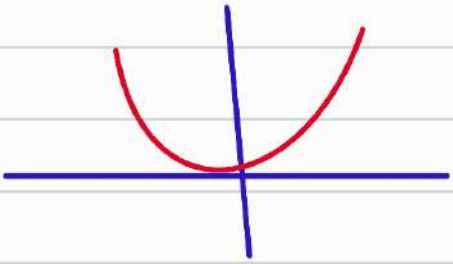


$m =$ سالب

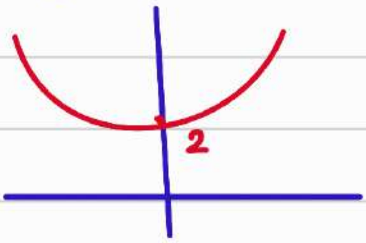
$m =$ موجب

∩ ∪ | الداله التربيعيه

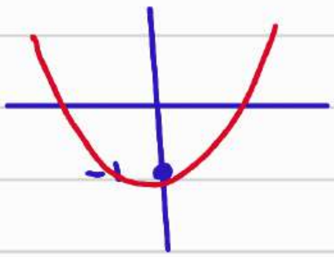
$y = x^2$



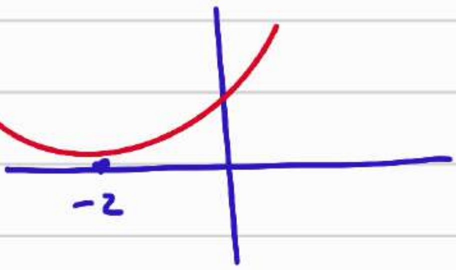
$y = x^2 + 2$



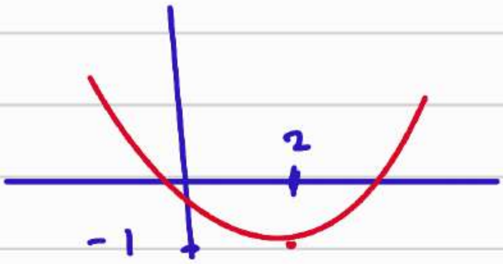
$y = x^2 - 1$



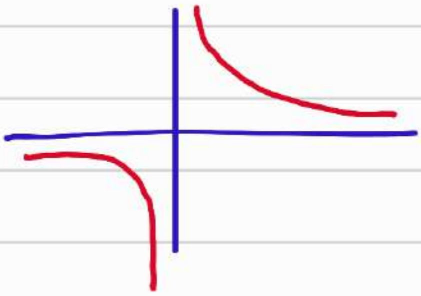
$y = (x+2)^2$



$y = (x-2)^2 - 1$

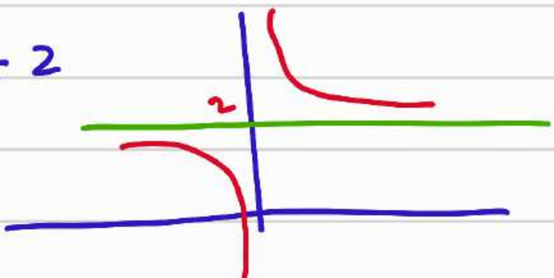


$y = \frac{1}{x}$

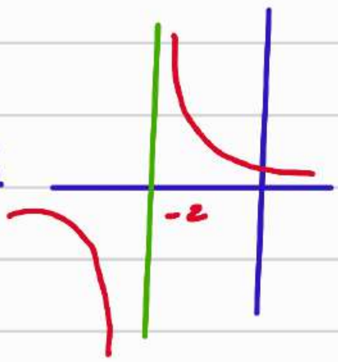


داله كسريه

$y = \frac{1}{x} + 2$

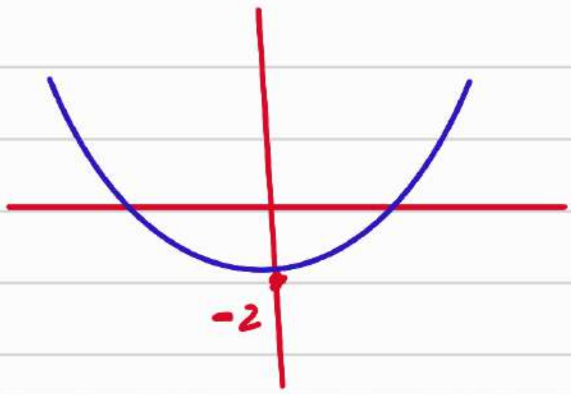


$y = \frac{1}{x+2}$

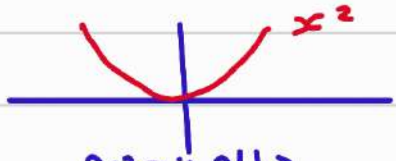


EXAMPLE 4 Sketch the graphs of

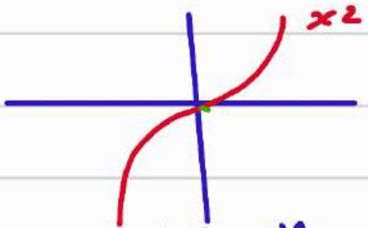
(a) $f(x) = x^2 - 2$ (



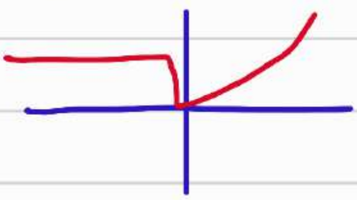
even odd function الدالة الزوجيه والفرديه



داله زوجيه
even



داله فرديه
odd



لا فرديه ولا زوجيه

الداله الزوجيه ← اذا عوضنا $(-x)$ و x في المعادله تكون الاجابه نفسها $f(x) = f(-x)$

الداله الفرديه ← اذا عوضت x و $-x$ يكون نفس الجواب بس مختلف بالاشارة $f(x) = -f(-x)$

في دوال كثيره الحدود

اذا كانت كل الاسس زوجيه او اعداد ثابته ← even

اذا كانت كل الاسس فرديه

$f(x) = x^4 + 2x^2 + 5$

كل الاسس زوجيه
even

$f(x) = x^5 + x^3 + x$

كل الاسس فرديه
odd

EXAMPLE 5 Is $f(x) = \dfrac{x^3 + 3x}{x^4 - 3x^2 + 4}$ even, odd, or neither?

زوجية فردية ولا واحدة

$$f(x) = \frac{x^3 + 3x}{x^4 - 3x^2 + 4}$$

$$f(-x) = \frac{(-x)^3 + 3(-x)}{(-x)^4 + 3(-x)^2 + 4} = \frac{-x^3 - 3x}{x^4 + 3x^2 + 4}$$

$$= -\left[\frac{x^3 + 3x}{x^4 + 3x^2 + 4}\right]$$

$f(-x) = -f(x)$ odd function

الدالة فردية

Problem Set 0.5

9. For $f(x) = 2x^2 - 1$, find and simplify $[f(a+h) - f(a)]/h$.

$$\frac{f(a+h) - f(a)}{h}$$

$$f(a+h) = 2(a+h)^2 - 1 = 2(a^2 + 2ah + h^2) - 1$$
$$= 2a^2 + 4ah + h^2 - 1$$

$$f(a) = 2a^2 - 1$$

$$\frac{f(a+h) - f(a)}{h} = \frac{\cancel{2a^2} + 4ah + h^2 \cancel{-1} - \cancel{2a^2} \cancel{+1}}{h}$$

$$= \frac{4ah + h^2}{h} = \frac{\cancel{h}(4a + h)}{\cancel{h}} = 4a + h$$

13. Find the natural domain for each of the following.

(a) $F(z) = \sqrt{2z + 3}$ (b) $g(v) = 1/(4v - 1)$

(c) $\psi(x) = \sqrt{x^2 - 9}$ (d) $H(y) = -\sqrt{625 - y^4}$

a) $f(z) = \sqrt{2z+3}$ $\quad 2z + 3 \geq 0$ $\quad z \geq \frac{-3}{2}$

$[-\frac{3}{2}, \infty)$ $\quad \frac{-3}{2} \;\; \infty$

b) $g(v) = \frac{1}{4v-1}$ $\quad 4v - 1 = 0$ $\quad v = \frac{1}{4}$

$\{v : v \neq \frac{1}{4}\}$ $\quad v \neq \frac{1}{4} : v \in \mathbb{R}$

c) $\psi(x) = \sqrt{x^2 - 9}$ $\quad x^2 - 9 \geq 0$

$x^2 \geq 9$ $\quad |x| \geq 3$

$(-\infty, -3] \cup [3, \infty)$

d) $H(y) = \sqrt{625 - y^4}$ $\qquad$ $625 - y^4 \geq 0$

$625 \geq y^4$ $\qquad$ $\sqrt[4]{y^4} \leq \sqrt[4]{625}$ $\qquad$ $|y| \leq 5$

$[-5, 5]$

$\{y \in R : -5 \leq y \leq 5\}$

In Problems 15–30, specify whether the given function is even, odd, or neither, and then sketch its graph.

