3.8
Antiderivatives

معكوس المشتقة

$$f(x) = 3x^2 \xrightarrow{\text{مشتقه}} f'(x) = 6x$$

$$f(x) = 3x^2 \xleftarrow{\text{معكوس المشتقة}} f'(x) = 6x$$

$$D_x F(x) = f(x)$$

$$F'(x) = f(x)$$

antiderivative of $f(x) = F(x)$

antiderivative $= \int 4x^3\, dx = x^4 + C$

$$\boxed{\int x^n\, dx = \frac{x^{n+1}}{n+1}}$$

$$\int x^3\, dx = \frac{1}{4} x^4 + C$$

$$\int 2x^5\, dx = \frac{2}{6} x^6 + C = \frac{1}{3} x^6 + C$$

EXAMPLE 1 Find an antiderivative of the function $f(x) = 4x^3$ on $(-\infty, \infty)$.

$\int$

$$\int 4x^3\,dx = \frac{4}{4}x^4 + C = x^4 + C$$

EXAMPLE 2 Find the general antiderivative of $f(x) = x^2$ on $(-\infty, \infty)$.

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

EXAMPLE 3 Find the general antiderivative of $f(x) = x^{4/3}$.

$$\int x^{4/3}\,dx = \frac{3\,x^{7/3}}{7} + C = \frac{3}{7}x^{7/3} + C$$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	$\int \sin x\,dx = -\cos x + C$
$\int dx = x + C$	$\int \cos x\,dx = \sin x + C$
$\int k\,f(x)\,dx = k\int f(x)\,dx$	$\int f(x) \mp g(x) = \int f \mp \int g$

EXAMPLE 4 Using the linearity of $\int$, evaluate

(a) $\int (3x^2 + 4x)\, dx$ (b) $\int (u^{3/2} - 3u + 14)\, du$ (c) $\int \left(1/t^2 + \sqrt{t}\right) dt$

a) $\int (3x^2 + 4x)\, dx = 3\int x^2\, dx + 4\int x\, dx$

$$3\frac{x^3}{3} + 4\frac{x^2}{2} + c$$

$$= x^3 + 2x^2 + c$$

b) $\int (u^{3/2} - 3u + 14)\, du = \int u^{3/2} du - 3\int u\, du + 14\int du$

$$\frac{2u^{\frac{5}{2}}}{5} - 3\frac{u^2}{2} + 14u + c$$

$$\frac{2}{5}u^{5/2} - \frac{3}{2}u^2 + 14u + c$$

c) $\int (t^{-2} + t^{1/2})\, dt = \int t^{-2} dt + \int t^{1/2} dt$

$$\frac{t^{-1}}{-1} + \frac{2t^{3/2}}{3} + c = \frac{-1}{t} + \frac{2}{3}t^{3/2} + c$$

$$\boxed{\int (f(x))^{r} \cdot (f'(x))\, dx = \frac{(f(x))^{r+1}}{r+1} + C}$$

$$\int (\underline{3x^4+2x})^{10} (12x^3+2)\, dx$$

$$(f(x))^{r} \quad (f'(x)) \quad = \frac{(3x^4+2x)^{11}}{11} + C$$

$$\int -4 (\underbrace{\cos x}_{\sin x})^3 \underline{\sin x}\, dx = -\cancel{4} \frac{(\cos x)^4}{\cancel{4}} + C$$

