Lecture 1
1.1 Propositional Logic- Homework

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

Question 1

a) Boston is the capital of Massachusetts. → Proposition (True)
b) Miami is the capital of Florida. → Proposition (False)
c) 2 + 3 = 5. → Proposition (True)
d) 5 + 7 = 10. → Proposition (False)
e) x + 2 = 11. → Not a proposition
f) Answer this question. → Not a proposition

5. What is the negation of each of these propositions?

Question 5

a) Mei has an MP3 player. → Negation: Mei does not have an MP3 player.
b) There is no pollution in New Jersey. → Negation: There is pollution in New Jersey.
c) 2 + 1 = 3. → Negation: 2 + 1 ≠ 3.
d) The summer in Maine is hot and sunny. → Negation: The summer in Maine is not hot or not sunny.

أ) مي لديها مشغل MP3. → النفي: مي لا تمتلك مشغل MP3.
ب) لا يوجد تلوث في نيوجيرسي. → النفي: يوجد تلوث في نيوجيرسي.
ج) 2 +1 =3. → النفي: 2 +1 ≠3.
د) الصيف في ولاية ماين حار ومشمس. → النفي: الصيف في ولاية ماين ليس حارًا أو ليس مشمسًا.

6. What is the negation of each of these propositions?

Question 6

a) Jennifer and Teja are friends. → Negation: Jennifer and Teja are not friends.
b) There are 13 items in a baker's dozen. → Negation: There are not 13 items in a baker's dozen.

أ) جينيفر وتيجا صديقتان. → النفي: جينيفر وتيجا ليستا صديقتين.
ب) هناك 13 عنصرًا في الدزينة. → النفي: لا يوجد 13 عنصرًا في الدزينة.

9. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

Question 9

a) Quixote Media had the largest annual revenue. → False
b) Nadir Software had the lowest net profit AND Acme Computer had the largest annual revenue. → True
c) Acme Computer had the largest annual revenue. → True
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue. → True
e) Nadir Software had the smallest net profit iff Acme Computer had the largest annual revenue. → True

أ (شركة Quixote Media خطأ. ← لديها أكبر إيرادات سنوية
ب (شركة Nadir Software و Acme Computer لديها أقل ربح صافٍ صحيح. ← لديها أكبر إيرادات سنوية
ج (شركة Acme Computer صحيح. ← لديها أكبر إيرادات سنوية
د (إذا كانت Quixote Media لديها أقل ربح صافٍ، فإن Acme Computer صحيح. ← لديها أكبر إيرادات سنوية
هـ) Nadir Software لديها أقل ربح صافٍ إذا وفقط إذا كانت Acme Computer صحيح. ← لديها أكبر إيرادات سنوية

11. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$
d) $p \rightarrow \neg q$ **e)** $\neg q \rightarrow p$ **f)** $\neg p \rightarrow \neg q$
g) $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

Question 11

a) ¬q → Sharks have not been spotted near the shore.
b) p ∧ q → Swimming is allowed and sharks have been spotted.
c) ¬p ∨ q → Swimming is not allowed, or sharks have been spotted.
d) p → ¬q → If swimming is allowed, then sharks have not been spotted.
e) ¬q → ¬p → If sharks have not been spotted, then swimming is not allowed.

f) ¬p → ¬q → If swimming is not allowed, then sharks have not been spotted.
g) p ↔ ¬q → Swimming is allowed iff sharks have not been spotted.
h) ¬p ∧ (p ∨ ¬q) → Swimming is not allowed, and either swimming is allowed or sharks have not been spotted.

أ) ¬q → لم يتم رصد أسماك القرش بالقرب من الشاطئ.
ب) p ∧ q → السباحة مسموحة وتم رصد أسماك القرش.
ج) ¬p ∨ q → السباحة غير مسموحة، أو تم رصد أسماك القرش.
د) p → ¬q → إذا كانت السباحة مسموحة، فإن أسماك القرش لم تُرصد.
هـ) ¬q → ¬p → إذا لم يتم رصد أسماك القرش، فإن السباحة غير مسموحة.
و) ¬p → ¬q → إذا لم تكن السباحة مسموحة، فإن أسماك القرش لم تُرصد.
ز) p ↔ ¬q → السباحة مسموحة إذا وفقط إذا لم يتم رصد أسماك القرش.
ح) ¬p ∧ (p ∨ ¬q) → (إما أنها مسموحة أو لم يتم رصد أسماك القرش، و)السباحة غير مسموحة.

17. Let p, q, and r be the propositions

p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

Question 17

a) r ∧ ¬p
b) (¬p ∧ q) ∧ r.
c) r → (q ↔ ¬p)
d) ¬q ∧ (¬p ∧ r)
e) q → (r ∧ ¬p).
f) (p ∧ r) → ¬q.

19. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$.

Question 19

a) If 1+1=2, then 2+2=5 → False
b) If 1+1=3, then 2+2=4 → True
c) If 1+1=3, then 2+2=5 → True
d) If monkeys can fly, then 1+1=3 → True

21. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

a) Coffee or tea comes with dinner.
b) A password must have at least three digits or be at least eight characters long.
c) The prerequisite for the course is a course in number theory or a course in cryptography.
d) You can pay using U.S. dollars or euros.