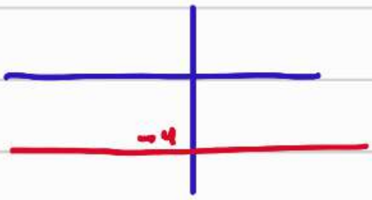
15. $f(x) = -4$ $\qquad$ **16.** $f(x) = 3x$

17. $F(x) = 2x + 1$ $\qquad$ **18.** $F(x) = 3x - \sqrt{2}$

19. $g(x) = 3x^2 + 2x - 1$ $\qquad$ **20.** $g(u) = \dfrac{u^3}{8}$

21. $g(x) = \dfrac{x}{x^2 - 1}$ $\qquad$ **22.** $\phi(z) = \dfrac{2z + 1}{z - 1}$

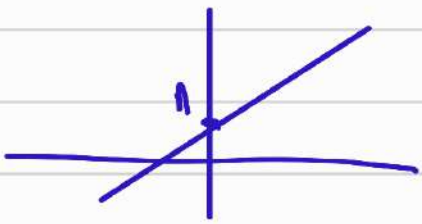
15) $f(x) = -4$ $\qquad$ even



17) $f(x) = 2x + 1$ → Neither even nor odd

$f(-x) = -2x + 1$ → لازوجيه و لافردية

$y = mx + b$



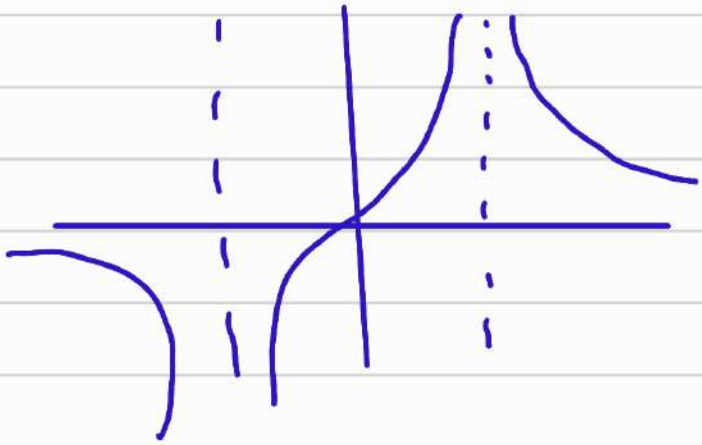
21) $g(x) = \dfrac{x}{x^2 - 1}$

$$g(-x) = \frac{-x}{(-x)^2 - 1} = -\frac{x}{x^2-1}$$

$$f(x) = -f(-x)$$

odd

x	0	1	-1	2	-2	3	-3
$g(x)$	0	∞	∞	$\frac{2}{3}$	$-\frac{2}{3}$	$3/8$	$-3/8$



0.6 Operations on Functions

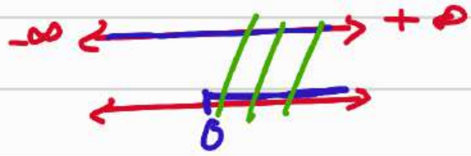
Sums, Differences, Products, Quotients, and Powers Consider functions f and g with formulas

$$f(x) = \frac{x-3}{2}, \qquad g(x) = \sqrt{x}$$

المجال للدالة الجديدة تكون تقاطع المجالين في حالة الجمع والطرح والضرب

$$(f+g)(x) = \frac{x-3}{2} + \sqrt{x} \qquad [0,\infty)$$

$x \geq 0$



Formula	Domain
$(f+g)(x) = f(x) + g(x) = \frac{x-3}{2} + \sqrt{x}$	$[0, \infty)$
$(f-g)(x) = f(x) - g(x) = \frac{x-3}{2} - \sqrt{x}$	$[0, \infty)$
$(f \cdot g)(x) = f(x) \cdot g(x) = \frac{x-3}{2}\sqrt{x}$	$[0, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x-3}{2\sqrt{x}}$	$(0, \infty)$

في حالة القسمة

مجال الدالة في حالة القسمة هو نفس المجال في حالة الجمع لكن نستثني منه اصفار المقام

$$\frac{f}{g}(x) = \frac{\frac{x-3}{2}}{\sqrt{x}} = \frac{x-3}{2\sqrt{x}} \qquad [0,\infty)$$

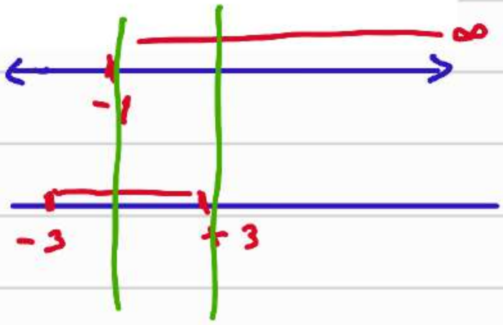
$$\sqrt{x}^2 = 0^2$$

$$x = 0$$

الاجابة $[0,\infty)$ ما عدا صفر

$(0,\infty$

EXAMPLE 1 Let $F(x) = \sqrt[4]{x+1}$ and $G(x) = \sqrt{9-x^2}$, with respective natural domains $[-1, \infty)$ and $[-3, 3]$. Find formulas for $F+G$, $F-G$, $F \cdot G$, F/G, and F^5 and give their natural domains.



$$(f+G)(x) = \sqrt[4]{x+1} + \sqrt{9-x^2}$$

domain $[-1, 3]$

$$(f-G)(x) = \sqrt[4]{x+1} - \sqrt{9-x^2} \qquad [-1, 3]$$

$$(f \cdot G)(x) = \sqrt[4]{x+1}\,\sqrt{9-x^2} \qquad [-1, 3]$$

$$\frac{f}{g}(x) = \frac{\sqrt[4]{x+1}}{\sqrt{9-x^2}} \qquad [-1, 3)$$

$\sqrt{9-x^2} = 0$ نبحث عن اصفار المقام

$9 - x^2 = 0$

$9 = x^2$

$\sqrt{9} = x$

$\pm 3 = x$

$$F^5(x) = \left(\sqrt[4]{x+1}\right)^5 = (x+1)^{5/4}$$

domain $[-1, \infty)$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x-3}{2}\right) = \sqrt{\frac{x-3}{2}}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right) = \frac{\sqrt{x}-3}{2}$$

Composition of function

$g \circ f(x) = g(f(x))$ نعوض f داخل g

$f \circ g(x) = f(g(x))$ نعوض g داخل f

$f(x) = \frac{x-3}{2}$ $g(x) = \sqrt{x}$

$$g \circ f(x) = \sqrt{\frac{x-3}{2}}$$

$$f \circ g(x) = \frac{\sqrt{x}-3}{2}$$

EXAMPLE 2 Let $f(x) = 6x/(x^2 - 9)$ and $g(x) = \sqrt{3x}$, with their natural domains. First, find $(f \circ g)(12)$; then find $(f \circ g)(x)$ and give its domain.