لحل التكامل نفرض ما داخل القوس $= u$ ونعوض بكل المعادله ب u

$$\int (3x^4+2x)^{10} (12x^3+2)\, dx$$

$$u = 3x^4+2x$$

$$\frac{du}{dx} = 12x^3+2$$

$$dx = \frac{du}{12x^3+2}$$

$$\int u^{10} \cancel{(12x^3+2)} \frac{du}{\cancel{12x^3+2}}$$

$$\int u^{10}\, du = \frac{u^{11}}{11} + C$$

$$= \frac{(3x^4+2x)^{11}}{11} + C$$

EXAMPLE 5 Evaluate

(a) $\int (x^4 + 3x)^{30}(4x^3 + 3)\, dx$ (b) $\int \sin^{10} x \cos x\, dx$

a) $\int (x^4+3x)^{30}(4x^3+3)\, dx$

$u = x^4+3x$

$\frac{du}{dx} = 4x^3+3$

$dx = \frac{du}{4x^3+3}$

$\int u^{30}\cancel{(4x^3+3)}\frac{du}{\cancel{4x^3+3}}$

$\int u^{30}\, du = \frac{u^{31}}{31} + C$

$= \frac{(x^4+3x)^{31}}{31} + C$

b) $\int \sin^{10} x \cos x\, dx = \frac{\sin^{11} x}{11} + C$

EXAMPLE 6 Evaluate

(a) $\int (x^3 + 6x)^5(6x^2 + 12)\, dx$ (b) $\int (x^2 + 4)^{10} x\, dx$

a) $\int (x^3+6x)^5(6x^2+12)\, dx$

$u = x^3+6x$

$\frac{du}{dx} = 3x^2+6$

$dx = \frac{du}{3x^2+6}$

$\int u^5 \overset{2}{\cancel{(6x^2+12)}}\frac{du}{\cancel{3x^2+6}}$

$2\int u^5\, du = 2\frac{u^6}{6} + C = \frac{1}{3}(x^3+6x)^6 + C$

b) $\int (x^2+4)^{10}\, x \, dx$

$$u = x^2+4$$
$$\frac{du}{dx} = 2x$$
$$dx = \frac{du}{2x}$$

$$\int u^{10}\, \cancel{x}\, \frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int u^{10}\, du = \frac{1}{2}\,\frac{u^{11}}{11} + C$$

$$= \frac{1}{22}(x^2+4)^{11} + C$$

Problem Set 3.8

Find the general antiderivative $F(x) + C$ for each of the following.

1. $f(x) = 5$
2. $f(x) = x - 4$
3. $f(x) = x^2 + \pi$
4. $f(x) = 3x^2 + \sqrt{3}$
5. $f(x) = x^{5/4}$
6. $f(x) = 3x^{2/3}$
7. $f(x) = 1/\sqrt[3]{x^2}$
8. $f(x) = 7x^{-3/4}$
9. $f(x) = x^2 - x$
10. $f(x) = 3x^2 - \pi x$
11. $f(x) = 4x^5 - x^3$
12. $f(x) = x^{100} + x^{99}$

2) $\int (x-4)\,dx = \frac{x^2}{2} - 4x + c$

5) $\int x^{5/4}\,dx = \frac{4}{9}x^{\frac{9}{4}} + c$

9) $\int (x^2 - x)\,dx = \frac{x^3}{3} - \frac{x^2}{2} + c$

11) $\int (4x^5 - x^3)\,dx = \frac{4x^6}{6} - \frac{x^4}{4} + c$

$= \frac{2}{3}x^6 - \frac{1}{4}x^4 + c$

15. $f(x) = \frac{3}{x^2} - \frac{2}{x^3}$

$\int (3x^{-2} - 2x^{-3})\,dx = \frac{3x^{-1}}{-1} - \frac{2x^{-2}}{-2} + c$

$= \frac{-3}{x} + \frac{1}{x^2} + c$

29. $\int (5x^2 + 1)(\underline{5x^3 + 3x - 8})^6 \, dx$

$$\int (\cancel{5x^2+1})(u)^6 \frac{du}{\cancel{15x^2+3}_{3}}$$

$u = 5x^3 + 3x - 8$

$\frac{du}{dx} = 15x^2 + 3$

$dx = \frac{du}{15x^2+3}$

$$\frac{1}{3}\int u^6 \, du = \frac{1}{3}\frac{u^7}{7} + c$$

$$= \frac{u^7}{21} + c = \frac{(5x^3+3x-8)^7}{21} + c$$

33. $\int x^2 \sqrt{x^3 + 4} \, dx$

$u = x^3 + 4$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$\int \cancel{x^2} (u)^{1/2} \frac{du}{3\cancel{x^2}}$$

$$\frac{1}{3}\int u^{1/2} \, du$$

$$\frac{2}{3} \frac{u^{3/2}}{3} + c = \frac{2}{9} u^{3/2} + C$$

$$= \frac{2}{9}(x^3+4)^{3/2} + C$$

25. $\int (\sin\theta - \cos\theta) \, d\theta = -\cos\theta - \sin\theta + C$