Question 21

a) Coffee or tea with dinner → Exclusive OR
b) Password ≥3 digits or ≥8 chars → Inclusive OR
c) Prerequisite: number theory or cryptography → Inclusive OR
d) Pay in dollars or euros → Exclusive OR

25. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements.]

a) It snows whenever the wind blows from the northeast.
b) The apple trees will bloom if it stays warm for a week.
c) That the Pistons win the championship implies that they beat the Lakers.
d) It is necessary to walk eight miles to get to the top of Long's Peak.
e) To get tenure as a professor, it is sufficient to be world famous.
f) If you drive more than 400 miles, you will need to buy gasoline.
g) Your guarantee is good only if you bought your CD player less than 90 days ago.
h) Jan will go swimming unless the water is too cold.
i) We will have a future, provided that people believe in science.

Question 25

a) If wind northeast, then snow.
b) If warm 1 week, then trees bloom.
c) If Pistons win, then they beat Lakers.
d) If reach top Long's Peak, then walked 8 miles.
e) If world famous, then professor tenure.
f) If drive >400 miles, then need gas.
g) If guarantee valid, then bought <90 days ago.
h) If water not cold, Jan swims.
i) If people believe in science, then we have a future.
j) If Carol on boat, then seasick.

أ. (إذا هبت الرياح من الشمال الشرقي، إذن يتساقط الثلج
ب. (إذا استمر الجو دافئًا أسبوعًا، إذن الأشجار تزهر
ج (إذا فاز فريق Pistons، إذن هزموا Lakers.
د (إذا وصلت لقمة Long's Peak، إذن مشيت 8 أميال.
هـ. (إذا كنت مشهورًا عالميًا، إذن تصبح أستاذًا دائمًا
و. (إذا قدت >400 ميل، إذن تحتاج وقود
ز. (إذا كان الضمان صالحًا، إذن اشتريت الجهاز قبل >90 يومًا
ح (إذا لم يكن الماء باردًا، فإن Jan تسبح.
ط. (إذا آمن الناس بالعلم، إذن سيكون لنا مستقبل
ي (إذا كانت Carol على قارب، إذن تصاب بدوار البحر.

27. Write each of these propositions in the form "p if and only if q" in English.

a) If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.
b) For you to win the contest it is necessary and sufficient that you have the only winning ticket.
c) You get promoted only if you have connections, and you have connections only if you get promoted.
d) If you watch television your mind will decay, and conversely.
e) The trains run late on exactly those days when I take it.

Question 27

a) Buy ice cream iff hot outside.
b) Win contest iff only winning ticket.

c) Get promoted iff have connections.
d) Watch TV iff mind decays.
e) Train late iff I take it.

أ. (شراء آيس كريم ↔ إذا كان الجو حارًا
ب. (الفوز بالمسابقة ↔ إذا كان لديك التذكرة الوحيدة الفائزة
ج. (الترقية ↔ إذا كان لديك علاقات
د. (مشاهدة التلفاز ↔ إذا تدهور العقل
هـ. (القطار يتأخر ↔ إذا ركبته

30. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

Question 30

a) If snow tonight → stay home.
Converse: If stay home → snow.
Contrapositive: If not stay home → no snow.
Inverse: If no snow → not stay home.

b) If sunny day → go beach.
Converse: If go beach → sunny.
Contrapositive: If not go beach → not sunny.
Inverse: If not sunny → not go beach.

c) If stay up late → sleep noon.
Converse: If sleep noon → stayed late.
Contrapositive: If not sleep noon → not stayed late.
Inverse: If not stay up late → not sleep noon.

أ. (إذا تساقط الثلج الليلة →أبقى في المنزل
العكس: إذا بقيت في المنزل →تساقط الثلج.
المقابل: إذا لم أبق في المنزل →لم يتساقط الثلج.
العكسي: إذا لم يتساقط الثلج →لن أبقى في المنزل.

ب. (إذا كان يومًا مشمسًا →أذهب للشاطئ
العكس: إذا ذهبت للشاطئ →يوم مشمس.
المقابل: إذا لم أذهب للشاطئ →ليس يومًا مشمسًا.
العكسي: إذا لم يكن يومًا مشمسًا →لن أذهب للشاطئ.

ج. (إذا سهرت لوقت متأخر → أنام حتى الظهر
العكس: إذا نمت حتى الظهر → سهرت.
المقابل: إذا لم أنم حتى الظهر → لم أسهر.
العكسي: إذا لم أسهر → لن أنام حتى الظهر.

32. How many rows appear in a truth table for each of these compound propositions?

Question 32

a) (q → ¬p) ∨ (¬p → ¬q) → 2 vars → 4 rows
b) (p ∨ ¬t) ∧ (p ∨ ¬s) → 3 vars → 8 rows
c) (p → r) ∨ (¬s → ¬t) ∨ (¬u → v) → 6 vars → 64 rows
d) (p ∧ r ∧ s) ∨ (q ∧ t) ∨ (r ∧ ¬t) → 5 vars → 32 rows

34. Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg p$
b) $p \leftrightarrow \neg p$
c) $p \oplus (p \vee q)$
d) $(p \wedge q) \rightarrow (p \vee q)$
e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

أ

34a) $p \rightarrow \neg p$

p	$\neg p$	$p \rightarrow \neg p$
F	T	T
T	F	F

34b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
F	T	F
T	F	F

34c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
F	F	F	**F**
F	T	T	**T**
T	F	T	**F**
T	T	T	**F**

34d) $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
F	F	F	F	**T**
F	T	F	T	**T**
T	F	F	T	**T**
T	T	T	T	**T**

- **Tautology** (always true).

34e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$q \rightarrow \neg p$	$p \leftrightarrow q$	Biconditional
F	F	T	T	**T**
F	T	T	F	**F**
T	F	F	F	**F**
T	T	F	T	**F**

34f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	XOR
F	F	T	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	F	T

- **Tautology** (always true).