SOLUTION

$$(f \circ g)(12) = f(g(12)) = f\left(\sqrt{36}\right) = f(6) = \frac{6 \cdot 6}{6^2 - 9} = \frac{4}{3}$$

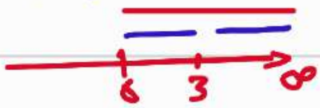
$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3x}\right) = \frac{6\sqrt{3x}}{\left(\sqrt{3x}\right)^2 - 9}$$

$f(x) = \frac{6x}{x^2-9}$ $g(x) = \sqrt{3x}$

✱ $g(12) = \sqrt{3(12)} = \sqrt{36} = 6$ $f(6) = \frac{6 \times 6}{6^2 - 9} = \frac{36}{27} = \frac{4}{3}$

$$\ast \quad f \circ g(x) = \frac{6\sqrt{3x}}{(\sqrt{3x})^2 - 9} = \frac{6\sqrt{3x}}{3x-9}$$

حساب مجال الدالة المركبة نحسب المجال للدالة النهائية
بعد التركيب ثم نتقاطعها مع مجال الدالة الاولى



$$\frac{6\sqrt{3x}}{3x-9}$$

$3x \geqslant 0 \qquad x \geqslant 0$

$3x - 9 = 0 \qquad x = 3$

$[0,3) \cup (3,\infty)$ المجال

EXAMPLE 3 Write the function $p(x) = (x+2)^5$ as a composite function $g \circ f$.

$$P(x) = (x+2)^5$$

$$g(x) = (x+2) \qquad f(x) = x^5$$

$$f \circ g(x) = (x+2)^5$$

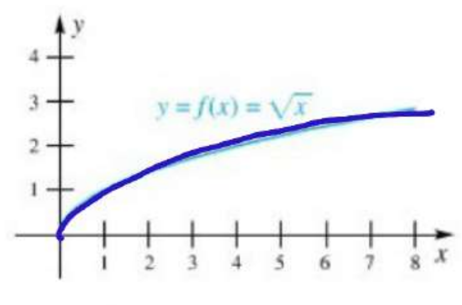


Figure 7

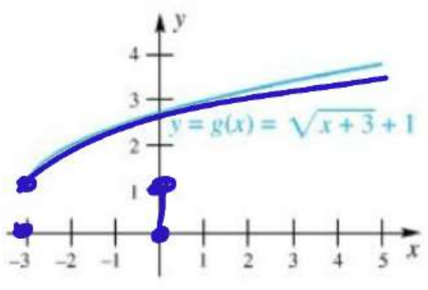


Figure 8

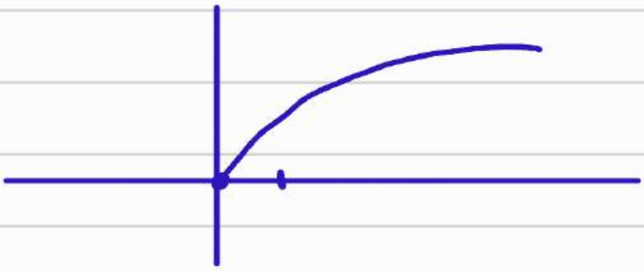
EXAMPLE 4 Sketch the graph of $g(x) = \sqrt{x+3} + 1$ by first graphing $f(x) = \sqrt{x}$ and then making appropriate translations.

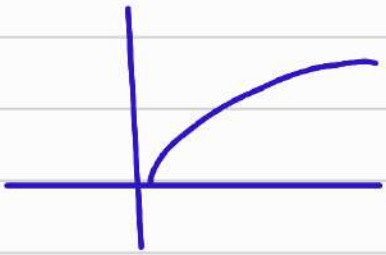
دالة الجذر

$$f(x) = \sqrt{x}$$

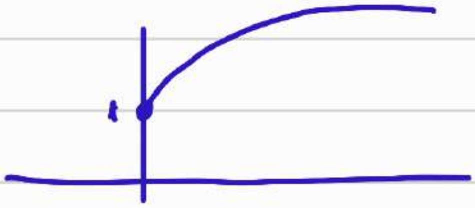
x	$f(x)$
0	0
4	2
9	3
16	4

دالة الجذر x معرفة فقط
على الاعداد الموجبة $[0,\infty)$

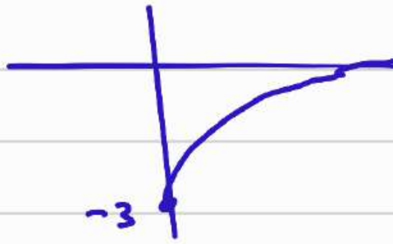




$\sqrt{x}$



$\sqrt{x}+1$



$\sqrt{x}-3$

$\sqrt{x-1}$

$\sqrt{x+2}$

$\sqrt{x+2}-4$

Problem Set 0.6

1. For $f(x) = x + 3$ and $g(x) = x^2$, find each value (if possible).

(a) $(f + g)(2)$ (b) $(f \cdot g)(0)$ (c) $(g/f)(3)$

(d) $(f \circ g)(1)$ (e) $(g \circ f)(1)$ (f) $(g \circ f)(-8)$

a) $(f+g)(2) = f(2) + g(2)$

$= 5 + 4 = 9$

b) $(f \cdot g)(0) = f(0) \cdot g(0)$

$= 3 \cdot 0 = 0$

c) $\left(\frac{g}{f}\right)(3) = \frac{g(3)}{f(3)} = \frac{9}{6} = \frac{3}{2}$

d) $f \circ g(1) \Rightarrow$ $g(1) = 1$

$f(1) = 4$

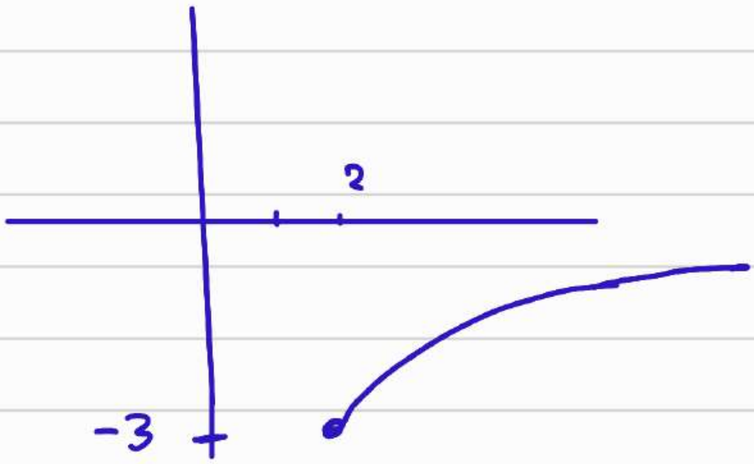
e) $g \circ f(1)$ $f(1) = 4$

$g(4) = 16$

f) $g \cdot f(-8)$ $f(-8) = 5$

$g(5) = 25$

15. Sketch the graph of $f(x) = \sqrt{x-2} - 3$ by first sketching $g(x) = \sqrt{x}$ and then translating. (See Example 4.)



11. Find f and g so that $F = g \circ f$. (See Example 3.)

(a) $F(x) = \sqrt{x+7}$ (b) $F(x) = (x^2 + x)^{15}$

a)

$f(x) = x + 7$

$g(x) = \sqrt{x}$.

$g \circ f(x)$

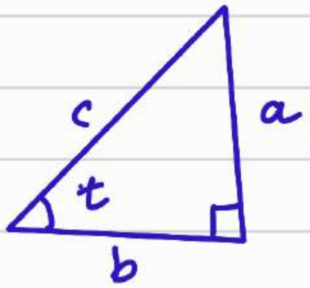
b)

$f(x) = x^2 + x$

$g(x) = x^{15}$

0.7

Trigonometric Functions



$$\sin t = \frac{a}{c} \qquad \cos t = \frac{b}{c}$$

$$\tan t = \frac{a}{b}$$

$$\csc t = \frac{1}{\sin t} \qquad \sec t = \frac{1}{\cos t}$$

$$\cot t = \frac{1}{\tan t} \qquad \tan t = \frac{\sin t}{\cos t}$$

$$\sin^2 t + \cos^2 t = 1$$

$\sin(-t) = -\sin(t)$ odd فردية

$\cos(-t) = \cos t$ even زوجية

$$\sin t = \sin(t + 2\pi)$$

$$\cos t = \cos(t + 2\pi)$$

EXAMPLE 5 Show that tangent is an odd function.

SOLUTION

$$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\tan t$$

$$\tan t = \frac{\sin t}{\cos t}$$

$$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\tan t$$

odd function

EXAMPLE 6 Verify that the following are identities.

$$\boxed{1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t}$$

a) $1 + \tan^2 t = \sec^2 t$

$\cos^2$ × مقامات

$$1 + \frac{\sin^2 t}{\cos^2 t} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

نعم تحققت المطابقة $= \sec^2 t$

b) $1 + \cot^2 t = \csc^2 t$

$\cot t = \frac{\cos t}{\sin t}$

$$1 + \frac{\cos^2 t}{\sin^2 t}$$

تحققت المتطابقة

$$\frac{\sin^2 t + \cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t} = \csc^2 t$$

Then

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

List of Important Identities We will not take space to verify all the following identities. We simply assert their truth and suggest that most of them will be needed somewhere in this book.

Trigonometric Identities The following are true for all x and y, provided that both sides are defined at the chosen x and y.

