Umm Al-Qura University

College of Computing

Department of Computer and Network Engineering

CEN0601: Discrete Models

Assignment #1

Term: Fall 2025

Rules for Submitting Homework Assignments:

. Use a text editor to compose your answer to this assignment .

. Accepted file formats are PDF and DOC.

. Homework is only accepted on the due date in the designated assignment on Blackboard (No late submissions).

. In your submitted file, you have to answer the mandatory four questions (2, 4, 8 and 9) labeled **(NOT OPTIONAL)** and a fifth question of your choice.

Question 1. **Classify as propositions and give truth values if well-defined.** For each sentence, say whether it is a proposition (P) or not (NP). If it *is* a proposition, give its truth value (**T/F**).

(a) "2 + 3 = 5." P T

(b) "Close the door." NP

(c) "The capital of Egypt is Miami." P F

(d) "$x^2 > 4$." NP

(e) "5 + 7 = 12?" NP

(f) "There is life on Mars." P F

(NOT OPTIONAL) Question 2. Suppose that the phones have the following specifications:

	RAM (MB)	ROM (GB)	Camera (MP)
Smartphone A	256	32	8
Smartphone B	288	64	4
Smartphone C	128	32	5

Determine the truth value of each proposition.

T (a) Smartphone B has the most RAM of these three smartphones. T

T (b) Smartphone C has more ROM or a higher resolution camera than Smartphone B. $F \vee T = T$

F (c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A. $T \wedge T \wedge F = F$

F (d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera. $(T \wedge T) \rightarrow F = F$

F (e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

$F \leftrightarrow T = F$

Question 3. Let the atomic propositions be

p: Swimming at the **Jeddah** shore is allowed.

q: Sharks have been spotted near the shore.

Express each compound proposition as an English sentence.

Helpful key for symbols:

$$\neg P = not\ P, \qquad P \land Q = P\ and\ Q, \qquad P \lor Q = P\ or\ Q\ (possibly\ both),$$

$$P \rightarrow Q = if\ P\ then\ Q, \quad P \leftrightarrow Q = P\ iff\ Q.$$

(a) $\neg q$
(b) $p \land q$
(c) $\neg p \lor q$
(d) $p \rightarrow \neg q$
(e) $\neg q \rightarrow p$
(f) $\neg p \rightarrow \neg q$
(g) $p \leftrightarrow \neg q$
(h) $\neg p \land (p \lor \neg q)$

(a) ¬q: "Sharks have **not** been spotted near the shore."
(b) p ∧ q: "Swimming is allowed **and** sharks have been spotted."
(c) ¬p ∨ q: "Either swimming isn't allowed, **or** sharks have been spotted."
(d) p → ¬q: "If swimming is allowed, then sharks have **not** been spotted."
(e) ¬q → p: "If sharks have **not** been spotted, then swimming is allowed."
(f) ¬p → ¬q: "If swimming isn't allowed, then sharks haven't been spotted."
(g) p ↔ ¬q: "Swimming is allowed **iff** sharks have not been spotted."
(h) ¬p ∧ (p ∨ ¬q) ≡ **¬p ∧ ¬q**: "Swimming isn't allowed **and** sharks haven't been spotted.

<u>(NOT OPTIONAL)</u> Question 4. **Symbolization with connectives.** Let p: "It is raining." q: "The ground is wet." r: "The sprinkler is on." Translate the English statements into propositional logic.

(a) "If it is raining, then the ground is wet." $p \rightarrow q$
(b) "The ground is wet if and only if it is raining or the sprinkler is on." $q \leftrightarrow (p \lor r)$
(c) "It is not raining, but the ground is wet and the sprinkler is not on." $\sim p \land q \land \sim r$
(d) "Either it is raining and the sprinkler is on, or the ground is not wet."

$(p \land r) \lor q$

Question 5. **Negations in good English** (avoid "It is not the case that"). Let p: "Alice submitted the homework." q: "Bob attended class."

(a) $\neg p$
(b) $\neg(p \vee q)$ $= \neg p \wedge \neg q$
(c) $\neg(p \wedge \neg q)$ $\neg p \vee q$
(d) $\neg(p \rightarrow q)$
$\hookrightarrow \neg(\neg p \vee q) = p \wedge \neg q$

(A) ¬p
"Alice did not submit the homework."

(B) ¬(p ∨ q)
"Alice did not submit the homework and Bob has not attended class."

(C) ¬(p ∧ ¬q)
"Alice did not submit the homework or Bob has attended class."