Odd–even identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Cofunction identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Addition identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Double-angle identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

Half-angle identities

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

Sum identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

Product identities

$$\sin x \sin y = -\tfrac{1}{2}[\cos(x + y) - \cos(x - y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)]$$

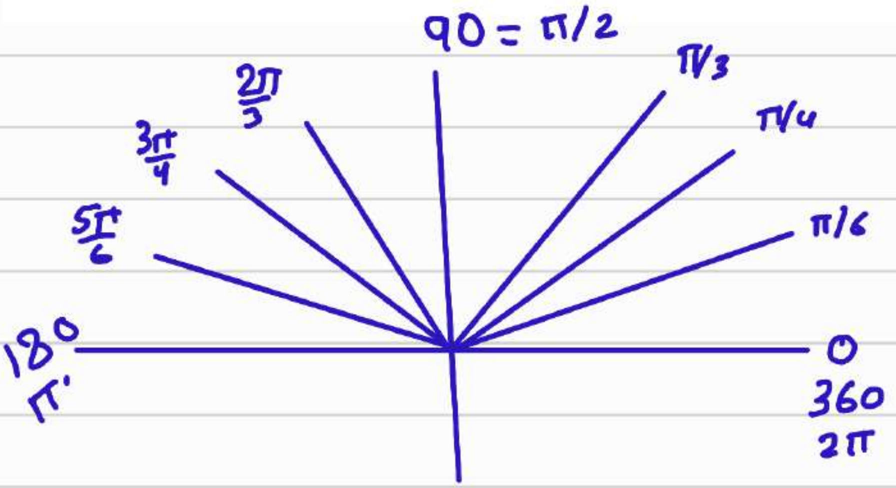
$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)]$$

Problem Set 0.7

9. Evaluate without using a calculator.

(a) $\tan \frac{\pi}{6}$ (b) $\sec \pi$ (c) $\sec \frac{3\pi}{4}$

(d) $\csc \frac{\pi}{2}$ (e) $\cot \frac{\pi}{4}$ (f) $\tan\left(-\frac{\pi}{4}\right)$



t	$\sin t$	$\cos t$
0	0	1
$\pi/6$ 30	$1/2$	$\sqrt{3}/2$
$\pi/4$ 45	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$ 60	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0
$2\pi/3$ 120	$\sqrt{3}/2$	$-1/2$
$3\pi/4$ 135	$\sqrt{2}/2$	$-\sqrt{2}/2$
$5\pi/6$ 150	$1/2$	$-\sqrt{3}/2$
π	0	-1

a) $\tan \frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $\sec(\pi) = \frac{1}{\cos\pi} = \frac{1}{-1} = -1$ $2 = \sqrt{2}\sqrt{2}$

c) $\sec \frac{3\pi}{4} = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{\frac{-\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$

d) $\csc \frac{\pi}{2} = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = \frac{1}{1} = 1$

e) $\cot\left(\frac{\pi}{4}\right) = \dfrac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

f) $\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -\dfrac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = -1$

11. Verify that the following are identities (see Example 6).

(a) $(1 + \sin z)(1 - \sin z) = \dfrac{1}{\sec^2 z}$

(b) $(\sec t - 1)(\sec t + 1) = \tan^2 t$

(c) $\sec t - \sin t \tan t = \cos t$

(d) $\dfrac{\sec^2 t - 1}{\sec^2 t} = \sin^2 t$

$(a+b)(a-b)$

a) $(1 + \sin z)(1 - \sin z) = \dfrac{1}{\sec^2 z}$

$= 1 - \sin^2 z$

$\cos^2 z + \sin^2 z = 1$

$\cos^2 z = 1 - \sin^2 z$

$\sin^2 z = 1 - \cos^2 z$

$= \cos^2 z$

$= \dfrac{1}{\sec^2 z}$

b) $(\sec t - 1)(\sec t + 1) = \tan^2$

$= \sec^2 t - 1$

$= \tan^2 t$ ✓

c) $\sec t - \sin t \tan t = \cos t$

$$\frac{1}{\cos t} - \sin t \frac{\sin t}{\cos t}$$

$$\frac{1}{\cos t} - \frac{\sin^2 t}{\cos t}$$

$$\frac{1-\sin^2 t}{\cos t} = \frac{\cos^2 t}{\cos t} = \cos t$$

d) $\frac{\sec^2 t - 1}{\sec^2 t} = \sin^2 t$

$$\frac{\tan^2}{\sec^2 t} = \frac{\frac{\sin^2 t}{\cancel{\cos^2 t}}}{\frac{1}{\cancel{\cos^2 t}}} = \sin^2 t$$

Find the exact values in Problems 27–31. Hint: Half-angle identities may be helpful.

27. $\cos^2 \frac{\pi}{3}$

28. $\sin^2 \frac{\pi}{6}$

29. $\sin^3 \frac{\pi}{6}$

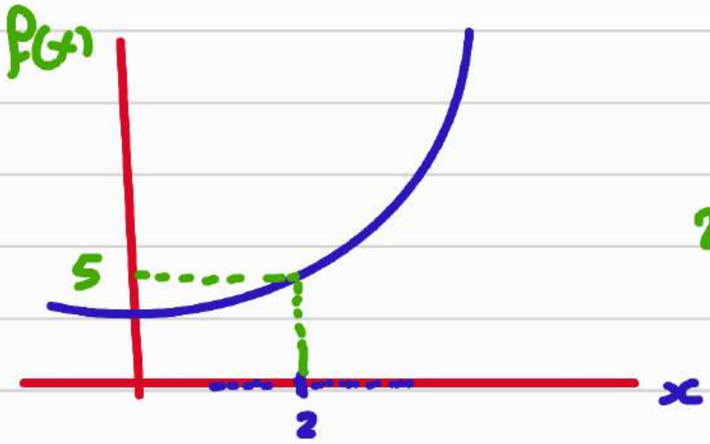
30. $\cos^2 \frac{\pi}{12}$

27) $\left(\cos \frac{\pi}{3}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

29) $\left(\sin \frac{\pi}{6}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

CHAPTER 1 Limits

1.1 Introduction to Limits



عندما x تقترب من 2
فان f(x) تقترب
من 5

$$\lim_{x \to 2} f(x) = 5$$

نحسب النهاية بالتعويض المباشر
في الدالة

$$f(x) = 3x - 1$$

$$\lim_{x \to 3} (3x - 1) = 8$$

$$y = \frac{x^3 - 1}{x - 1}$$

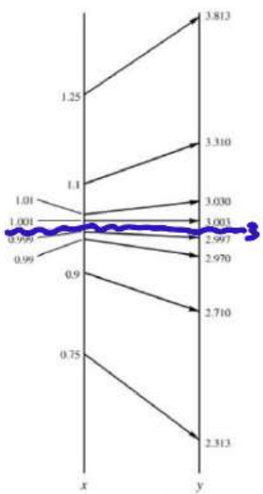
$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \frac{1 - 1}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{\cancel{(x - 1)}(x^2 + x + 1)}{\cancel{(x - 1)}}$$

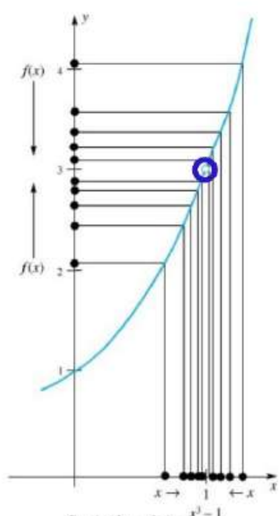
$$1 + 1 + 1 = 3$$

x	$y = \frac{x^3 - 1}{x - 1}$
1.25	3.813
1.1	3.310
1.01	3.030
1.001	3.003
↓	↓
$\boxed{1.000}$	$\boxed{?}$
↑	↑
0.999	2.997
0.99	2.970
0.9	2.710
0.75	2.313

Table of values



Schematic diagram



Graph of $y = f(x) = \frac{x^3 - 1}{x - 1}$

$\lim_{x \to c} f(x) = L$ $\qquad\qquad$ $f(x) = L$

اذا نتج من التعويض المباشر عدد L

فانه يساوي قيمة النهاية لكن اذا كانت الاجابة $= \frac{0}{0}$

فان النهاية تحتاج التحليل

EXAMPLE 1 Find $\lim_{x \to 3}(4x - 5)$. $= 4(3) - 5 = 7$

EXAMPLE 2 Find $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$.

نعوض تعويض مباشر

$\frac{(3)^2 - 3 - 6}{3 - 3} = \frac{0}{0}$

لا ينفع التعويض المباشر

يجب تحليل الدالة

للتخلص من الصفر

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}}$$

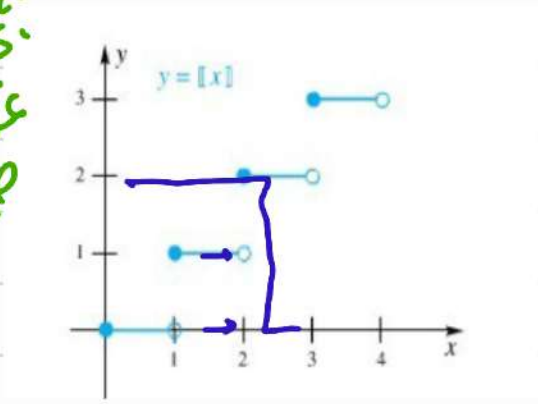
الان نعوض

$$\lim_{x \to 3} (x - 2) = 1$$

EXAMPLE 5 **(No limit at a jump)** Find $\lim_{x \to 2} [x]$.

SOLUTION Recall that $[x]$ denotes the greatest integer less than or equal to x (see Section 0.5). The graph of $y = [x]$ is shown in Figure 7. For all numbers x less than 2 but near 2, $[x] = 1$, but for all numbers x greater than 2 but near 2, $[x] = 2$. Is $[x]$ near a single number L when x is near 2? No. No matter what number we propose for L, there will be x's arbitrarily close to 2 on one side or the other, where $[x]$ differs from L by at least $\frac{1}{2}$. Our conclusion is that $\lim_{x \to 2} [x]$ does not exist. If you check back, you will see that we have not claimed that every limit we can write must exist. ■

دالة اكبر عدد صحيح



$$f(x) = [x] = [2] = 2$$

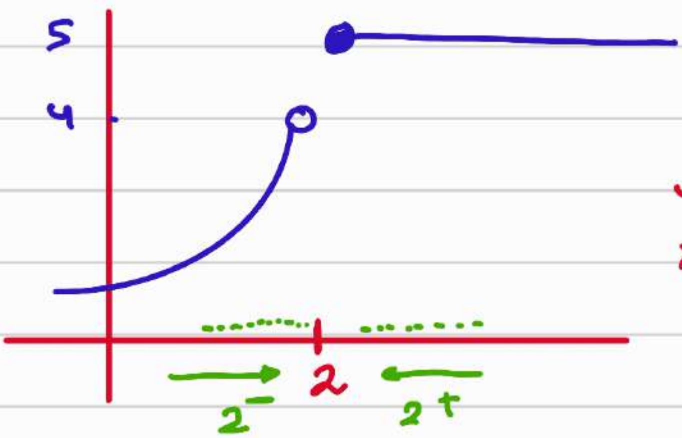
$$[3.1] = 3$$

$$[2.9] = 2$$

$$\lim_{x \to 2^-} f(x) = = 1$$

$$\lim_{x \to 2^+} f(x) = 2$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2} f(x) = \text{not exist}$$



left hand limt

$$\lim_{x \to 2^-} f(x) = 4$$ يسار

Right hand limt

$$\lim_{x \to 2^+} f(x) = 5$$

$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) = \lim \text{ not exist}$$

اذا لم تساوي النهايتين اذا النهاية غير موجودة

اذا تساوت الطرفين فان النهاية موجوده

Problem Set 1.1

In Problems 1–6, find the indicated limit.

1. $\lim_{x\to 3}(x-5)$
2. $\lim_{t\to -1}(1-2t)$
3. $\lim_{x\to -2}(x^2+2x-1)$
4. $\lim_{x\to -2}(x^2+2t-1)$
5. $\lim_{t\to -1}(t^2-1)$
6. $\lim_{t\to -1}(t^2-x^2)$

1) $\lim_{x\to 3}(x-5) = 3-5 = -2$

3) $\lim_{x\to -2}(x^2+2x-1)$

$= (-2)^2 + 2(-2) - 1 = -1$

In Problems 7–18, find the indicated limit. In most cases, it will be wise to do some algebra first (see Example 2).

7. $\lim_{x\to 2}\frac{x^2-4}{x-2}$
8. $\lim_{t\to -7}\frac{t^2+4t-21}{t+7}$
9. $\lim_{x\to -1}\frac{x^3-4x^2+x+6}{x+1}$
10. $\lim_{x\to 0}\frac{x^4+2x^3-x^2}{x^2}$
11. $\lim_{x\to -t}\frac{x^2-t^2}{x+t}$
12. $\lim_{x\to 3}\frac{x^2-9}{x-3}$

7) $\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{0}{0}$

$\lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$\lim_{x\to 2}(x+2) = 4$

8) $\lim_{t\to -7}\frac{t^2+4t-21}{t+7} = \frac{49-28-21}{-7+7} = \frac{0}{0}$

$\lim_{t\to -7}\frac{\cancel{(t+7)}(t-3)}{\cancel{t+7}} = \lim_{t\to -7}(t-3) = -10$

11) $\lim_{x\to -t}\frac{x^2-t^2}{x+t} = \frac{0}{0}$

$\lim_{x\to -t}\frac{\cancel{(x+t)}(x-t)}{\cancel{x+t}} = \lim_{x\to -t} x-t = -t-t$

$= -2t$

43. Find each of the following limits or state that it does not exist.

(a) $\lim_{x\to 1} \frac{|x-1|}{x-1}$ (b) $\lim_{x\to 1^-} \frac{|x-1|}{x-1}$

$$\lim_{x\to 1^-} \frac{|x-1|}{x-1} = \frac{|0-1|}{0-1} = \frac{1}{-1} = -1$$

$$\lim_{x\to 1^+} \frac{|x-1|}{x-1} = \frac{|2-1|}{2-1} = \frac{1}{1} = +1$$

a) $\lim_{x\to 1} \frac{|x-1|}{x-1}$ does not exist

b) $\lim_{x\to 1^-} \frac{|x-1|}{x-1} = -1$

1.3

Limit Theorems

Theorem A **Main Limit Theorem**

Let n be a positive integer, k be a constant, and f and g be functions that have limits at c. Then

1. $\lim_{x \to c} k = k$; $\lim_{x \to 2} 5 = 5$
2. $\lim_{x \to c} x = c$; $\lim_{x \to 2} x = 2$
3. $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$; $\lim_{x \to 3} 5x = 5 \lim_{x \to 3} x = 15$
4. $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$;
5. $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$;
6. $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$; $\lim_{x \to 3} f(x) + g(x)$ $\lim f(x) + \lim g(x)$
7. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, provided $\lim_{x \to c} g(x) \neq 0$;
8. $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$; $\lim_{x \to 2} (2x+4)^5 = \left(\lim_{x \to 2} 2x+4\right)^5$
9. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$, provided $\lim_{x \to c} f(x) > 0$ when n is even.

EXAMPLE 1 Find $\lim_{x \to 3} 2x^4$.

$$2 \lim_{x \to 3} x^4 = 2(3)^4 = 162$$

EXAMPLE 2 Find $\lim_{x\to 4}(3x^2 - 2x)$.

$$\lim_{x\to 4} 3x^2 - \lim_{x\to 4} 2x$$

$$3(4)^2 - 2(4) = 48 - 8 = 40$$

EXAMPLE 3 Find $\lim_{x\to 4}\frac{\sqrt{x^2+9}}{x}$.

$$\frac{\lim_{x\to 4}\sqrt{x^2+9}}{\lim_{x\to 4} x} = \frac{\sqrt{\lim_{x\to 4}(x^2+9)}}{\lim_{x\to 4} x}$$

$$\frac{\sqrt{16+9}}{4} = \frac{5}{4}$$

EXAMPLE 4 If $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = 8$, find

$$\lim_{x\to 3}\left[f^2(x)\cdot\sqrt[3]{g(x)}\right]$$

$$\lim_{x\to 3} f^2(x) \cdot \lim_{x\to 3}\sqrt[3]{g(x)}$$

$$\left(\lim_{x\to 3} f(x)\right)^2 \cdot \sqrt[3]{\lim_{x\to 3} g(x)}$$

$$(4)^2 \cdot \sqrt[3]{8}$$

$$16 \cdot 2 = 32$$

EXAMPLE 5 Find $\lim\limits_{x\to 2} \dfrac{7x^5 - 10x^4 - 13x + 6}{3x^2 - 6x - 8}$.

$$\frac{7(2)^5 - 10(2)^4 - 13(2) + 6}{3(2)^2 - 6(2) - 8}$$

$$\begin{array}{r} 7 \\ 32 \\ \hline 224 \end{array}$$

$$\frac{7(32) - 10(16) - 13(2) + 6}{12 - 12 - 8} = \frac{44}{-8} = \frac{-11}{2}$$

$$\begin{array}{r} 224 \\ 180 \\ \hline 44 \end{array}$$

$(x-1)^2 = x^2 - 2x + 1$

$(A \pm B)^2 = A^2 \pm 2AB + B^2$

$A^2 - B^2 = (A+B)(A-B)$

EXAMPLE 6 Find $\lim\limits_{x\to 1} \dfrac{x^3 + 3x + 7}{x^2 - 2x + 1} = \lim\limits_{x\to 1} \dfrac{x^3 + 3x + 7}{(x-1)^2}$.