(D) ¬(p → q)
"Alice submitted the homework and Bob has not attended class."

Question 6. **Evaluate with given truth assignments.** Let *p* = T, *q* = F, *r* = T. Compute each value.

F (a) $\neg p \vee q$ $\quad F \vee F = F$
T (b) $(p \wedge q) \vee r$ $\quad (T \wedge F) \vee T = F \vee T = T$
T (c) $p \rightarrow (q \vee r)$ $\quad T \rightarrow (F \vee T) = T \rightarrow T = T$
T (d) $(p \oplus q) \leftrightarrow r$ $\quad (T \oplus F) \leftrightarrow T \Rightarrow T \leftrightarrow T = T$

Question 7. **Add missing parentheses using standard precedence.** Standard precedence: ¬ highest, then ∧, then ∨, then →, then ↔. Insert parentheses so the following is unambiguous:

$$(((\neg p) \vee (q \wedge r)) \rightarrow s) \leftrightarrow t.$$

<u>(NOT OPTIONAL)</u> Question 8. Let *S(x)* be the statement "Sensor *x* is active" and *R(x, t)* be the statement "The reading of sensor *x* is accurate at time *t*." The domain for *x* consists of all onboard sensors, and the domain for *t* consists of all recent time steps.

(a) Express the following statement using predicates and quantifiers: "There is a sensor that is active, but its reading is not accurate at all recent time steps." $\exists_x [S(x) \wedge \forall_t \neg R(x,t)]$

(b) Form the negation of the quantified statement you wrote in part (a) so that no negation appears to the left of any quantifier.

$$\exists_x [S(x) \wedge \neg \exists_t R(x,t)$$

$$\neg \forall_x [\neg S(x) \vee \exists_t R(x,t)]$$

$$\forall x [S(x) \rightarrow \exists_t R(x,t)]$$

(NOT OPTIONAL) Question 9. **Valid Arguments using Rules of Inference.** Show that the following premises imply the conclusion, detailing the rule of inference used at each step (e.g., Modus Ponens, Universal Instantiation, Hypothetical Syllogism, … etc.).

Premise 1: Every algorithm that uses optimization techniques is efficient.
Premise 2: If an algorithm is efficient, then it has a low runtime complexity.
Premise 3: The new pathfinding algorithm uses optimization techniques.
Conclusion: The new pathfinding algorithm has a low runtime complexity.

Question 10. **Direct Proof.** Show that if N is an odd integer, then $N^2 + N$ is an even integer.

if N is odd $\longrightarrow$ N^2+N even

assume N is odd $N = 2k+1$

$$\rightarrow N^2+N = (2k+1)^2 + 2k+1$$

$$= 4k^2 + 4k + 1 + 2k + 1$$

$$= 4k^2 + 6k + 2$$

$$= 2(\underbrace{2k^2 + 3k + 1}_{r})$$

$$= 2r \quad \text{even}$$

N^2+N even

Question 11: **Proof by Contraposition.** Prove the following statement using proof by contraposition: If *T* is an integer such that *3T + 5* is an even integer, then *T* is an odd integer.

$p \longrightarrow q$

$3T+5$ even $\longrightarrow T$ odd

$-q \longrightarrow p$

T even $\longrightarrow 3T+5$ odd

assume T even

$T = 2k$

$\longrightarrow 3(2k)+5 = 6k+5$

$(6k+4)+1$

$2(3k+2)+1$

$2r+1$

odd

$3T+5$ is odd

$-q \longrightarrow -p$ true then $p \rightarrow q$

$$p \leftrightarrow q \begin{cases} p \to q \\ q \to p \end{cases}$$

Question 12: **Proof of Equivalence (Biconditional Proof).** Show that the following two statements about an integer *x* are equivalent: *p*: *x* is an odd integer, *q*: $x^2 + 1$ is an even integer.

x is odd $\Longleftrightarrow$ x^2+1 even

Case 1 $\quad p \to q$ ✓

x is odd $\longrightarrow$ x^2+1 is even

$$x = 2k+1$$

$$x^2+1 = (2k+1)^2+1 = 4k^2+4k+1+1$$

$$= 4k^2+4k+2 = 2(2k^2+2k+1)$$

x^2+1 is even

Case 2 $\quad q \to p$ ✓

x^2+1 even $\longrightarrow$ x is odd

use Proof by contraposition

x is even $\longrightarrow$ x^2+1 odd

$$x = 2k \qquad x^2+1 = (2k)^2+1 = 4k^2+1$$

x^2+1 is odd