$$\frac{(1)^3 + 3(1) + 7}{1^2 - 2(1) + 1} = \frac{1 + 3 + 7}{1 - 2 + 1} = \frac{11}{0}$$

النهاية غير موجودة

EXAMPLE 7 Find $\lim\limits_{x\to 1} \dfrac{x-1}{\sqrt{x} - 1}$. $= \dfrac{1-1}{\sqrt{1} - 1} = \dfrac{0}{0}$ ✓

يجب مساواه التبسيط

نضرب البسط والمقام بمرافق المقام

$$\lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{x-1}} = 2$$

EXAMPLE 8 Find $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 + x - 6}$.

$$= \frac{4+6-10}{4+2-6} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{(x \cancel{-2})(x+5)}{(\cancel{x-2})(x+3)} = \frac{7}{5}$$

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$

Squeeze therarem

$$f < g < h$$

$$\lim_{x \to c} = L \qquad \lim_{x \to c} h(x) = L$$

EXAMPLE 9 Assume that we have proved $1 - x^2/6 \le (\sin x)/x \le 1$ for all x near but different from 0. What can we conclude about $\lim_{x \to 0} \frac{\sin x}{x}$?

$$1 - \frac{x^2}{6} \le \frac{\sin x}{x} \le 1$$

$$\lim_{x \to 0} 1 - \frac{x^2}{6} \qquad \lim_{x \to 0} 1 =$$

$$= 1 \qquad = 1$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Problem Set 1.3

In Problems 1–12, use Theorem A to find each of the limits. Justify each step by appealing to a numbered statement, as in Examples 1–4.

1. $\lim_{x\to 1}(2x+1)$

2. $\lim_{x\to -1}(3x^2-1)$

$$\lim_{x\to 1} 2x + \lim_{x\to 1} 1 = 2(1)+1 = 3$$

5. $\lim_{x\to 2}\dfrac{2x+1}{5-3x}$

6. $\lim_{x\to -3}\dfrac{4x^3+1}{7-2x^2}$

$$\frac{\lim_{x\to 2} 2x+1}{\lim_{x\to 2} 5-3x} = \frac{2(2)+1}{5-3(2)} = \frac{5}{-1} = -5$$

9. $\lim_{t\to -2}(2t^3+15)^{13}$

10. $\lim_{w\to -2}\sqrt{-3w^3+7w^2}$

$$\left(\lim_{x\to -2} 2t^3+15\right)^{13} = \left[2(-2)^3+15\right]^{13} = \left[-16+15\right]^{13}$$

$$[-1]^{13} = -1$$

In Problems 13–24, find the indicated limit or state that it does not exist. In many cases, you will want to do some algebra before trying to evaluate the limit.

13. $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 4}$ **14.** $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2}$ $\frac{0}{0}$ رقم / 0

15. $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1}$ **16.** $\lim_{x \to -1} \frac{x^2 + x}{x^2 + 1}$

13) $\lim_{x \to 2} \frac{2^2 - 4}{2^2 + 4} = \frac{0}{8} = 0$

15) $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} = \frac{(-1)^2 - 2(-1) - 3}{-1 + 1} = \frac{0}{0}$

$\lim_{x \to -1} \frac{(x - 3)\cancel{(x + 1)}}{\cancel{(x + 1)}} = (-1 - 3) = -4$

-pvq

19. $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$ **20.** $\lim_{x \to -3} \frac{x^2 - 14x - 51}{x^2 - 4x - 21}$

$\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1} = \frac{1^2 + 1 - 2}{1^2 - 1} = \frac{0}{0}$

$\lim_{x \to 1} \frac{(x + 2)\cancel{(x - 1)}}{(x + 1)\cancel{(x - 1)}} = \frac{3}{2}$

In Problems 25–30, find the limits if $\lim_{x\to a} f(x) = 3$ *and* $\lim_{x\to a} g(x) = -1$ *(see Example 4).*

25. $\lim_{x\to a} \sqrt{f^2(x) + g^2(x)}$ **26.** $\lim_{x\to a} \frac{2f(x) - 3g(x)}{f(x) + g(x)}$

27. $\lim_{x\to a} \sqrt[3]{g(x)}\,[f(x) + 3]$ **28.** $\lim_{x\to a} [f(x) - 3]^4$

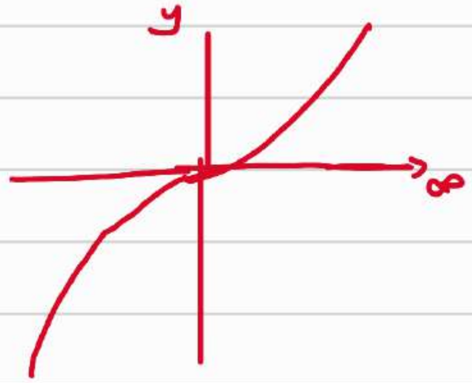
$$\sqrt[3]{\lim_{x\to a} g(x)} \cdot \left[\lim_{x\to a} f(x) + \lim_{x\to a} 3\right]$$

$$-1 \cdot [3 + 3] = -6$$

1.5 Limits at Infinity; Infinite Limits

$\lim\limits_{x \to \infty}$

$$\lim_{x \to \infty} x = \infty$$



$$\lim_{x \to -\infty} x = -\infty$$

$x \to \infty$ $\qquad \dfrac{\text{بسط}}{\text{مقام}}$

$\lim\limits_{x \to \infty} \dfrac{\text{بسط}}{\text{مقام}}$

- درجة البسط اكبر من المقام اذا النهاية $= \infty$
- درجة البسط اقل من درجة المقام اذا النهاية صفر
- اذا تساوت درجة البسط مع درجة المقام تكون النهاية $= \dfrac{\text{معامل البسط}}{\text{معامل المقام}}$

$$\lim_{x \to \infty} \frac{x^5+2x}{x^3+1} = \infty$$

$$\lim_{x \to \infty} \frac{x^3+1}{x^5+2x} = 0$$

$$\lim_{x \to \infty} \frac{2x^5+4}{3x^5-2} = \frac{2}{3}$$

$\frac{1}{\infty} = 0$

$\frac{1}{x} \quad \frac{1}{x^2} \quad \frac{1}{x^3}$

EXAMPLE 1 Show that if k is a positive integer, then

$$\lim_{x\to\infty} \frac{1}{x^k} = 0 \quad \text{and} \quad \lim_{x\to-\infty} \frac{1}{x^k} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^k} = \frac{1}{x^4} = \frac{1}{\infty^4} = 0$$

EXAMPLE 2 Prove that $\lim_{x\to\infty} \frac{x}{1+x^2} = 0.$

لحساب النهاية عندما $x \to \infty$ نقسم جميع الحدود
على اكبر حد

$$\lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{1}{x^2} + \frac{x^2}{x^2}} = \frac{\frac{1}{x}}{\frac{1}{x^2} + 1}$$

$$= \frac{\overbrace{\cancel{\tfrac{1}{\infty}}}^{\to \text{zer}}}{\underbrace{\cancel{\tfrac{1}{\infty^2}}}_{\text{zo}} + 1} = \frac{0}{0+1} = 0$$

EXAMPLE 3 Find $\lim_{x\to-\infty} \frac{2x^3}{1+x^3}.$

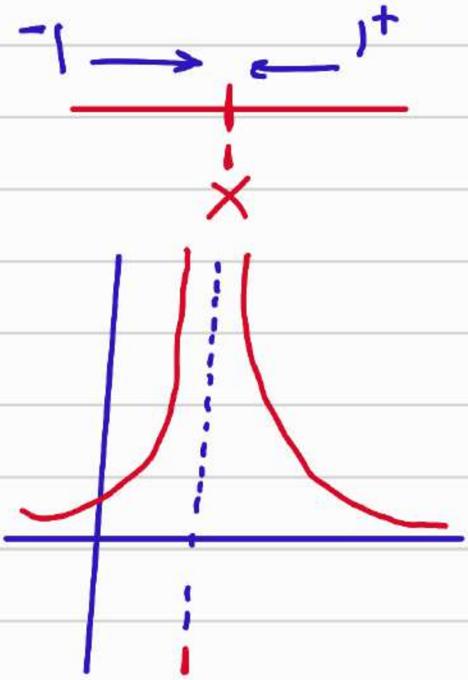
نقسم الكل
على x^3

$$\lim_{x\to-\infty} \frac{\frac{2\cancel{x^3}}{\cancel{x^3}}}{\frac{1}{x^3} + \frac{x^3}{x^3}} = \lim_{x\to-\infty} \frac{2}{\frac{1}{x^3} + 1}$$

$$= \frac{2}{\frac{1}{-\infty} + 1} = 2$$

EXAMPLE 5 Find $\lim_{x\to 1^-} \frac{1}{(x-1)^2}$ and $\lim_{x\to 1^+} \frac{1}{(x-1)^2}$.

$$\lim_{x\to 1} \frac{1}{(x-1)^2} = \frac{1}{0} =$$ النهاية غير موجودة



$$\lim_{x\to 1^+} \frac{1}{\underset{1.001}{(x-1)^2}} = \frac{1}{\text{صغير جداً}} = \infty$$

$$\lim_{x\to 1^-} \frac{1}{(0.999-1)^2} = \frac{1}{\text{صغير جداً}} \quad \infty$$

EXAMPLE 6 Find $\lim_{x\to 2^+} \frac{x+1}{x^2-5x+6}$. $\underset{x\to 2}{=} \frac{3}{4-10+6} = \frac{3}{0}$

النهاية غير موجودة

$-\infty$

$$\lim_{x\to 2^+} \frac{\overset{\text{من } 2}{x+1}}{\underset{\text{من } -1}{(x-3)}\underset{\text{من } 0}{(x-2)}} = -\frac{2}{0} = \infty$$

Asymptote خط التقارب

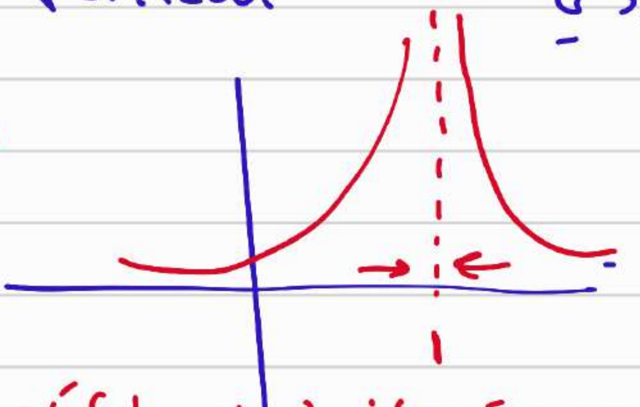
horizontal خط تقارب افقي

هو خط افقي يقترب
منه الدالة تحت
لا تلمسه

$\lim_{x \to \infty} f(x)$ نحسب خط التقارب الافقي من خلال

vertical خط التقارب العمودي

$x = 1$

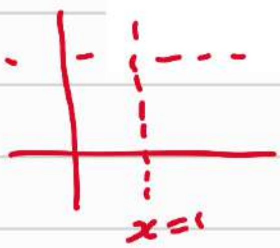


خط التقارب العمودي يقع عند اصفار المقام (c)
ويحقق القاعدة التالية

$\lim_{x \to c^+} f(x) = \infty$ $\qquad$ $\lim_{x \to c^-} f(x) = \infty$

EXAMPLE 7 Find the vertical and horizontal asymptotes of the graph of $y = f(x)$ if

$$f(x) = \frac{2x}{x-1}$$



$\boxed{x = 1}$ $\qquad$ $x - 1 = 0$ $\qquad$ العمودي

$\lim_{x \to 1^-} \frac{2x}{x-1} = \infty$ $\qquad$ $\lim_{x \to 1^+} \frac{2x}{x-1} = \infty$

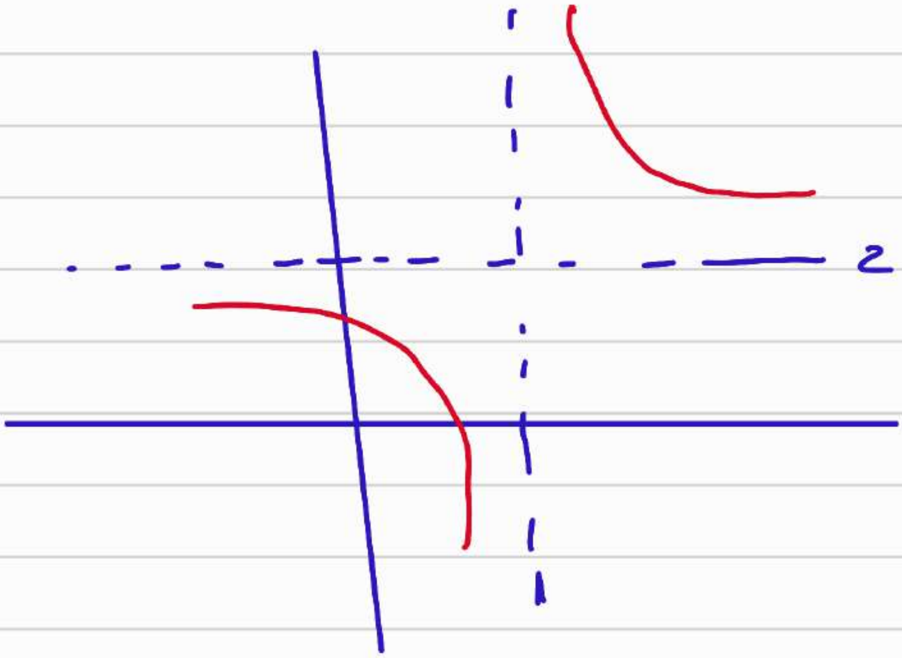
خط التقارب الافقي

$$\lim_{x \to \infty} \frac{2x}{x-1} = \lim_{x \to \infty} \frac{\frac{2x}{x}}{\frac{x}{x} - \frac{1}{x}}$$

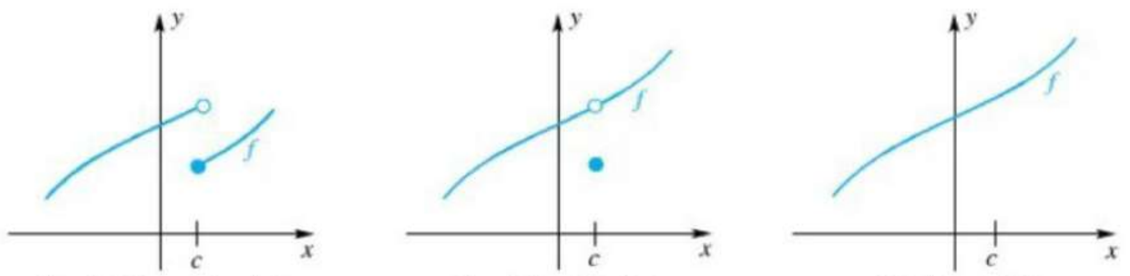
$$= \lim_{x \to \infty} \frac{2}{1 + \frac{1}{x}}$$

$$\frac{2}{1 + \cancel{\frac{1}{\infty}}} \text{ zero}$$

$$= 2$$



1.6 Continuity of Functions



$\lim_{x \to c} f(x)$ does not exist. — $\lim_{x \to c} f(x)$ exists, but $\lim_{x \to c} f(x) \neq f(c)$. — $\lim_{x \to c} f(x) = f(c)$

النهايه غير موجودة
غير متصل

النهايه موجودة
$f(c) \neq \lim_{x \to c}$
غير متصل

له نهايه
متصل

ماهي شروط الاتصال عند النقطة

① الداله معرفه عند c $\quad f(C)$ موجودة

$f(C) = \_$

② النهايه موجودة

$\lim_{x \to c} f(x) = -$

③ $\lim_{x \to c} f(x) = f(c)$

اذا تحققت الشروط

$f(x)$ is continous at $x = C$

EXAMPLE 1 Let $f(x) = \dfrac{x^2-4}{x-2}$, $x \neq 2$. How should f be defined at $x = 2$ in order to make it continuous there?

تحتاج لتعريفي 2 في الدالة

كم يجب ان نكون قيمة $f(2)$ حتى تصبح

الداله متصلة عند 2

$f(2) = 4$?? $= f(2)$ ①

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 4 \quad ②$$

③

$$f(2) = \lim_{x\to 2} f(x)$$

$$4 = 4$$

يجب ان تكون $f(2) = 4$ حتى تكون الداله متصلة

② removable discontinuity

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

عندها الداله متصلة.

---

① جميع كثيرات الحدود متصلة على جميع الاعداد

$f(x) = x^2 + 2x + 5$

② دوال الجذور الزوجيه $\sqrt{\ }$ $\sqrt[4]{\ }$ $\sqrt[6]{\ }$

متصله فقط على ما $x \geq 0$

③ دوال القيمه المطلقه $|x|$ متصله على جميع الاعداد

④ الدوال الكسريه غير متصله عند اصفار المقام $\dfrac{1}{x-1}$

⑤ داله جمع او ضرب او قسمه $f+g$ $f \cdot g$ $\dfrac{g}{f}$

EXAMPLE 2 At what numbers is $F(x) = (3|x| - x^2)/(\sqrt{x} + \sqrt[3]{x})$ continuous?

$$F(x) = \frac{3|x| - x^2}{\sqrt{x} + \sqrt[3]{x}}$$

$|x|$ and x^2 $\sqrt[3]{x}$ $\Rightarrow$ متصل على كل الاعداد

$\sqrt{x}$ $\Rightarrow$ متصل على كل الاعداد الموجبة

The function $F(x)$ continous at All positive numbers

EXAMPLE 3 Determine all points of discontinuity of $f(x) = \dfrac{\sin x}{x(1-x)}$, $x \neq 0, 1$. Classify each point of discontinuity as removable or nonremovable.

نبحث عن اصفار المقام للحصول على نقاط الانقطاع

$x(1-x) = 0$ $\boxed{x = 0}$ $1 - x = 0$ $\boxed{x = 1}$

$$\boxed{\text{الداله متصله عند جميع الاعداد ماعدا } x=0 \quad x=1}$$

$\lim\limits_{x \to 1} \dfrac{\sin x}{x(1-x)} = \dfrac{\text{رقم}}{\text{صفر}}$ = the limit is not exit

non removable discontinuity

$\lim\limits_{x \to 0} \dfrac{\sin x}{x(1-x)} = \dfrac{0}{0}$ نحاول التحليل

$\cancel{\lim\limits_{x \to 0} \dfrac{\sin x}{x}}^{1} \cdot \lim\limits_{x \to 0} \dfrac{1}{1-x} = 1$ النهاية موجودة

removable contin--

$h(x) = f(g(x)) = f \circ g(x)$

اذا كانت g متصله و f متصله اذا التركيب تكوينة متصل

EXAMPLE 4 Show that $h(x) = |x^2 - 3x + 6|$ is continuous at each real number.

$g(x) = x^2 - 3x + x$ متصل على جميع الاعداد

$f(x) = |x|$ متصله على جميع الاعداد

$h(x) = f(g(x))$ متصل على جميع الاعداد

EXAMPLE 5 Show that is continous except at 3 , -2

$$h(x) = \sin \frac{x^4 - 3x + 1}{x^2 - x - 6}$$

$g(x) = \dfrac{x^4 - 3x + 1}{x^2 - x - 6}$ $\qquad f(x) = \sin x$

متصل

$h(x) = f \circ g(x)$

Sin x متصل اذا نبحث في اتصال g(x)

g(x) داله كسريه اذا نبحث عن اصفار المقام

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

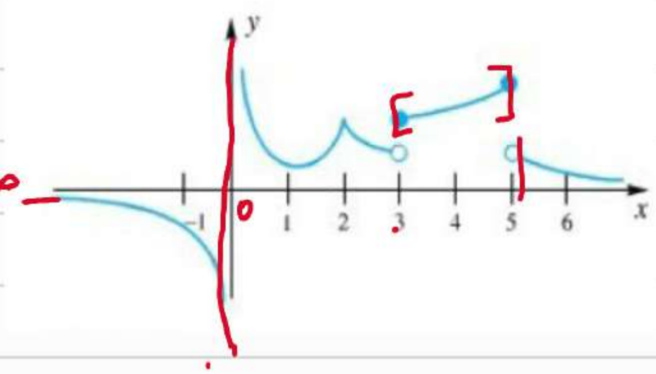
$x - 3 = 0 \qquad x = 3$

$x + 2 = 0 \qquad x = -2$

داله g متصله عند جميع الارقام ماعدا $3, -2$

EXAMPLE 6 Using the definition above, describe the continuity properties of the function whose graph is sketched in Figure 7.

فترات الاتصال



$(-\infty, 0)$

$(0, 3)$

$[3, 5]$

$(5, \infty)$

EXAMPLE 7 What is the largest interval over which the function defined by $g(x) = \sqrt{4 - x^2}$ is continuous?

نبحث عن الفترات التي تجعله فيه الجذر موجب

$4 - x^2 \geq 0$

$4 \geq x^2$ $\qquad \pm 2 \geq |x|$

فترة الحل $\qquad [-2, +2]$

$-2 \longleftrightarrow +2$

$g(x) = \sqrt{4 - x^2} \qquad =$

لتحديد الاتصال عند فترة :

1- الفترة تحسب ضمن الحل

2- النهاية موجودة عند الأطراف (الطرف الأول لندرس النهاية من اليمين والطرف الأخير لندرس النهاية من اليسار

$$\lim_{x \to a^+}$$

$$\lim_{x \to b^-}$$

$$\lim_{x \to -2^+} \sqrt{4 - x^2} = \sqrt{4 - (-2)^2} \approx 0$$

$\approx {}^{-1.9}_{\;4}$

$$\lim_{x \to 2^-} \sqrt{4 - x^2} = \sqrt{4 - 4} = 0$$

النهاية موجودة و الدالة معرفة على الفترة

الاتصال صحيح حد $(-2,2)$

Problem Set 1.6

In Problems 1–15, state whether the indicated function is continuous at 3. If it is not continuous, tell why.

1. $f(x) = (x - 3)(x - 4)$ **2.** $g(x) = x^2 - 9$

① $f(3) = (3-3)(3-4) = 0$

② $\lim_{x \to 3} (x-3)(x-4) = 0$

③ $\lim_{x \to 3} f(x) = f(3)$ Continous

9. $h(x) = \dfrac{x^2 - 9}{x - 3}$

① $f(3) = \dfrac{3^2-9}{3-0} = \dfrac{0}{0}$

f(x) is not continouse at x=3
becuse f(x) is not exist

11. $r(t) = \begin{cases} \dfrac{t^3 - 27}{t - 3} & \text{if } t \neq 3 \\ 27 & \text{if } t = 3 \end{cases}$

$(A^3 - B^3)$
$= (A - B)(A^2 + AB + B^2)$

① $f(3) = 27$

② $\lim_{x \to 3} \dfrac{t^3 - 3^3}{t - 3} = \dfrac{0}{0}$

$\lim_{x \to 3} \dfrac{\cancel{(t-3)}(t^2 + 3t + 9)}{\cancel{t-3}} = 9 + 9 + 9 = 27$

③ $f(3) = \lim_{x \to 3} f(x)$ continous

$27 = 27$

13. $f(t) = \begin{cases} t - 3 & \text{if } t \leq 3 \\ 3 - t & \text{if } t > 3 \end{cases}$

① $f(3) = 0$ ✓

② $\lim_{t \to 3^+} 3 - t = 0$

$\lim_{t \to 3^-} t - 3 = 0$

continouse

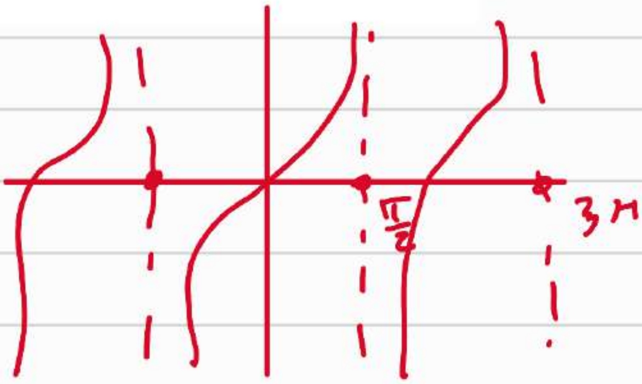
③ $\lim_{x \to 3} f(x) = f(3)$

$0 = 0$

In Problems 24–35, at what points, if any, are the functions discontinuous?

27. $r(\theta) = \tan\theta$

$\frac{\pi}{2} + 2\pi n$



31. $G(x) = \dfrac{1}{\sqrt{4 - x^2}}$

الجذر مادخل الجذر اكبر من صفر

$4 - x^2 \geq 0$ $4 \geq x^2$ $\pm 2 \geq |x|$

$\sqrt{4 - x^2} = 0$ $4 - x^2 = 0$ $\sqrt{4} = \sqrt{x^2}$

± 2 ماعدا $x = \pm 2$

معرف فقط على هذه الفترة $(-2, +2]$

$(-\infty, -2] \cup (2, \infty)$